Problem #1 (40 marks)

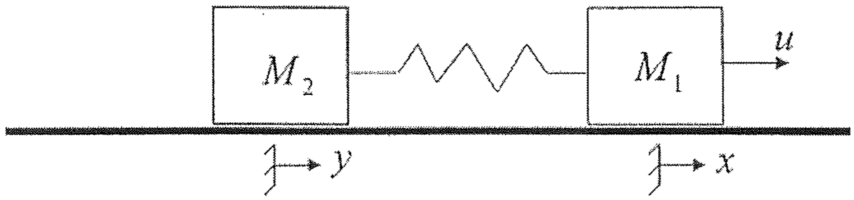


Figure 1 – Mass spring model

Figure 1 shows a car of mass M_1 towing a trailer of mass M_2. The car-trailer system can be modeled as a 2 mass-spring system on a frictionless surface. The spring force usually saturates for large displacements. The spring force F_S is modeled as a nonlinear function of the spring relative displacement d given by $F_S = k \cdot a\tan(d)$ (see Figure 2).

a) Draw the free body diagrams and derive the equations of motion.

b) Linearize the system around the relative spring displacement $d=0$. **HINT:** $\dfrac{da\tan(z)}{dz} = \dfrac{1}{1+z^2}$

NOTE: If you could not solve parts a) and b), consider a linear spring with spring constant k throughout the rest of this question.

c) Draw the block diagram for the linearized system.

d) Draw the signal flow graph for the linearized system.

e) Using Mason's rule, obtain the transfer function $G(s)=X(s)/U(s)$ (if you use another method to obtain the transfer function it will only be worth 50% of the marks).

f) Compute the sensitivity function $S_k^G(s)$ and its steady state gain.

Compute the percentage change in $G(j\omega)$ caused by 10% variation in k at $\omega = 1\ rad/\sec$, by letting $k=1,\ M_1 = 1\ and\ M_2 = 2$.

g) Is the system $G(s)$ stable? Why?

h) Can you stabilize the system using a proportional controller? Show the root locus. ← ~.

i) Using root locus, design a dynamic compensator $K(s)$ that stabilizes the system $X(s)/U(s)$. Show the transfer function $K(s)$. You do not need to compute explicit values for the parameters of the controller but you are required to sketch the root locus.

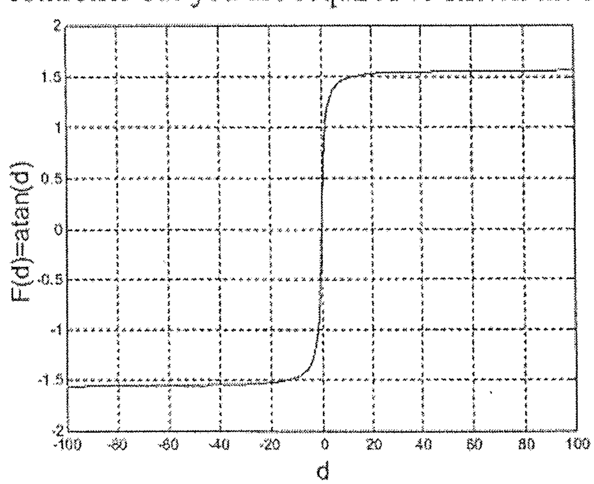


Figure 2 – Plot of $F(d)=atan(d)$

Problem #2 (25 marks)

a) The characteristic equation of a unity negative feedback system is given by:

$$1 + \frac{K(s+40)}{s(s+10)}$$

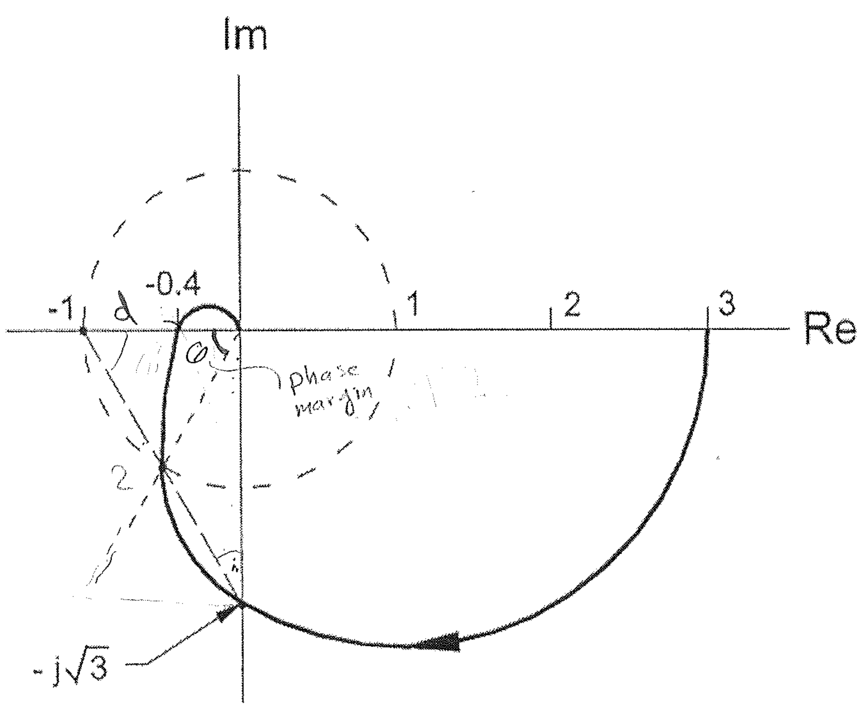
Draw the root locus showing all relevant steps.

b) The open-loop transfer function of a unity negative feedback system is given by:

$$G(s) = \frac{2}{s-1}$$

(i) Draw the polar plot for the system. Show all relevant calculations.
(ii) Using the Nyquist stability criterion, explain if the closed-loop system is stable or unstable.

c) For the polar plot shown below, determine the gain margin in decibels, and calculate the phase margin.



d) With respect to a minimum phase unity negative feedback system, explain physically the significance of (1) phase margin, and (2) gain margin.

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CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE

DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING
MECH 371/4 sec. X FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term **Instructor: H. Hong** **Date: March 4, 2003**

Answer all four (4) questions. **Closed book exam.** **Total time: 1 hour 15 minutes**

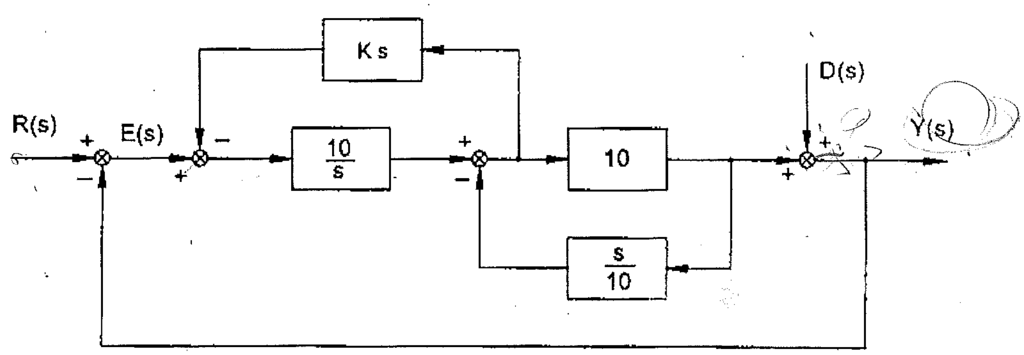
For all questions:
Clearly show all equations and mathematical steps.
Answers without mathematical steps will be given a mark of zero.

Problem 1.→ 23; Problem 2.→ 10; Problem 3.→ 10; Problem 4.→ 12 = 55 marks total.

Problem 1. [1→ 3, 2→ 2, 3→ 6, 4→ 4, 5→ 6, 6→ 2 = 23 marks]

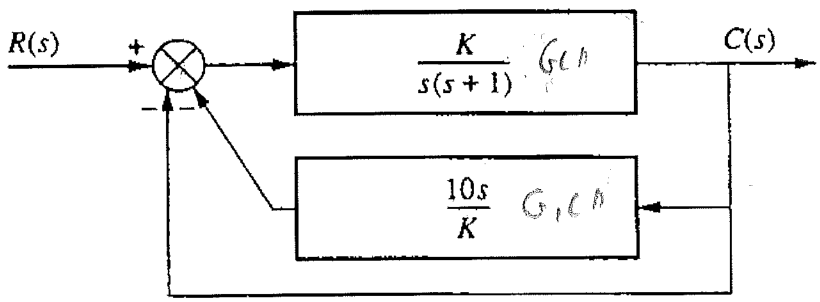
A negative feedback control system has a block diagram as shown below.

(1) Draw the signal flow graph for the system.
(2) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(3) For a unit step input, find the value for the gain K so that the system has 20% overshoot.
What is the natural frequency ω_n of the system?
What is the settling time T_S of the system?
(4) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency), ω_d (damped natural frequency), and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.
(5) Find the total steady-state response due to a unit step input R(s) and a unit step disturbance D(s).
(6) Determine the sensitivity of the transfer function T(s) with respect to gain K, ie: S_K^T



Problem 2. [10 marks]

For the feedback control system shown below:
(1) Determine the type number of the system.
(2) Using the concept of system type number, determine the steady state error to a unit step input.
(3) Using the concept of system type number, determine the steady state error to a unit ramp input.
(4) Using the concept of system type number, determine the steady state error to a unit parabola input.



Problem 3. [10 marks]

Using the Routh-Hurwitz test, determine the stability of the closed-loop transfer function:

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

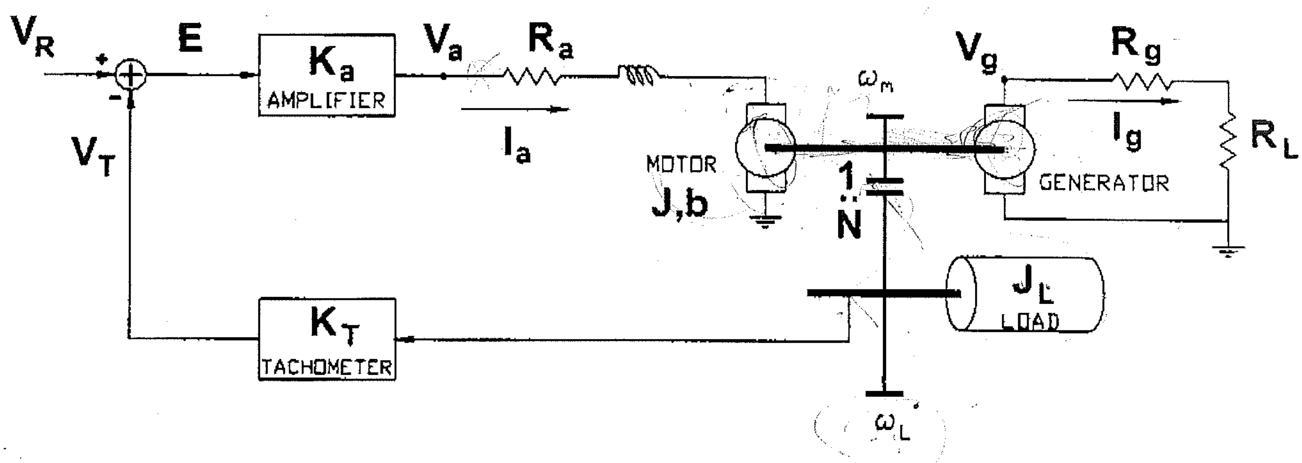
Determine, if any,
(1) the marginally stable roots, and their numerical values.
(2) the total number of unstable roots.

