- ❖ Risk and the investor
  - Expected return
  - Variance
- ❖ Types of investors
  - Risk averse
  - Risk lover
  - Risk neutral
- ❖ Manage risk
  - Risk pooling
  - Risk spreading

❖ *Risk and the investor*

- In many activities such as gambling, deciding whether to purchase car insurance, health insurance, house insurance or even holding or losing a job, the outcome is not certain and involves some risk.
- In this chapter, we introduce:
  - Simple methods about how to evaluate the outcome of risky events.
  - How people would react differently to risky outcomes.
  - How could we manage or decrease the risk.

○ *Expected return*

- Assume you bet 5$ on the toss of a coin such that the chance of heads or tails is 50-50.
- The outcome is uncertain.
- Calculate the expected outcome or *expected value (EV)* from gambling. The expected value determines how much on average you would earn/lose if you gamble.
- Fair gamble: On average yields a zero profit. *(EV = 0)*

$$EV = Probability_{Heads} * (outcome)_{Heads} + Probability_{Tails} * (outcome)_{Tails}$$

$$EV = 0.5 * (+ \$5) + 0.5 * (- \$5) = \$ 0$$

- Another example: Suppose there is a 50% chance to win $10 and a 50% chance to lose $5. What is the expected value from this gamble?

  EV = 0.5*(+ $10) + 0.5*(- $5) = + $2.5
  The average earning from this gamble is $2.5. This is not a fair gamble. To make it a fair gamble, participating in this gamble has a fee of $2.5.

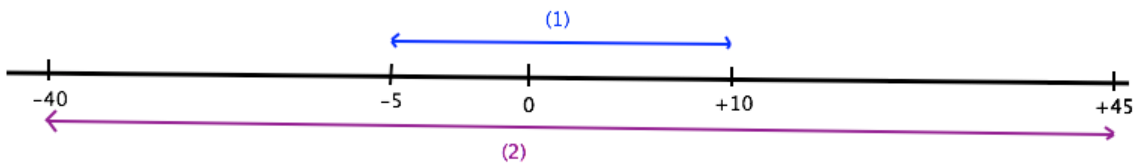
- *Variance:*

- It measures the dispersion of outcomes. Bigger dispersion (bigger variance) shows higher risk.

- Assume the following two gambles:

1. 50% → + $10
   50% → - $5
   EV = +$2.5

2. 50% → +$45
   50% → -$40
   EV = 0.5 ($45) + 0.5*(-40) = +$2.5



- The outcomes in gamble 2 are more dispersed than gamble 1. Gamble 2 is riskier and has a bigger variance.
- There are three types of behaviour in response to risk.

❖ *Types of investors*

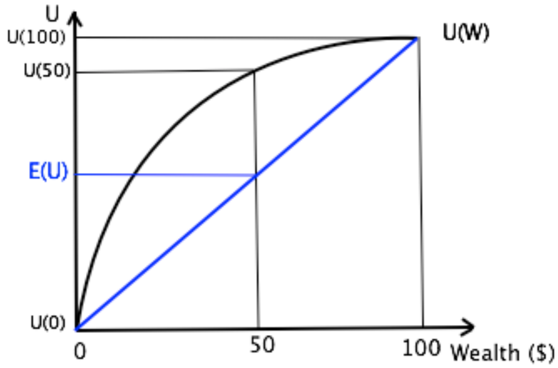
You are offered either $50 with full certainty or a gamble that pays $100 with 50% chance and zero with 50% chance. What would you choose?

- ***Risk averse* people**: prefer to avoid risk, but may choose to bet or gamble if the odds are sufficiently in their favour despite their dislike of risk.

  - Most investors are risk averse. They have a diminishing marginal utility for money.

  - This person will avoid a fair gamble and *may* take a gamble with EV > 0.

Diminishing marginal utility of each additional $ gambled



Each additional utility, is worth less and less

Utiltity for a risk averse person

- The expected return from this gamble is:

  EV = 0.5 * 0 + 0.5 * 100 = 50

- Expected utility from gambling is:

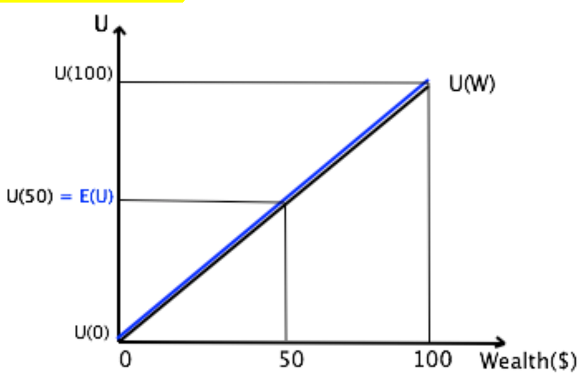
  E(U) = 0.5 * U(100) + 0.5 * U(0)

- Refer to the graph. The risk averse person prefers $50 with certainty since:

  U(50) > E(U)

This person may prefer to gamble if he is offered much less than $50.

- ***Risk neutral* people**: Only interested in taking a risk if the odds yield an average profit and ignore the dispersion in possible outcomes.
  - These investors value each additional dollar, in terms of utility, at a constant rate.



