Concordia University
Engineering and Computer Science

Department of Mechanical and Industrial Engineering

FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

COURSE		NUMBER	SECTION
FUNDAMENTALS OF CONTROL SYSTEMS		MECH 371/4	F
EXAMINATION: FINAL EXAM	DATE: December 14, 2011	TIME & PLACE Room:	# OF PAGES: 4+ 1 cover page
PROFESSOR: W.F. Xie		LAB INSTRUCTOR:	

MATERIALS ALLOWED ■ NO O YES (PLEASE SPECIFY)

CALCULATORS ALLOWED O NO ■ YES

CLOSED BOOK EXAM
NO MATERIALS ALLOWED
NONPROGRAMABLE CALCULATORS

SPECIAL INSTRUCTIONS:

ANSWER ALL FIVE (5) QUESTIONS.

ALL RELEVANT PROCEDURE STEPS MUST BE SHOWN CLEARLY.
OTHERWISE ZERO MARKS WILL BE GIVEN.

PLEASE* RETURN EXAM QUESTIONAIRE *WITH YOUR ANSWER BOOKLET.

Name: ______________________________ **I.D.:** ______________________________
Surname, given names

QUESTION #	MARKS per QUESTION	MARKS
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

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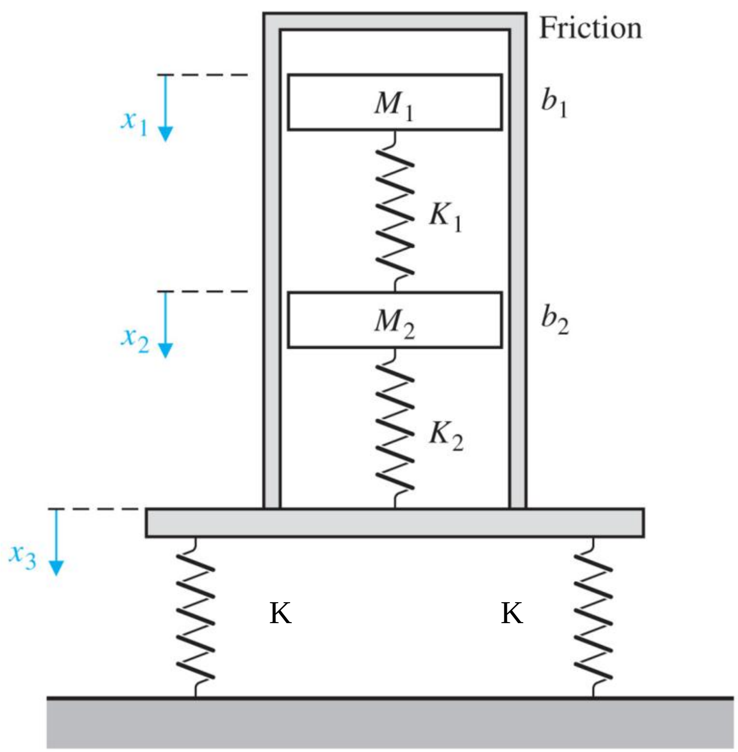
Problem #1 (20 marks)

A mechanical system is shown in the following figure, which is subjected to a known displacement $x_3(t)$ with respect to the reference.

(a) Determine the two independent equations of motion with regard to $x_1(t)$ and $x_2(t)$.

(b) Obtain the equations of motion in terms of the Laplace transforms, assuming that the initial conditions are zero.

(c) Obtain the relationship $T_{13}(s) = \dfrac{X_1(s)}{X_3(s)}$



<u>Problem #2</u> (20marks)

The open-loop transfer function of a unity negative feedback system is given by

$$GH(s) = \frac{50}{s(s+2)(s+4)}$$

(a) Plot the Nyquist diagram.
(b) Investigate the stability of the system based on Nyquist diagram.
(c) Use Routh-Hurwitz stability criterion to validate the stability.