HINT: The negative number -4 is a vector written as $-4+0j$ or as $4e^{+180j}$.

Problem 4. [12 marks]

A permanent magnet DC-motor drives a rotational load through a speed reduction gear ratio. The motor also directly drives a generator that supplies current to a resistive load. The load speed is measured by a tachometer for velocity feedback control. The error between the reference and tachometer voltage drives an amplifier that supplies power to the armature of the DC-motor.

Draw the block diagram for the feedback control system shown in the schematic diagram below. Provide justification to any assumptions made.



<u>Given equations with respect to schematic diagram:</u>

$E(s) = V_R(s) - V_T(s)$	error = reference voltage minus tachometer voltage
$V_T(s) = K_T\omega_L(s)$	tachometer voltage = tachometer gain times load speed
$V_a(s) = K_a E(s)$	amplifier voltage = amplifier gain times error voltage
$V_a(s) = R_a I_a(s) + V_b(s)$	amplifier (or armature) voltage = armature resistance times armature current plus back EMF [armature inductance neglected]
$V_b(s) = K_b\omega_m(s)$	back EMF = back EMF gain times motor speed
$T(s) = K_m I_a(s)$	motor generated torque = motor constant times armature current
$T_m(s) = T(s) - T_d(s)$	torque driving motor = motor generated torque less torque driving disturbances
$T_m(s) = J\dot{\omega}_m(s) + b\omega_m$	torque driving motor = armature inertia times motor speed plus damping times motor speed
$T_d(s) = T_{d1}(s) + T_{d2}(s)$	disturbance torque = torque driving inertia load plus torque driving generator

<u>Equations that must be derived from schematic diagram:</u>

$\omega_L(s) = ??$ ✓ — load speed $\omega_L(s) \Rightarrow$ motor speed $\omega_m(s)$ with gear reduction. Let gear ratio be N>1

$T_{d1}(s) = ??$ ✓ — load torque $T_{d1}(s) \Rightarrow$ rotational load inertia J_L; omit damping; load angular acceleration $\dot{\omega}_L(s)$; gear ratio N>1

$V_g(s) = ??$ — generator voltage $V_g(s) \Rightarrow$ generator voltage gain K_g; generator (or motor) speed $\omega_m(s)$

$V_g = K_g \omega_m$

$I_g(s) = ??$ — generator current $I_g(s) \Rightarrow$ generator armature resistance R_g; resistive load R_L; generator voltage $V_g(s)$; neglect generator inductance

$T_{d2}(s) = ??$ — generator torque $T_{d2}(s) \Rightarrow$ generator constant K_{GG}; generator current $I_g(s)$

$$I_g = \frac{1}{R_g + R_L} V_g$$

Problem #3 (15 marks)

A unity feedback system has a plant transfer function given by:

$$G_p = \frac{1}{s(s+1)}$$

a) Design a cascade proportional derivative (PD) compensator so that the following design specifications are met:

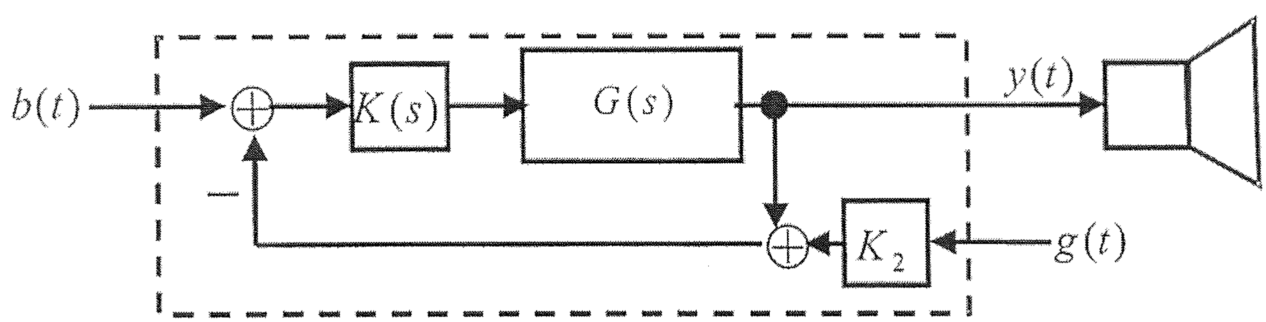
$$\textit{percent overshoot} \quad P.O. = 4.32\ \%$$

$$\textit{natural frequency} \quad \omega_n = 14.14\ \frac{rad}{\sec}$$

b) Using the concept of system "type number", determine for the compensated system, the steady-state error for (1) unit step input, (2) unit ramp input, and (3) unit parabolic input.

Problem #4 (20 marks)

You have decided to jam a little bit in a musical practice session with your friend. You play the bass (low frequencies) and he/she plays the guitar (high frequencies). You plan to design a mixing table so that it amplifies your bass sound and attenuates your friend's guitar sound. The design is shown in the block diagram below, where G(s) is a low-pass filter, b(t) is the bass signal, and g(t) is the guitar signal. Note that b(t) is considered as the input signal, and g(t) is considered as noise.



Given that $G(s) = \dfrac{20000\pi}{s + 20000\pi}$, using Bode plot techniques and the approximations that

$|K(j\omega)G(j\omega)| >> 1$ for $\omega < 100\pi\ rad/\sec$

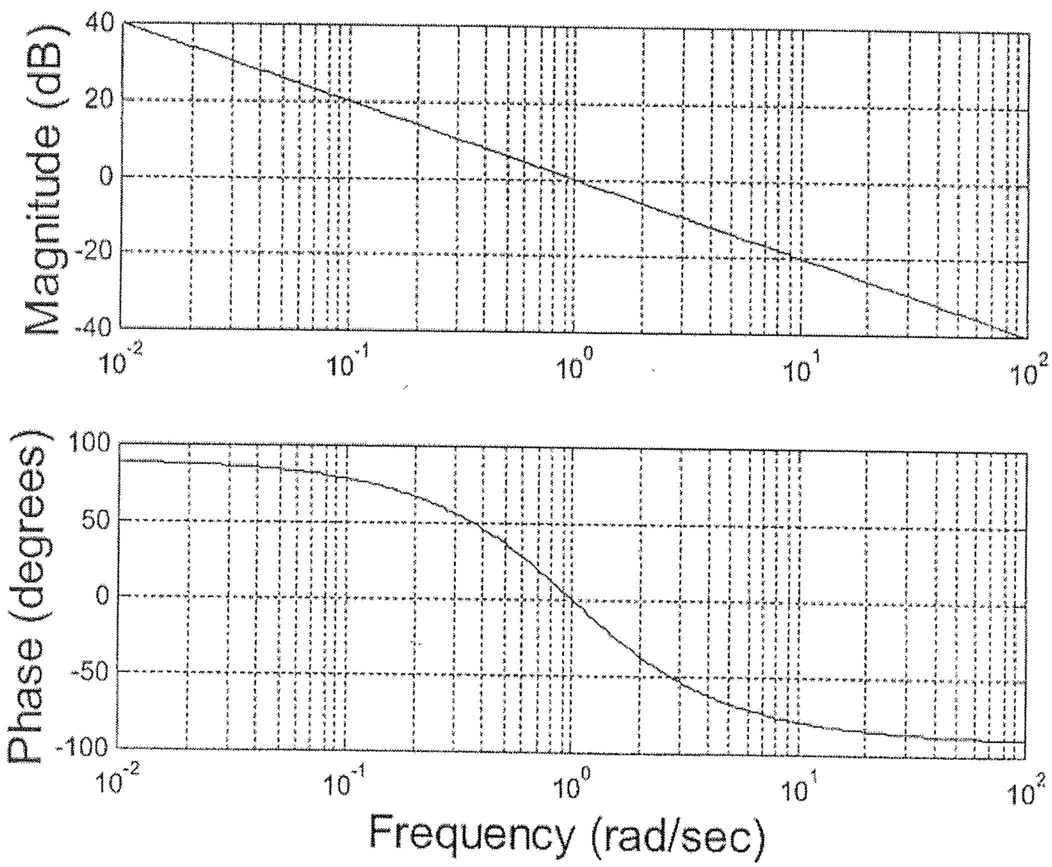
and $|K(j\omega)G(j\omega)| << 1$ for $\omega > 200\pi\ rad/\sec$, design *K(s)* such that:

(1) The phase margin of the system is 90°,
(2) The crossover frequency (open-loop) is $\omega_c = 200\pi\ rad/\sec$,
(3) $\left|\dfrac{e(j\omega)}{b(j\omega)}\right| \le 0.1$ for $\omega \le 20\pi\ rad/\sec$ (bass-low frequencies), where e(s)=b(s)-y(s),
(4) $\left|\dfrac{y(j\omega)}{g(j\omega)}\right| \le 0.1$ for $\omega \ge 2000\pi\ rad/\sec$ (guitar-high frequencies) for (worst case) $K_2 = 1$.

Plot the asymptotic magnitude and phase diagrams of the original and compensated system on the same Bode plot. To confirm that the above 4 conditions have been met, clearly describe and/or indicate your proofs on the Bode plot.

BONUS Question (10 marks)

Obtain the transfer function G(s) of the system whose Bode diagram is shown below. In your answer booklet, on the same diagram, plot the asymptotic magnitude and phase Bode plots of all the individual terms of G(s), and the combined asymptotic Bode plot of the transfer function G(s).



MECH 371/4

1.12

FUNDAMNETALS OF CONTROL SYSTEMS Section X

Mid-Term Thursday March 7, 2002

Answer All 4 Questions (total 45 marks)

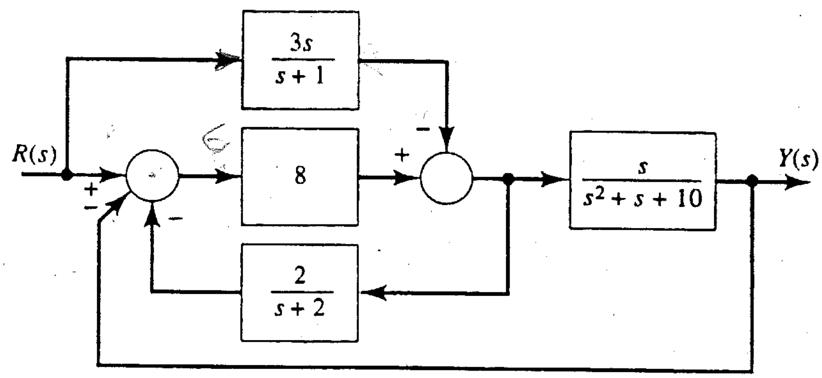
Question #1 (10 marks)

For the system shown in the block diagram:

a) Draw the signal flow graph.

b) Using Mason's loop rule, determine the overall system transfer function $\dfrac{Y(s)}{R(s)}$.