Same utility whether he earns 50

Utiltity for a risk neutral person

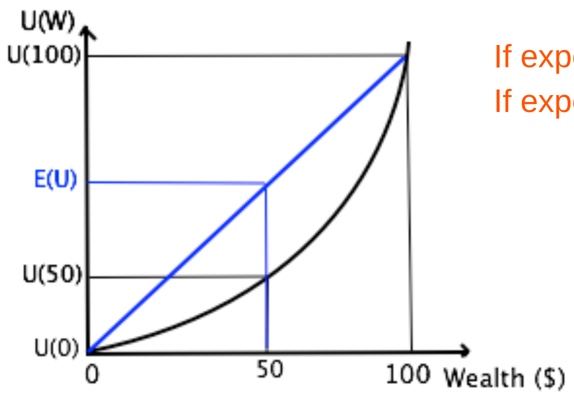
- Refer to the graph. The risk neutral person is indifferent between taking the $50 or gamble. Since

  U(50) = E(U)

where the expected utility from gambling is E(U) = 0.5 * U(100) + 0.5 * U(0)

→ This person prefers gambling to anything less than $50 with certainty, since U($<50) < E(U).

- ***Risk lovers***: willing to take additional risk even if the investment has relatively low or no expected return.
  - These investors value each additional dollar, in terms of utility, at an increasing rate.



Utility for a risk lover person

If expected value is 0 or positive = risk lover would take risk
If expected value is negative = risk lover uncertain

- Refer to the graph. The risk-lover prefers gambling rather than accepting $50 with no risk, since,

U(50) < E(U)

where E(U) is the expected utility from gambling.

This person may choose to not gamble if offered much more than $50.

- ❖ *Manage risk*

Here, we want to introduce two ways that individuals and companies decrease risk.

- ○ *Risk spreading*

  - An insurer reduces its risk by spreading it to other insurance companies.
  - For example, insuring an oil and gas company or a nuclear facility by *one* insurance company is very risky. This insurer may choose to reduce its risk exposure by spreading it to other insurers. Therefore, more than one insurer would insure very risky projects.

- ○ *Risk pooling*

  - It aggregates independent risks to make the aggregate less uncertain.
  - Example: car insurance companies, house insurance companies, credit cards, portfolio diversification.
    - The insurer brings together individuals to reduce the risk.
      - The insurer can divide the cost of an accident over a large number of people so the premium payment for each individual decreases.

  - Here, we assume that the risk incurred by each individual is independent of the risk incurred by others.
      - This assumption is not always fully satisfied. For example, the economic situation may impact investment returns in financial markets. Also, there could be correlation between returns on different stocks. This is called systemic risk.

  - Example: Suppose Marc and Zoë are looking for jobs independently. Each gets an interview for a job that pays $5000. Each has a 50% chance of getting the job and a 50% chance of not getting the job (and ending up with $0 income).

- Suppose they are single: Each has a wide income variation: from $0 to $5000.
- Now, suppose they are married:

		Zoe	
		$5000 (50%)	$0 (50%)
Marc	$5000 (50%)	EV = (0.5*5000)+(0.5*5000)= **5000**	EV = (0.5*5000)+(0.5*0) = **2500**
	$0 (50%)	EV = (0.5*0)+(0.5*5000)= **2500**	EV = (0.5*0)+(0.5*0)= **0**

- By pooling their incomes (ie, by being married), each reduces the variation in monthly income.
    - There is 25% chance of them ending up with \$0 or \$5000.
    - There is 50% for them to live with \$2500.

- Another example: *Portfolio diversification*

- A portfolio is a combination of assets
- Diversification reduces the total risk of a portfolio by *pooling risks* across several assets.
    - Do not put all your eggs in one basket.
- The *degree of risk* can be measured by the *variance*.
    - Variance is a measure of how dispersed the outcomes are.
    - It is a weighted sum of the squared deviations from the mean.

$$Variance = P_1 * (return_1 - EV)^2 + P_2 * (return_2 - EV)^2 + P_3 * (return_3 - EV)^2 + \cdots + P_N * (return_N - EV)^2 = \sum_{i=1}^{N} P_i * (return_i - EV)^2$$

where P_i is the probability of each outcome and *EV* is the expected value of the investment.

- Example: an investor wants to invest \$200 and has three options (refer to the table below)
    - The returns on these stocks are independent of each other.
    - Calculate the expected return (EV) for each strategy.
    - Determine which strategy has the lowest risk.

Investment strategies		Possible outcomes				Expected value of investment (EV)	Variance
Oil	200	220 (50%)		200 (50%)		(0.5*220)+(0.5*200)=210	100
Bank	200	220 (50%)		200 (50%)		(0.5*220)+(0.5*200)=210	100
Oil Bank	100 100	110 110 (25%)	100 100 (25%)	110 100 (25%)	100 110 (25%)	(0.25*220)+(0.25*200)+(0.5*210)=210	50

$$Variance_{Oil} = Variance_{Bank} = 0.5 * (220 - 210)^2 + 0.5 * (200 - 210)^2 = 100$$

$$Variance_{Oil-Bank} = 0.25 * (220 - 210)^2 + 0.25 * (200 - 210)^2 + 0.5 * (210 - 210)^2 = 50$$