<u>Problem #3</u> (20marks)

A unity feedback system has the process

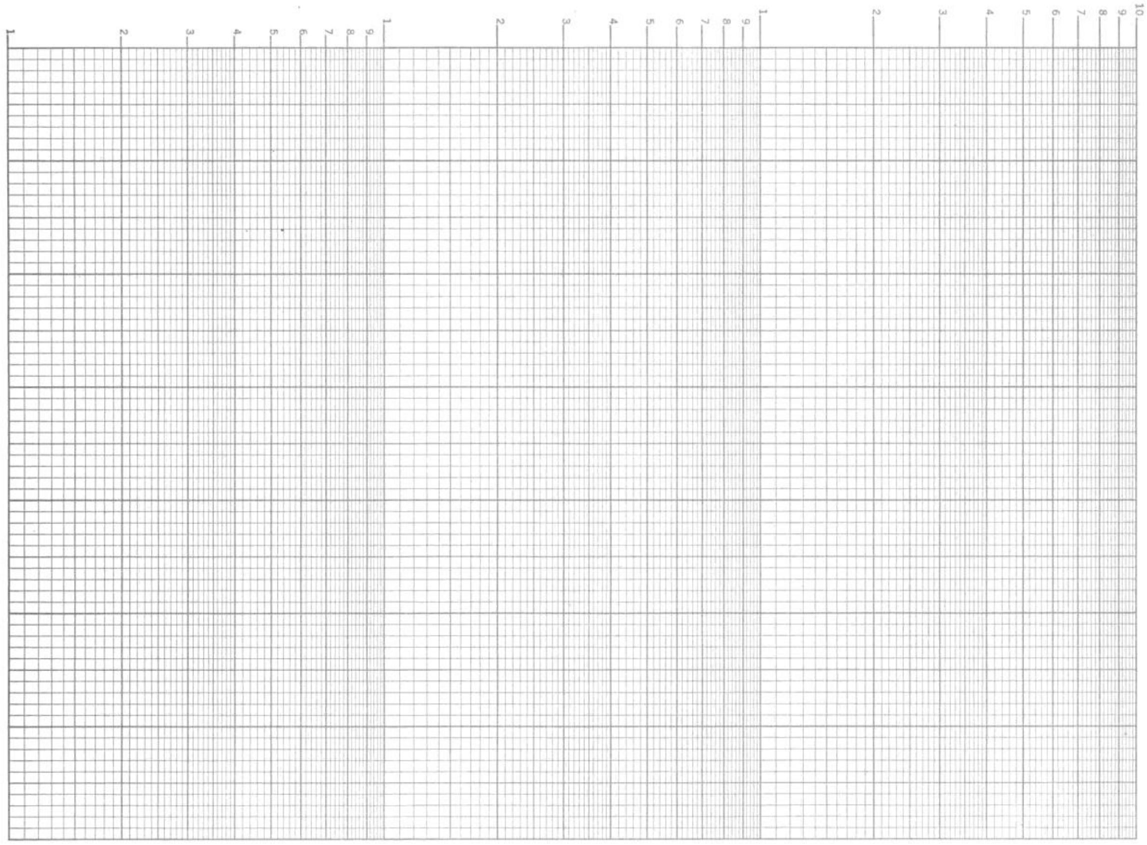
$$G(s) = \frac{K(s+10)}{s(s+5)}$$

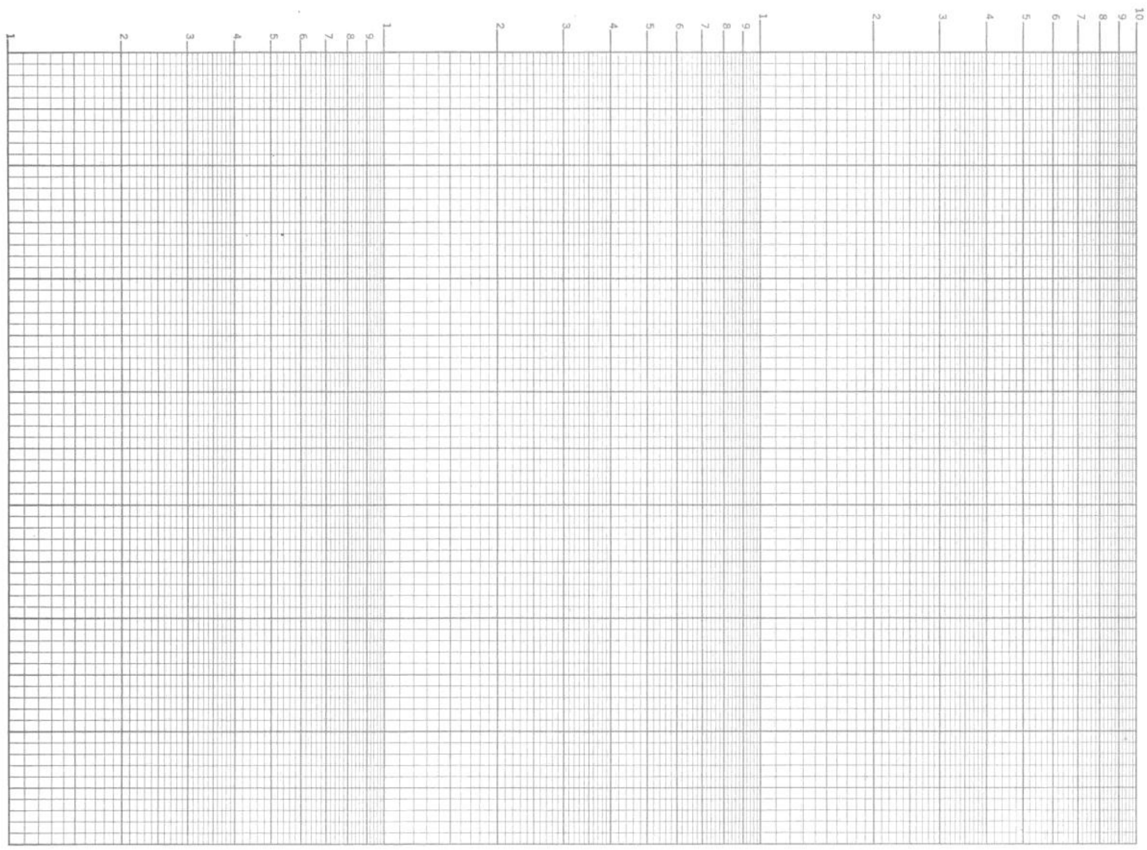
a). Sketch the root locus of the system as $0 < K < \infty$.
b). Prove the points $s_{1,2} = -5 \pm 5j$ are located on the root locus and determine the gain K at the points.
c) Compute the settling time and percent overshoot of the system for a unit step input when K is equal to above (b) value. The formulae are given on Page 4.

<u>Problem #4</u> (20 marks)
A unity feedback system has a plant transfer function given by:

$$G(s) = \frac{67.4(s+6)}{s(s+2)(s^2+10s+64)}$$

(a) On the semi-log paper provided, plot the asymptotic Bode diagram. Show all relevant steps. Indicate the relevant slopes on the diagram.
(b) (1) From the asymptotic Bode diagram, determine phase-crossover frequency (ω_p) gain margin, gain-crossover frequency (ω_g) and phase margin. Show these values on the diagram clearly.
(2) Is the closed loop system stable or unstable? Explain your answer clearly.