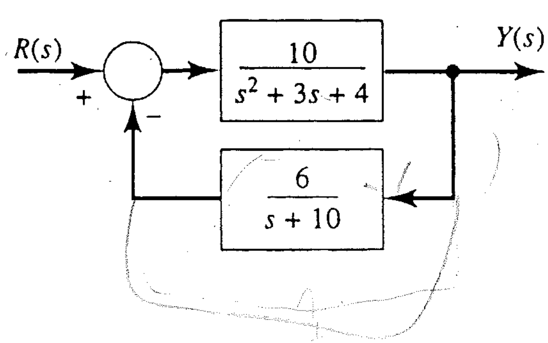
Show all mathematical steps.



Question #2.A (10 marks)

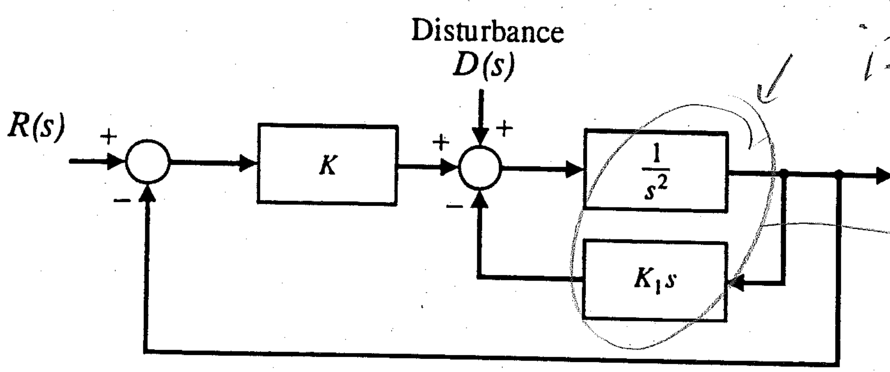
a) Determine the type number of the system.
b) Using the concept of system type number, determine the steady state error to a unit step input.
c) Using the concept of system type number, determine the steady state error to a unit ramp input.
d) Using the concept of system type number, determine the steady state error to a unit parabola input.

Show all equations and mathematical steps.
Answers without mathematical proofs will be given a mark of zero.



Question #2.B (5 marks)

For the system shown, the command signal $R(s)$ and disturbance signal $D(s)$ are both unit step inputs. Determine the gain K so that the steady state output $y(t)$ is 1.01 .

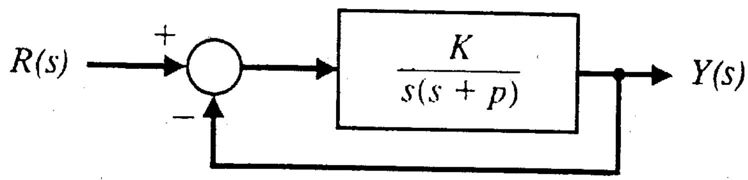


$\frac{1/s}{1+\frac{4}{s}} = \frac{\frac{1}{s}}{\frac{s+4}{s}} = \frac{1}{s(s+4)}$

Question #3 (10 marks)

For the system shown, it is required that the settling time $T_S \le 4$ seconds and the percent overshoot $P.O. \le 4.3\%$. Determine:

a) The gain K and parameter p necessary to satisfy the above performance requirements.
b) On the s-plane, plot the region of valid characteristic root locations.
c) Determine the equation for the sensitivity of the overall system transfer function with respect to parameter p. ie: determine S_p^T



Question #4 (10 marks)

The characteristic equation of a system is given by: $s^6 + s^5 + 5s^4 + s^3 + 2s^2 - 2s - 8$

Using the Routh-Hurwitz test, determine:

a) If any, the marginally stable roots, and their numerical values.
b) If any, the total number of unstable roots.

HINT: The negative number -4 is a vector written as $-4+0j$ or as $4e^{+180j}$.

MECH 371/4

1.12

FUNDAMNETALS OF CONTROL SYSTEMS Section X

Mid-Term Thursday March 7, 2002

Answer All 4 Questions (total 45 marks)

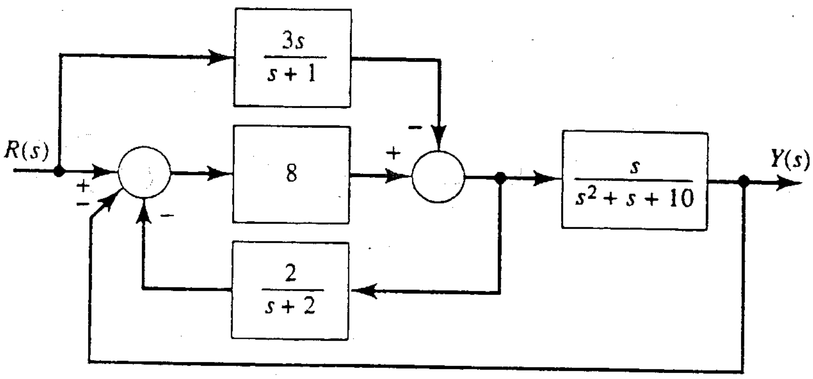
Question #1 (10 marks)

For the system shown in the block diagram:

a) Draw the signal flow graph.

b) Using Mason's loop rule, determine the overall system transfer function $\frac{Y(s)}{R(s)}$.

Show all mathematical steps.



Question #2.A (10 marks)

a) Determine the type number of the system.
b) Using the concept of system type number, determine the steady state error to a unit step input.
c) Using the concept of system type number, determine the steady state error to a unit ramp input.
d) Using the concept of system type number, determine the steady state error to a unit parabola input.

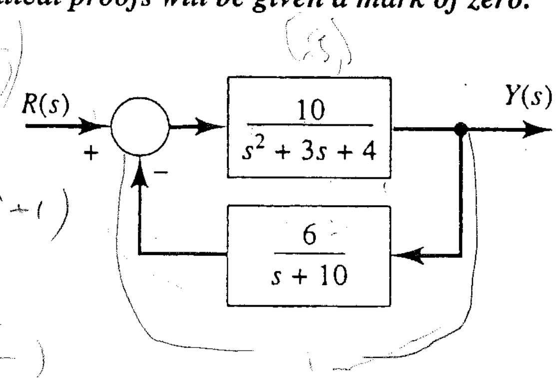
Show all equations and mathematical steps.
Answers without mathematical proofs will be given a mark of zero.

(s^3+s^2+s+4)

$s(s^2+s+1)$

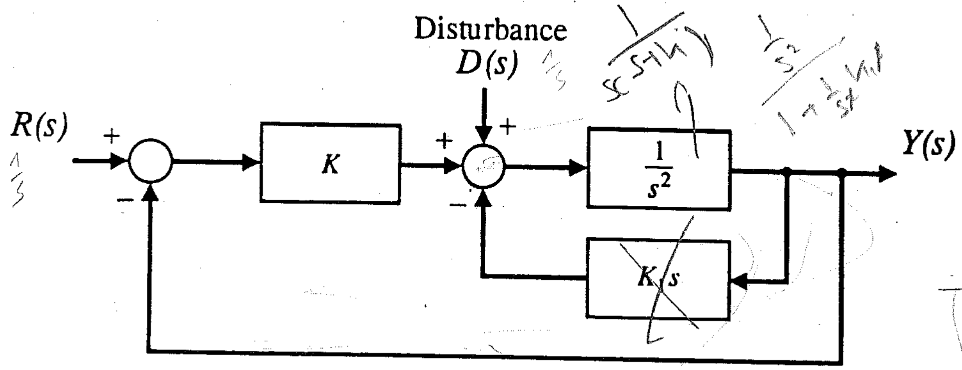
$s=0$

$\frac{1}{s^3(s^2 \ldots)}$



Question #2.B (5 marks)

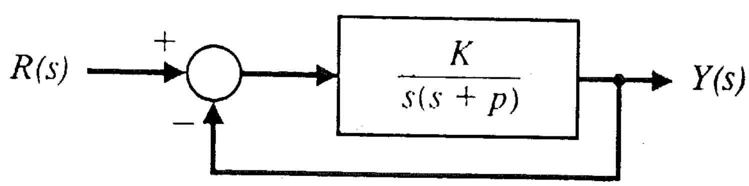
For the system shown, the command signal $R(s)$ and disturbance signal $D(s)$ are both unit step inputs. Determine the gain K so that the steady state output $y(t)$ is 1.01 .



Question #3 (10 marks)

For the system shown, it is required that the settling time $T_S \le 4$ seconds and the percent overshoot $P.O. \le 4.3\%$. Determine:

a) The gain K and parameter p necessary to satisfy the above performance requirements.
b) On the s-plane, plot the region of valid characteristic root locations.
c) Determine the equation for the sensitivity of the overall system transfer function with respect to parameter p. ie: determine S_p^T



Question #4 (10 marks)

The characteristic equation of a system is given by: $s^6 + s^5 + 5s^4 + s^3 + 2s^2 - 2s - 8$

Using the Routh-Hurwitz test, determine:

a) If any, the marginally stable roots, and their numerical values.
b) If any, the total number of unstable roots.

HINT: The negative number -4 is a vector written as $-4+0j$ or as $4e^{+180j}$.

$T_1 = \dfrac{\frac{1}{s(s+4)}}{1 + \frac{1}{s(s+4)}} = \dfrac{1}{s^2+4s+1}$

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
ENGR 372/4 sec. Y FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term **Instructor: Dr. H. Hong** **Date: March 8, 2001**

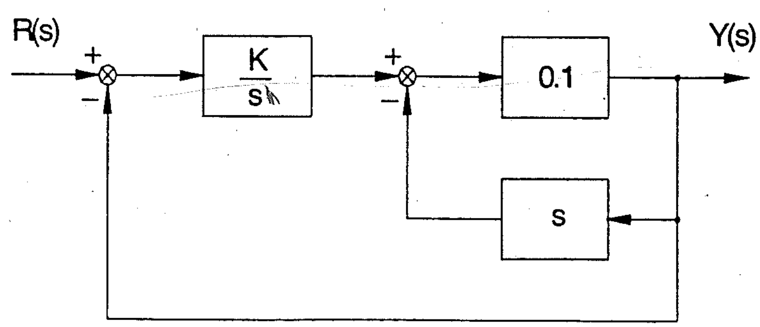
Answer all three (3) questions. **Closed book exam.** **Total time: 1 hour 15 minutes**

Problem 1.→23; Problem 2.→7; Problem 3.→5; = 35 marks total.

Problem 1. [a→1, b→7, c→3, d→2, e→1, f→3, g→3, h→3 = 23 marks]

A negative feedback control system has a block diagram as shown below.

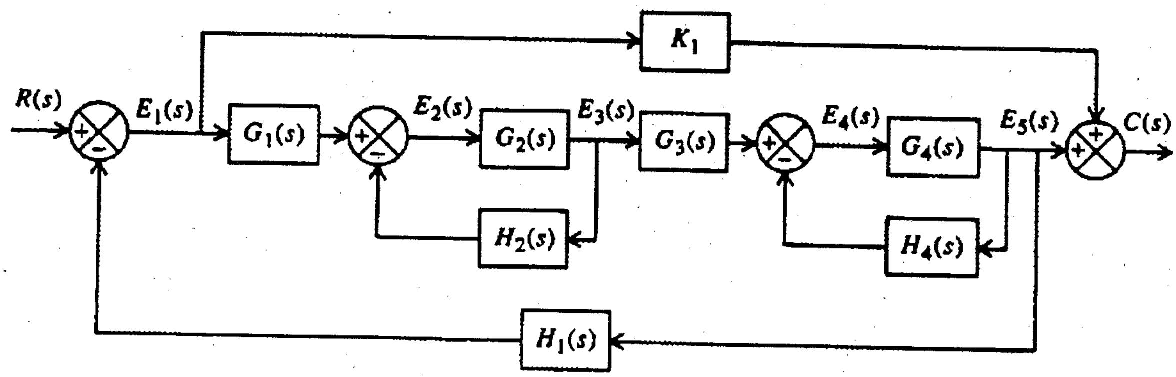
(1) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(2) For a unit step input, find the value for the gain K so that the system has 20% overshoot.
What is the natural frequency ω_n of the system?
What is the settling time T_s of the system?
(3) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency) and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.
(4) Using the Routh-Hurwitz stability criterion determine the range of gain K for the system to be stable?
(5) What is the "type number" of the system?
(6) For a unit step input, determine the position error constant K_p, and the steady-state error. Show all mathematical steps leading to your answer.
(7) For a unit ramp input, determine the velocity error constant K_v, and the steady-state error. Show all mathematical steps leading to your answer.
(8) For a unit acceleration input, determine the acceleration error constant K_a, and the steady-state error. Show all mathematical steps leading to your answer.



Problem 2. [a→1½, b→5½ = 7 marks]

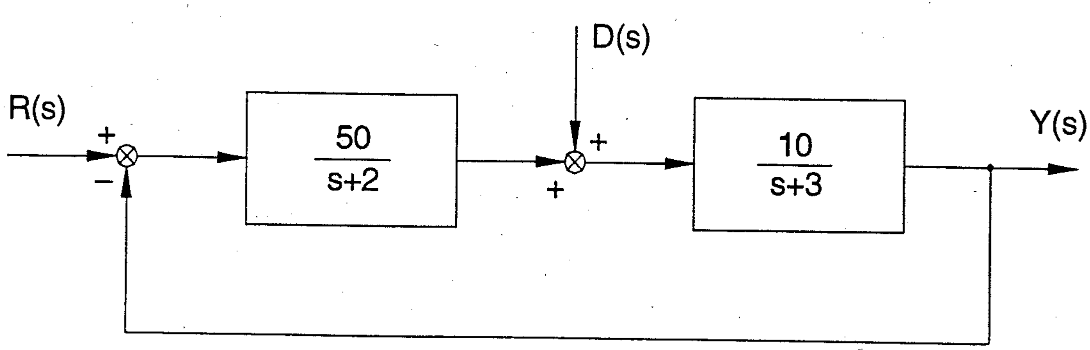
For the system represented in the block diagram below, find:

(1) the signal flow graph representation.
(2) the transfer function T(s) = C(s)/R(s), by using (only) Mason's signal-flow gain formula.



Problem 3. [5 marks]

For the system shown below find the total steady-state error due to a unit step input R(s) and a unit step disturbance D(s).



$$Y = R(s)\,\frac{50}{s+2} + D(s)\,\frac{10}{s+3}$$

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Concordia UNIVERSITY

COURSE	NUMBER	SECTION
FUNDAMENTALS OF CONTROL SYSTEMS	ENGR 372/4	U,X,Y,W

EXAMINATION	DATE	TIME	# OF PAGES
FINAL EXAM	APRIL 24, 2001	14:00-17:00 Hrs	3

INSTRUCTOR

S.Hashtrudi Zad, H.Hong, J.V.Svoboda, N.Suresh

MATERIALS ALLOWED: ☒ NO ☐ YES (PLEASE SPECIFY)

CALCULATORS ALLOWED: ☐ NO ☒ YES

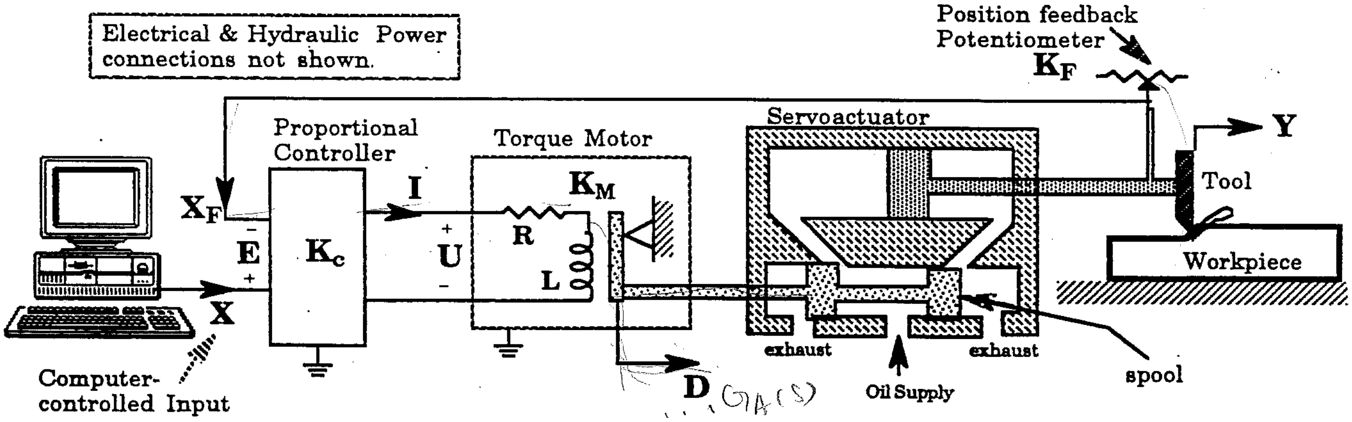
ONLY non-programmable calculators will be allowed.

SPECIAL INSTRUCTIONS:

Attempt all problems. All solution steps must be shown clearly. Identify your final answers clearly.

PROBLEM # 1 [10 Points]

Shown below is the simplified schematic representation of a one-axis closed loop drive for a CNC[Computer Numerical Controlled] machine tool. The tool position **y**(t) is controlled by the computer-generated command voltage **x**(t). The proportional controller with voltage gain $\mathbf{K_c}$ compares the command **X**(s) with the position-feedback signal $\mathbf{X_F}$(s) derived from a linear feedback-potentiometer. The controller voltage output **U**(s) drives an electromechanical torque-motor represented by an R-L load impedance and a gain factor $\mathbf{K_M}$. The torque-motor moves the spool of a servo-actuator, resulting in the tool movement **Y**. The functional relationships, in the s-domain, between the various system components are also given below.



Position Error signal $\quad E(s) = X(s) - X_F(s)$

Controller Output $\quad U(s) = K_c E(s)$, $K_c = 10$ volt/volt

Motor current $I(s) = \dfrac{U(s)}{R + sL}$, $R = 100\,\Omega$, $L = 1$ Henry

Spool Displacement $\quad D(s) = K_M I(s)$, $K_M = 0.1$ metre/amp

Tool Position $\quad Y(s) = G_A(s)D(s)$, $G_A(s) = \dfrac{100}{s(1 + 0.25\,s)}$ metre/metre

Feedback signal $\quad X_F(s) = K_F . Y(s)$, $K_F = 1$ volt/metre

(i) Draw the block diagram or signal flow graph of the control system, showing all variables and parameters clearly.

(ii) Obtain the open-loop transfer function $G(s)H(s)$ and the closed-loop transfer function $T(s)$ where $T(s) = Y(s) / X(s)$

(iii) Draw the pole-zero diagram of $G(s)H(s)$, and hence show that the dynamics of the torque-motor can be neglected in comparision with that of the servoactuator.

(iv) With the simplification provided in **(iii)** above, show that the overall CNC machine tool drive model is a unity- gain, critically- damped second- order system with an undamped natural frequency of **2 radians/sec.**[Hint: **Remember to include the DC gain of the neglected term in the open-loop transfer function when deriving the overall approximate transfer function]**

PROBLEM # 2 [12 Points]

(A) Consider a unity-feedback control system with the open-loop transfer function :

$$G(s) = \frac{2s^2 + 2s + 1}{(s^2 - 1)(s - \alpha)} \quad , \text{where} \quad \alpha > 0$$

(i) Use the Routh-Hurwitz Criterion to find the range of α for which the closed-loop system will be stable.

(ii) Find the value of α at which the system will begin to oscillate and also the frequency of the oscillation.

(B) A unity feedback system has the open-loop transfer function :

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)}$$

6 points

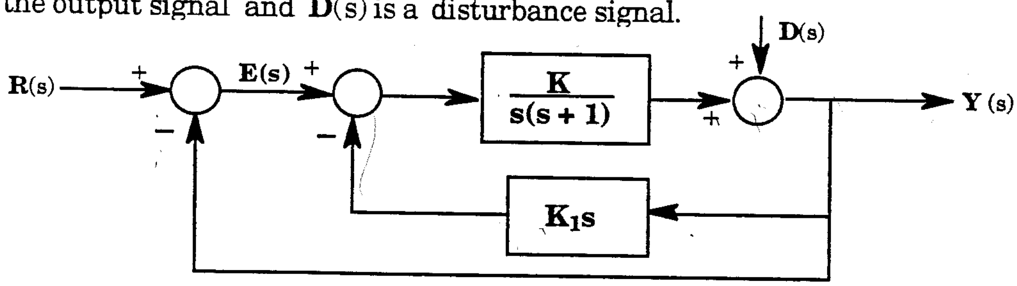
(i) Draw the root locus of the closed-loop system for $K > 0$, showing all the relevant steps clearly.

(ii) Show that the point $s_1 = -0.588 + j\,0.658$ lies on the root locus and determine the gain K at the point s_1.

(iii) Assuming the gain K is set to the value obtained in **(ii)** above, determine an estimate of the settling time T_s $(= 4/\zeta\omega_n)$ of the step response of the closed-loop system.

PROBLEM # 3 [12 Points]

The block diagram of a feedback control system is shown below, where $R(s)$ is the input signal, $Y(s)$ is the output signal and $D(s)$ is a disturbance signal.



$\frac{Y(s)}{R}$

$\frac{D(s)}{R}$

(i) Find values for K and K_1 such that the poles of the closed- loop system will be located at the points $-1.61 \pm j\,3.14$ on the s-plane.

(ii) Assume $d(t) = 0$. Determine the steadystate error e_{ss}, <u>using error coefficients</u>, if the input signal is $r(t) = 1 + t$ [<u>Note:</u> e_{ss} is the steadystate value of signal $E(s)$]

(iii) Determine the steadystate output $y_{ss}(t)$, $t \to \infty$, for a simultaneous unit step input and a unit-step disturbance input.