(c) Introduce a constant gain K controller (i.e.: proportional controller) to obtain a phase margin of 45 degrees. Find out the numerical value of gain K controller.





Problem #5 (20 marks)

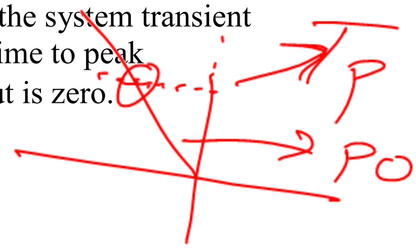
A unity negative feedback control system has the plant transfer function given by:

2–3:15pm Monday → old/review problems

$$G_P = \frac{10}{(s+3)(s+5)}$$

$G_c \rightarrow n_p \geq n_z$ + PD + PID

(a) Using the root locus method, design a proportional controller so that the system transient response due to a step input *r(t)* has exactly 4.3 % overshoot.
(b) Using the root locus method, design a proper controller so that the system transient response due to a step input *r(t)* has exactly 4.3 % overshoot, time to peak $T_P = 1.57 sec$, and the steady state error e_{ss} due to the step input is zero.



Given information:

Percent overshoot $P.O. = 100\, e^{-\zeta\pi / \sqrt{1-\zeta^2}}$, $\theta = \cos^{-1}\zeta$

Time to peak $T_P = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

Settling time $T_s = \dfrac{4}{\zeta\omega_n}$

where ω_n, ω_d and ζ is the system natural frequency, system damped natural frequency, and system damping ratio, respectively.