(iv) Determine the sensitivity of the transfer function $T(s) = Y(s) / R(s)$ to the velocity-feedback constant K_1, <u>ie</u> derive $S^{T}_{K_1}$

PROBLEM # 4 [10 Points]

The open-loop transfer function of a unity-feedback system is given by :

$$G(s) = \frac{K}{s\,(s+4)\,(s+10)}$$

The gain **K** is to be chosen such that the steadystate error of the closed-loop system to a unit ramp input is $\left(\frac{1}{12}\right)$

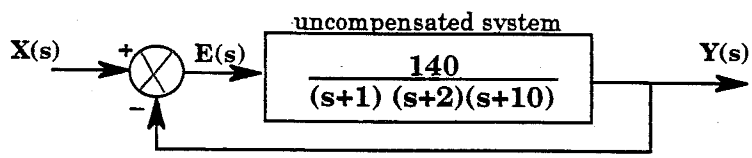
(i) Determine **K** and sketch the corresponding Bode ***magnitude*** plot of $G(j\omega)$.

(ii) Calculate the ***phase margin*** ϕ°_{pm} if it is known that the unity-gain crossover frequency $\omega_1 = 5.85$ radians/sec , for the **K** chosen in **(i)**.

(iii) Sketch the polar plot of the open loop transfer function $G(j\omega)$,for the **K** chosen in **(i)**, and determine the ***gain margin*** GM of the system using the polar plot

PROBLEM # 5 [11 Points]

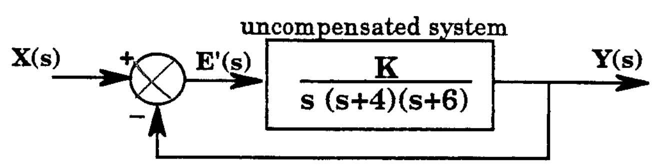
(A) The block diagram of a third-order feedback control system is shown below.



(i) Design a **cascade** phase-lag compensator ~~which will~~ which will reduce the steadystate error of the uncompensated system to a unit step input by a factor of **10**. Use a compensator of the form $\frac{s+0.1}{s+p_c}$

(ii) Draw the block diagram of the compensated system.

(B) The system whose block diagram is shown below , has dominant poles located at $-1.2 \pm j2$.



(i) Determine the gain **K** required for the above dominant poles . Also determine the damping ratio ζ and the settling time T_s of the approximated second-order system.

(ii) It is desired to improve the system transient response by means of a **cascade** PD compensator of the form $G_c(s) = K_c(s + z_c)$. Design a suitable compensator [ie first determine z_c and then K_c] to reduce the settling time T_s $(= 4/\zeta\omega_n)$ of the original system by a factor of **3** , ***without*** changing the damping ratio ζ .{ Hint: Use a graphical design procedure in the s-plane, applying the angle-criterion as well as the magnitude-criterion of the root-locus }

(iii) Using a rough sketch of the root-locus of the compensated system , comment on the validity of the assumption that the new complex closed-loop poles are still dominant.

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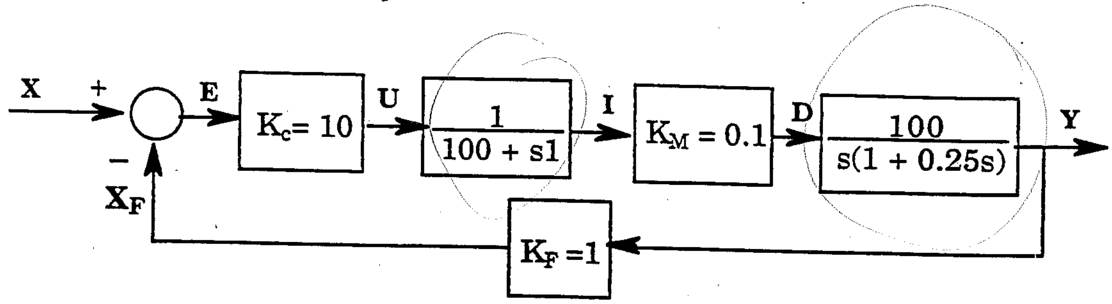
ENGR 372 2001 Solutions

#1 : The block diagram of the system is:

(i)



(ii)

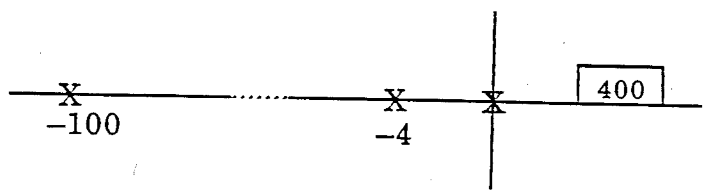
$$G(s) = \frac{K_c K_M G_A}{R + sL} = \frac{1.G_A}{100 + s1} = \frac{1}{s(1+0.01s)(1+0.25s)} = \frac{400}{s(s+4)(s+100)}$$

$$H(s) = K_F = 1$$

ie $\quad \mathbf{G(s)H(s)} = \dfrac{\mathbf{400}}{\mathbf{s(s+4)(s+100)}}$

and $\mathbf{T(s)} = \dfrac{G(s)}{1 + G(s)H(s)} = \dfrac{400}{400 + s(s+4)(s+100)} = \dfrac{\mathbf{400}}{\mathbf{s^3 + 104s^2 + 400s + 400}}$

(iii) The p-z diagram of the original (non-approximated) GH is shown below:



The OLTF pole at s= −100 (due to the torque-motor) is x25 away than the pole at s=− 4 (due to the servoactuator). Thus, we may neglect the OLTF pole at s=−100 as far as its effect on the transient response is concerned.

Comment (**not required as an answer!**)
[In the stable range,the CLTF poles originating from the OLTF poles at s=0 & s=−4 will be dominant , since the corresponding root locus branches will veer to the right,while the branch from −100 proceeds to the left. We may evaluate the gain for critical damping by locating the breakout point between 0 & −1 : From GH, setting dK/ds=0 gives s≈ −1.98 as the breakout point.The gain at this point is (1.98) (100−1.98) (4−1.98) =392 which is a little less than the specified K of 400.At K=400, the damping will be slightly less than critical]

(iv) The GH function may be approximated as

$$G(s)H(s) \approx \frac{4}{s(s+4)} \qquad H = K_F = 1$$

and $\therefore$ $T(s) = \dfrac{G(s)}{1+G(s)} \approx \dfrac{4}{s^2 + 4s + 4}$

ie $\quad 2\zeta\omega_n = 4$ and $\omega_n = \sqrt{4} = $ **2radians/sec** ; $\zeta = 1$ or the Closed-loop system is **Critically damped**

#2 : **(A)** $G(s) = \dfrac{2s^2 + 2s + 1}{(s^2 - 1)(s - \alpha)}$, where $\alpha > 0$

The CE is : $2s^2 + 2s + 1 + s^3 - \alpha s^2 - s + \alpha = 0$

or $s^3 + (2 - \alpha)s^2 + s + (1 + \alpha) = 0$

The RH array is

s^3	1	1
s^2	$(2-\alpha)$	$(1+\alpha)$
s^1	A	-----
s^0	$(1+\alpha)$	

$A = \dfrac{1-2\alpha}{2-\alpha}$

$\dfrac{(2-\alpha)-(1-\alpha)}{2-\alpha}$ $= \dfrac{1-2\alpha}{2-\alpha} > 0$ $\alpha < \frac{1}{2}$ ... > 0

For stability, we must have $1+\alpha > 0$, $2-\alpha > 0$, and from $A > 0$, $(1-2\alpha) > 0$
These conditions imply $\alpha < 2$, $\alpha < 0.5$ and $\alpha > -1$ respectively

(i) Since $\alpha > 0$, the range of α for stability is **$\underline{0 < \alpha < 0.5}$**

(ii) If $\alpha = 0.5$, a row of zeros results in the RH array: The auxiliary equation is $1.5s^2 + 1.5 = 0$ or $s = \pm j1$, ie $\omega_{osc} = \underline{1 \text{ radian/sec}}$

(B) (i) The OLTF is **G(s) = K (s+2) / s (s+1) (s+ 3)** characteristic eq.

$n = 3$, $m = 1$ $\quad n-m = 2$ --------- 2 asymptotes at $\pm 90°$ and
$\sigma_A = [(0-1-3)-(-2)]/2 = -1$

$\dfrac{\Sigma P - \Sigma Z}{n-m}$ then $\dfrac{(2r+1)\,\pi}{n-m}$ $r = 0 \ldots n-m-1$

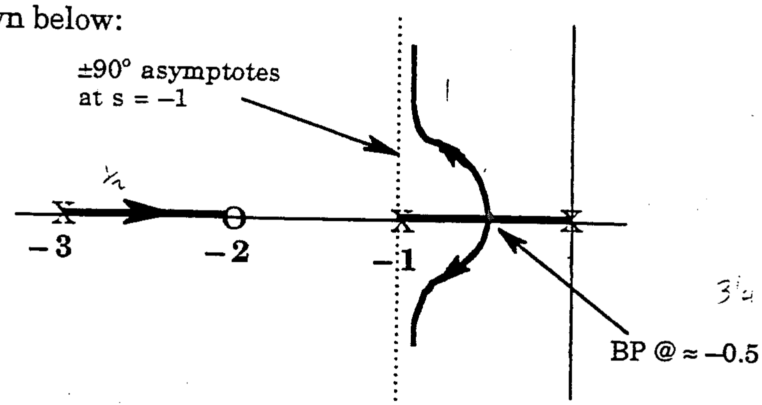
The Breakout Point between s=0 and s=–1 can be found by trial & error by locating the value of s , between 0 & –1 , for which the magnitude of K, $|K| = |s(s+1)(s+3)/(s+2)|$, reaches a maximum :
Using trial values,

s	|K|
–0.1	0.25
–0.3	0.33
–0.4	0.39
–0.5	0.42
–0.6	0.41
–0.7	0.37

Maximum here indicates approximate location of BP at s = – 0.5

$\dfrac{(2r+1)\times 180°}{n_p - n_z}$ $0, 1\ (n_p - n_z - 1)$ $= 90°, 270°$

The root locus is shown below:

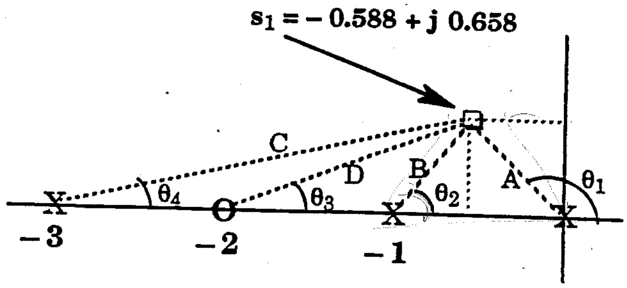


3/4

3/4

(ii) If the point s1= −0.588 + j0.658 is to lie on the root-locus, the angle criterion must $\sum(\angle zeros) - \sum(\angle poles) = \pm 180°$ must be satisfied at the point.

ie Referring to the p-z diagram shown, we must have:
$\phi = \theta_3 - \theta_1 - \theta_2 - \theta_4 = -180°$ where the angles are as shown.



ie $\phi = \tan^{-1}[0.658/(2-0.588)] - [180° - \tan^{-1}(0.658/0.588)] - \tan^{-1}[0.658/(1-0.588)] - \tan^{-1}[0.658/(3-0.588)]$

$= 25° - 131.8° - 57.9° - 15.3° = -180°$

∴ The angle criterion is satisfied and s_1 lies on the root locus.

(iii) Using the magnitude criterion: $K(@\,s_1) = A.B.C/D$

ie $K(@\,s_1) = (0.882)(0.776)(2.5)/1.558 \approx \underline{1.0995}$

Settling time $T_s = 4/\zeta\omega_n = 4/0.588 = \underline{6.8027}$ sec

---

#3 : **(i)** Reducing the complete system [& with D(s) = 0)] the transfer function $T_R = Y_R/R$ is

$$\frac{Y_R}{R} = \frac{\frac{K}{s(s+1)}}{1 + \frac{K}{s(s+1)}(1+K_1 s)} = \frac{K}{s^2 + s\underbrace{(1+KK_1)}_{2\zeta\omega_n} + \underbrace{K}_{\omega_n^2}}$$

The characteristic equation (CE) is : $s^2 + (1 + KK_1)s + K = 0$ — ½

Since the closed-loop pole locations are complex, we know that the system is underdamped and the roots of the CE are:

$$-\zeta\omega_n \pm j\omega_d = -\zeta\omega_n \pm j\omega_n\sqrt{(1-\zeta^2)} = -1.61 \pm j\,3.14$$

$S^2 + 3.22S + 12.45$

Thus, $\zeta\omega_n = 1.61$ and $\omega_d = \omega_n\sqrt{(1-\zeta^2)} = 3.14$

and $\omega_n = \sqrt{[(\zeta\omega_n)^2 + (\omega_d)^2]} = \approx 3.53$ — 1 $K = (\omega_n)^2 \approx \underline{12.46}$ — ½

Also, $1 + KK_1 = 2\zeta\omega_n = 2(1.61) = 3.22$ or $KK_1 = 2.22$

1 $\therefore K_1 = 2.22/12.46 = \underline{0.178}$

$(\zeta = \zeta\omega_n/\omega_n = 1.61/3.53 = 0.456)$

½

(ii) For the unity feedback system, $G(s) = K/s(s+1+KK_1)$ → **TYPE 1 system**

$s^2 + s + KK_1 s$

The 'position' error constant is $K_p = \lim_{s\to 0} G(s) = \infty$ — ½ → $s^2 + s$

The 'velocity' error constant is $K_v = \lim_{s\to 0} s\,G(s) = K/(1+KK_1)$ —

ie $K_v = 12.46/[1+12.46(0.178)] = 3.872$

For $R(s) = 1/s + 1/s^2$, $\quad e_{ss} = 1/(1+K_p) + 1/K_v = 0 + 1/3.872 = \underline{0.258}$ $\quad \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ sum

(iii) $\dfrac{Y_R}{R} = \dfrac{K}{s^2+s(1+KK_1)+K}$, $\quad \dfrac{Y_D}{D} = \dfrac{s(s+1)}{s^2+s(1+KK_1)+K}$

$$Y = Y_R + Y_D = \frac{K}{s^2+s(1+KK_1)+K}\cdot\frac{1}{s} + \frac{s(s+1)}{s^2+s(1+KK_1)+K}\cdot\frac{1}{s}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$$y_{ss} = y(\infty) = \lim_{s\to 0} sY(s) = 1 + 0 = 1$$

$\frac{1}{2}$ $\frac{1}{2}$

(iv) For $T = T_R = Y/R \quad S_{T:K_1} = (K_1/T)\,\partial T/\partial K_1$

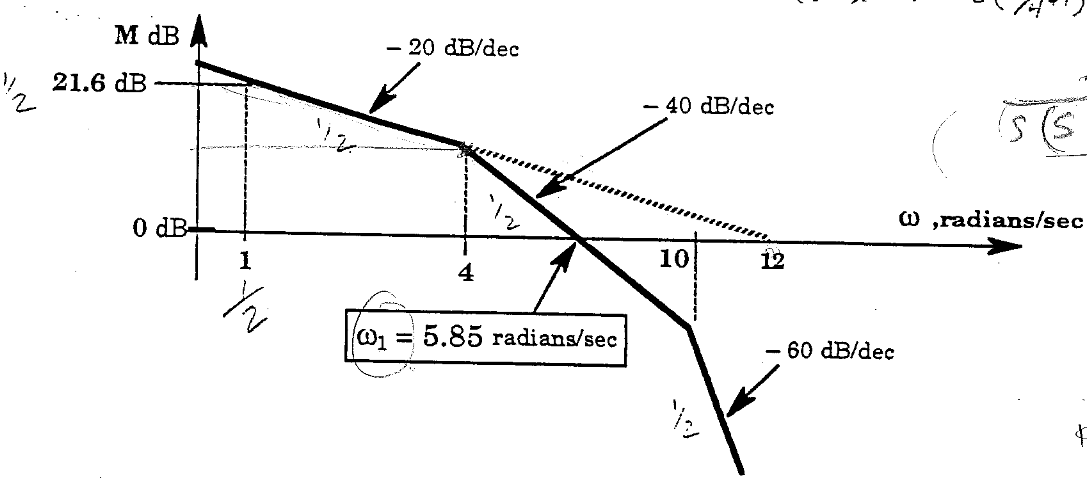
$$T = \frac{Y_R}{R} = \frac{K}{s^2+s(1+KK_1)+K} = \frac{N}{D}$$

$$S^T_{K_1} = \frac{K_1}{T}\frac{\partial T}{\partial K_1} = \frac{K_1 D}{N}\cdot\frac{0 - sK^2}{D^2} = \frac{-sKK_1}{s^2+s(1+KK_1)+K}$$

#4: (i) e_{ss} (unit ramp) $= 1/12$ corresponds to $K_v = 12 \Rightarrow K_b = K/40 \;\therefore\; K = \underline{480}$ $\frac{1}{2}$

$\therefore K_b\,(\text{dB}) = 20 \log 12 \approx$ **21.6 dB**

$$\frac{480}{s(s+4)(s+10)} = \frac{(480/4\times10)}{s(s/4+1)(s/10+1)} = \frac{12}{s(\frac{s}{4}+1)(\frac{s}{10}+1)}$$



$$\frac{K}{s\frac{(s+4)}{4}\frac{(s+10)}{40}}$$

$\frac{K}{40} = 12 - 0$

$\boxed{K = 480}$

Kb = 20 log 12 = 21.6

(ii) The phase shift of $G(j\omega)$ at $\omega_1 = 5.85$ radians/sec is

$\phi(\omega_1) = -90° - \tan^{-1}(\omega_1/4) - \tan^{-1}(\omega_1/10) = -90° - 55°.64 - 30°.33$
$\quad 1 \qquad \frac{1}{2} \qquad \frac{1}{2}$
$\approx -176°$

ie Phase Margin $\phi_{pm} = 180° - 176° = \underline{4°}$ $\quad 1$

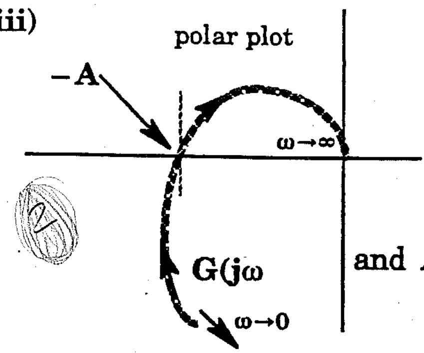
$GM = \dfrac{1}{|GH|}$

GM

$\angle\phi(\omega) = -90 - \tan^{-1}\frac{\omega}{4} - \tan^{-1}\left(\frac{\omega}{10}\right) = X$

$\phi_{PM} = 180 - X$

(iii)



$$G(s) = \frac{480}{s(s+4)(s+10)} = \frac{480}{s^3+14s^2+40s}$$

and $$G(j\omega) = \frac{480}{-14\omega^2 + j(40\omega - \omega^3)}$$

Setting $\text{Im}G(j\omega) = 0$ yields phase crossover $\omega_\pi = \sqrt{40} = 6.32$ radians/sec

and $A = 480/(14)(40) = 0.857$ or **GM** $= 1/0.857 \approx \underline{\mathbf{1.17}}$ (or $\underline{\mathbf{1.34}}$ **dB**)

½

$40\omega - \omega^2 = 0$
$\omega^2 = 40\omega$
$\omega^2 = 40$
$\boxed{\omega = 40}$

#5 : (A) (i) For the uncompensated system $\mathbf{K_{pu}} = 140/20 = \mathbf{7}$ ½

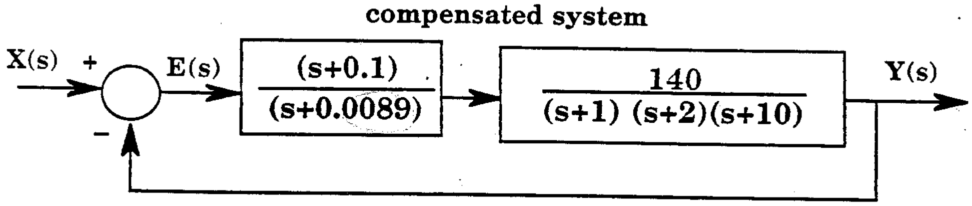
and $e_{ssu} = 1/(1+\mathbf{K_{pu}}) = 1/8 = 0.125$ ½

We require $e_{ssc} = e_{ssu}\,\mathbf{/10} = 1/80 = 1/(1+\mathbf{K_{pc}})$, where the new error-constant $\mathbf{K_{pc}} = (\mathbf{z_c}/\mathbf{p_c})\mathbf{K_{pu}} = \mathbf{7z_c/p_c}$. ½

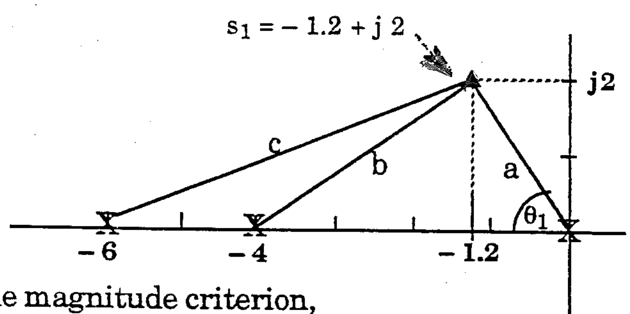
$\therefore$ $\mathbf{K_{pc}} = 80 - 1 = \mathbf{79 = 0.7/p_c}$ or $\mathbf{p_c} = 0.7/79 = \underline{\mathbf{0.0089}}$

compensated system

(ii)



(B) (i) The p-z diagram of the uncompensated system is shown with the dominant pole positions indicated by ▲ [Only the upper pole is shown here]



$\theta_1 = \tan^{-1}(2/1.2) = 59°.03$

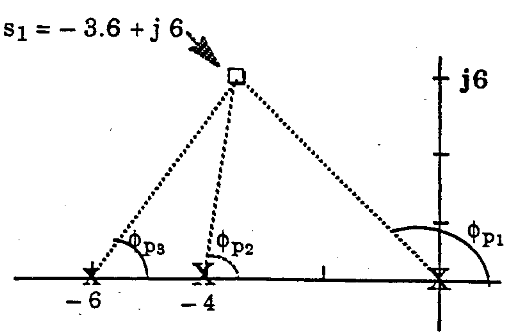
$\omega_n = 2.33$

From the magnitude criterion,

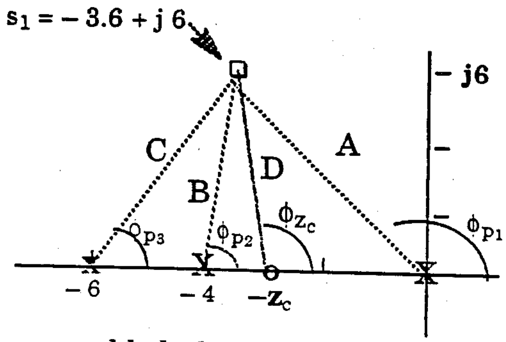
$\mathbf{K} = a.b.c = \sqrt{(1.2^2+2^2)}\,.\sqrt{(2.8^2+2^2)}\,.\sqrt{(4.8^2+2^2)} = (2.3324)\,(3.4409)\,(5.2) \approx \underline{\mathbf{41.73}}$

The damping ratio $\zeta = \cos 59°.03 = \underline{\mathbf{0.515}}$, Settling time $\mathbf{T_s} = 4/1.2 = \underline{\mathbf{3.33}}$ sec

(ii) To reduce $\mathbf{T_s}$ by a factor of **3** while keeping ζ unchanged, we need to have dominant poles at $s_1 = -(4/1.11) \pm j\,(4/1.11)\tan 59°.03 = \mathbf{-3.6 \pm j6}$ ½



Referring to the p-z diagram above, the angle $\angle G(s_1) = -[\ \phi_1 + \phi_2 + \phi_3]$ where ϕ_1, ϕ_2 and ϕ_3 are the angles shown above.
From the diagram, $\phi_1 = 180° - \tan^{-1}(6/3.6) = 121°$, $\phi_2 = \tan^{-1}(6/0.4) = 86°.2$ and $\phi_3 = \tan^{-1}(6/2.4) = 68°.2$ and $\therefore \angle G(s_1) = -275°.4 \neq -180°$
Therefore, the excess (positive) angle ϕ_z to be supplied by the compensator must be $275°.4 - 180° = \mathbf{95°.4}$. For a PD compensator, this excess angle is supplied by a suitably located zero (at $s = -\mathbf{z_c}$).In this case, $\mathbf{z_c}$ can be found from the equation: $\tan(180° - 95°.4) = 6/(3.6 - \mathbf{z_c})$ or $\mathbf{z_c} = 3.6 - 0.567 = \underline{\mathbf{3.03}}$
The p-z diagram of the compensated system is shown below:

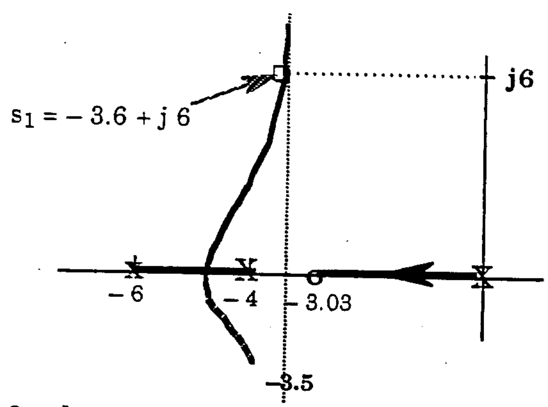


With the compensator added the system gain at $s = s_1$ is found from the magnitude criterion as $\mathbf{K'} = \mathbf{KK_c} = ABC/D$:

$$\mathbf{K'} = \sqrt{(3.6^2 + 6^2)}\,.\sqrt{(0.4^2+6^2)}\,.\sqrt{(2.4^2+6^2)}\,/\sqrt{(0.57^2+6^2)}$$
$$= (6.997)\,(6.013)\,(6.462)\,/\,6.027 \approx 271.9\,/6.027 = \mathbf{45.11} = \mathbf{41.73K_c}$$

The compensator gain is therefore $\mathbf{K_c} = 45.11/41.73 \approx \underline{\mathbf{1.08}}$

(iii) The root-locus of the compensated system is sketched below:
$n - m = 2$ ....$\pm 90°$ asymptotes at $\sigma = (-10 + 3.03)/2 \approx -3.5$



Comment:

For $\mathbf{K} = 271.9$, the complex roots are at $-3.6 \pm j6$.
From the "product rule " ('Grant's Rule), $\sum$ OLTF poles = $\sum$ CLTF poles, the location $s = -r_3$ of the third CLTF pole can be found:

$$-6 - 4 - 0 = -3.6 - 3.6 - r_3 \text{ and } r_3 = \mathbf{2.8}$$

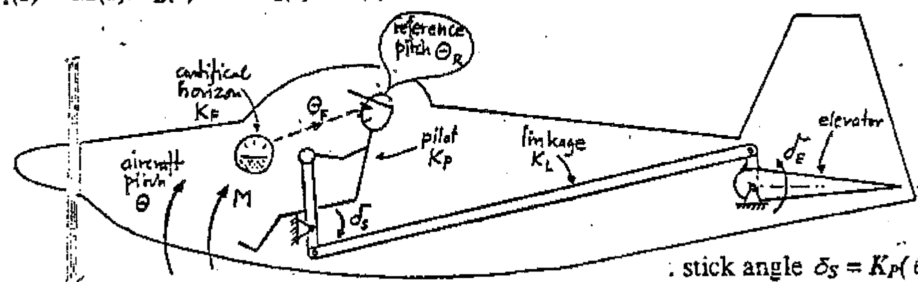
Now, the OLTF zero at $s = -3.03$ is also a CLTF zero and the presence of the CLTF pole at $s = -2.8$ provides an approximate p-z cancellation {(ie the residue for the (s+2.8) pole term in the inverse Laplace transform of Y(s) is likely to be small} Thus the pole-pair at $-3.6 \pm j6$ is likely to be dominant.

zzzzzzzzzzzzzzzzzzzzzzzzz

.96

PROBLEM #1 (10 Points)

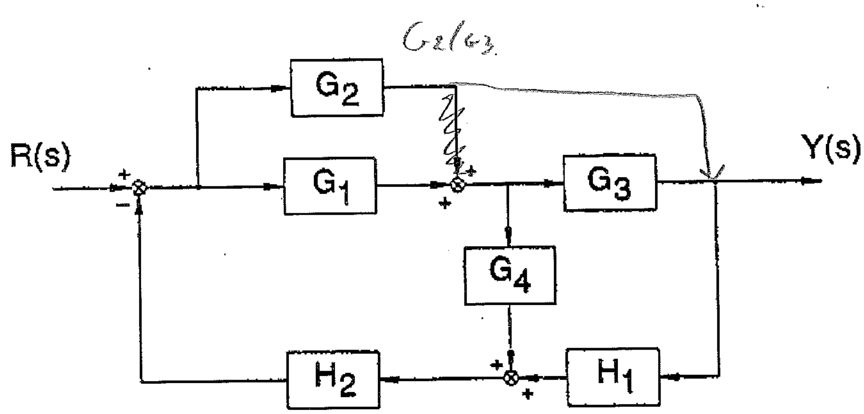
(A) Shown below is the schematic representation for the pitch control of an airplane. The pilot, modeled by the gain K_P, responds with the control stick angle δ_S to the discrepancy between the reference pitch angle θ_R and the indicated pitch angle θ_F. The angle θ_F is indicated by the artificial horizon instrument (gain K_F) which measures the aircraft pitch angle θ. The stick angle δ_S is transformed into the elevator angle δ_E by the control linkage with gain K_L, thus generating the pitching moment M, which in turn effects the pitch angle θ. The aircraft pitch dynamics are represented by the transfer functions:
$G_1(s) = M(s)/\delta_E(s)$ and $G_2(s) = \theta(s)/M(s)$.



- stick angle $\delta_S = K_P(\theta_R - \theta_F)$
- elevator angle $\delta_E = K_L \cdot \delta_S$
- pitching moment $M = G_1 \cdot \delta_E$
- aircraft pitch $\theta = G_2 \cdot M$
- indicated pitch $\theta_F = K_F \cdot \theta$

a) Draw the block diagram or the signal flow graph of the system
b) Determine the transfer function $T(s) = \theta(s)/\theta_R(s)$

(B) For the system defined by the block diagram below, determine: the closed-loop transfer function $T(s) = Y(s)/R(s)$



PROBLEM #2 (11 Points)

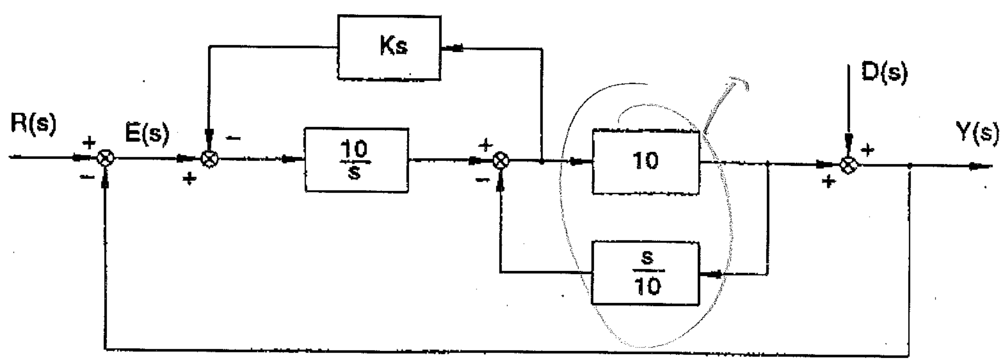
The open-loop transfer function of a unity feedback system is given by:

$$G(s) = \frac{K(s+5)}{s(s+1)(s+2)} \qquad \text{where } K > 0$$

a) Plot the root locus of the closed-loop system for $K > 0$, showing all relevant steps clearly.
b) Use the Routh-Hurwitz criterion to find the range of K for which the closed-loop system is stable.

PROBLEM #3 (10 Points)

The block diagram of a feedback control system is shown below, where *R(s)* is the input signal, *Y(s)* is the output signal, and *D(s)* is a disturbance signal.



a) What is the type number of the system? Show all relevant procedure steps.
b) Assume $D(s) = 0$. Using the definition of the type number of a system, determine the gain K such that for a unit-ramp input, the steady-state error $e_{ss} = 0.1$ is achieved.
[Note: e_{ss} is the steady-state value of signal *E(s)*]
c) Determine the steady-state output $y_{ss}(t)$, $t \to \infty$:, for a simultaneous unit-step input and unit-step disturbance input. Use the gain value of K determined from part (b).
d) Determine the sensitivity of the transfer function $T(s) = Y(s)/R(s)$ with respect to gain K.
ie: derive S_K^T.

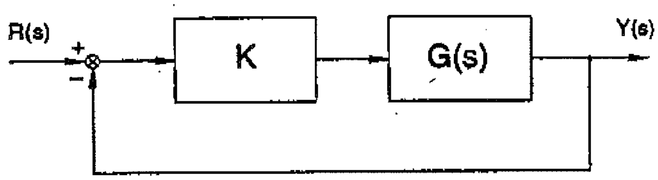
PROBLEM #4 (12 Points)

(A) A closed-loop system with unity feedback has an open-loop transfer function

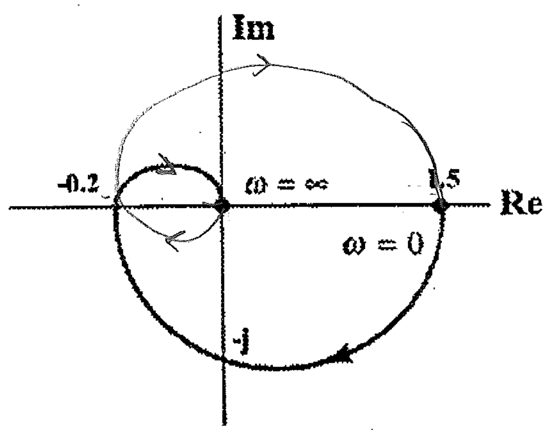
$$G(s) = \frac{K(s+1)}{s^2(s+10)}$$

a) Sketch the Bode magnitude diagram for the frequency range $0.1 < \omega < 100$, assuming $K = 1$.
b) Determine the gain K so that the (gain) crossover frequency of $G(s)$ becomes 2 rad/sec.
c) For the gain selected in part (b), determine the phase margin of the system.

(B) In the following unity feedback system, the plant $G(s)$ is stable, and gain $K > 0$.



For $K = 1$, the polar plot of $G(s)$, for $\omega > 0$, is depicted in the following figure.



Using the Nyquist criterion, determine:

a) The gain margin of the system.
b) The range of K which results in a stable closed-loop system.

PROBLEM #5 (12 Points)

A unity feedback system has the forward-path transfer function:

$$G(s) = \frac{K_P}{(s+0.5)(s+2)} \qquad \text{where } K_P = 19$$

To improve performance of the system, it is proposed to use a cascade phase-lead compensator with the transfer function:

$$G_C(s) = \frac{K_C(s+6)}{s+p_C}$$

in order to obtain dominant closed-loop poles at -2±j3.3.

Determine:

a) The compensator pole p_C and the compensator gain K_C (Hint: overall gain $K = K_C K_P$).
b) Provide a rough sketch of the root locus for the compensated system.
c) Sketch the complete block diagram of the compensated system.

done

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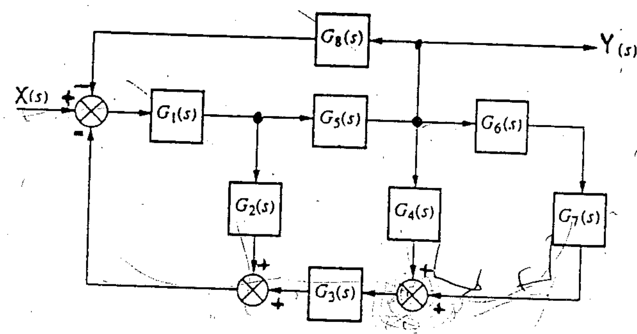
CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
ENGR 372/4 SEC X FUNDAMENTALS OF CONTROL SYSTEMS

CLASS TEST **Instructor: Dr. J. Svoboda** **Date: 7 March 2000**

Problem 1 For the system shown, find the following:
(a) signal flow graph representation
(b) transfer function T(s) = Y(s)/X(s)
(Use **either** block diagram algebra or Mason's rule)



flow graph ?

Problem 2 A closed-loop system is described by a transfer function:

3rd order

$$T(s) = \frac{6}{(s+12)(s^2+2s+3)}$$

(a) Plot the system's poles and obtain a second order approximation to the system. Justify the approximation.
(b) For the second order system obtained in (a) find the following:
Damping ratio ζ, undamped natural frequency ω_n and steady state gain K
(c) Sketch systems unit step response, with all relevant parameters (T_p, % OS, T_s) calculated and indicated in the plot.

Problem 3 The open-loop transfer function G(s) of a unity feedback control system is:

$g(s) = 1+G/1$

$$G(s) = \frac{12K}{s(s+4)(s+6)}$$

$g(s) = 1+G/1$

$g(s) = 1 + \frac{12K}{s(s+4)(s+6)}$

(a) Using Routh-Hurwitz criterion, determine the range of K for which the closed-loop system will be stable
(b) For K=10, find the steady-state error e_{SS} in response to a ramp input of amplitude 60.

5.2

①

068

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
ENGR 372/4 sec. Y FUNDAMENTALS OF CONTROL SYSTEMS

Mid-Term | Instructor: Dr. H. Hong | Date: March 9, 2000

Answer all three (3) questions. Closed book exam. Total time: 1 hour 15 minutes

Problem 1.→23; Problem 2.→7; Problem 3.→5; = 35 marks total.

Problem 1. [a→1, b→7, c→3, d→2, e→1, f→3, g→3, h→3 = 23 marks]

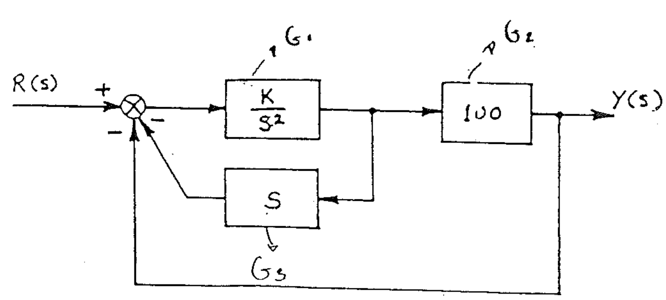
A negative feedback control system has a block diagram as shown below.

Ch.5

(a) Determine the closed-loop transfer function T(s) = Y(s)/R(s).
(b) For a unit step input, find the value for the gain K so that the system has 20% overshoot.
What is the natural frequency ω_n of the system?
What is the settling time Ts of the system?
(c) Sketch the closed-loop poles on the s-plane. Clearly indicate ω_n (natural frequency) and θ (angular representation of the damping ratio ζ), and their numerical values, on the figure.
(d) Using the Routh-Hurwitz stability criterion determine the range of gain K for the system to be stable?

?

(e) What is the "type number" of the system?
(f) For a unit step input, determine the position error constant Kp, and the steady-state error. Show all mathematical steps leading to your answer.
(g) For a unit ramp input, determine the velocity error constant Kv, and the steady-state error. Show all mathematical steps leading to your answer.
(h) For a unit acceleration input, determine the acceleration error constant Ka, and the steady-state error. Show all mathematical steps leading to your answer.



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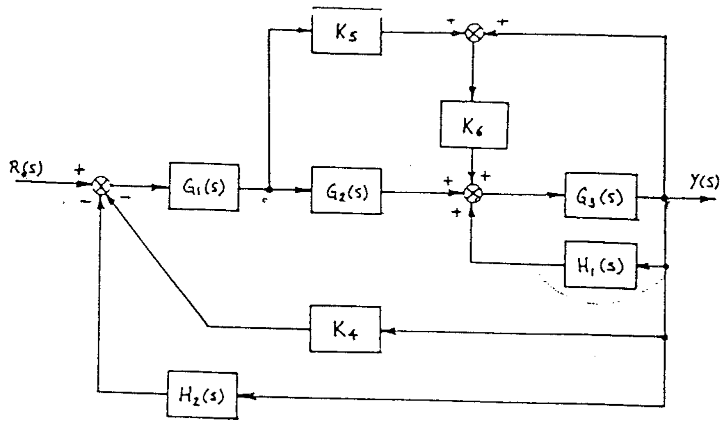
Problem 2. [a→1½, b→5½ = 7 marks]

For the system represented in the block diagram below, find:

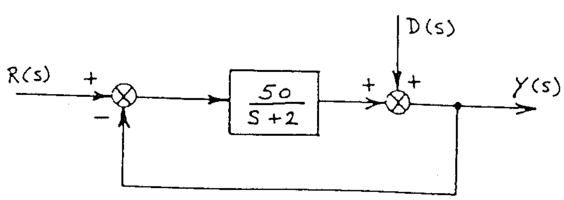
(a) the signal flow graph representation.
(b) the transfer function $T(s) = Y(s)/R(s)$, by using (only) Mason's signal-flow gain formula.

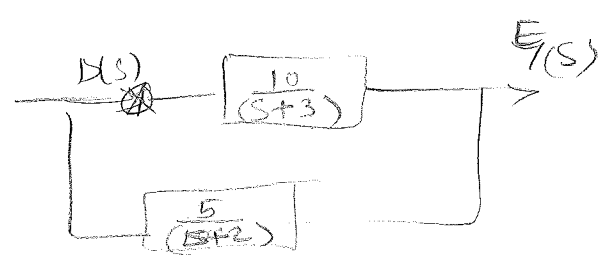


Problem 3. [5 marks]

For the system shown below find the total steady-state error due to a unit step input R(s) and a unit step disturbance D(s).

$E(s) = R(s) - Y(s)$





ENGR 372 Section W, 1999 - PRACTICE PROBLEM SET #2

#1. Draw the root locus of the closed loop systems whose OLTFs are as follows: [Give all construction steps systematically].Also comment on the stability of each system.

(a) $G(s) = K(s+3)(s+5)/(s^2+2s+3)$ (b) $GH(s) = K(s+6)/s(s+3)$

(c) $GH(s) = K/(s-4)(s+8)$ (d) $G(s) = K/s(s^2+4s+5)$

#2. The OLTF of a unity feedback system is given by : $G(s)= K(s+5)/(s^2+\alpha s+6)$, the nominal value of α being 4.

(a) If K=12, what are the resulting closed loop system performance specs ?

(b) Environmental changes can produce an increase in α from its nominal value and it is desired to observe the resulting change in CLTF root locations. Draw the root locus for increase in α from 0 to ∞.

(c) For $\alpha = 4.5$, determine the value of K required to obtain a settling time of 2.5 sec. What is the corresponding P.O and steadystate step error ?

✓#3. A unity feedback system has the OLTF $G(s)= K/s(s+4)(s+8)$. It is desired to have a steadystate ramp error of 8.7%. Determine whether this is possible and if so, the value of K needed. What is the best ramp error achievable ?

#4. Sketch the polar frequency- response plot for the following OLTFs using the asymptotic tendencies : $s \to 0$ and $s \to \infty$ (ie $\omega \to 0$ and $\omega \to \infty$):

(a) $GH(s)= K(s+4)/(s^2+5s+2)$ (b) $G(s)= 15(s+8)/(s+12)(s^2+5s+2)$

(c) $GH(s)= K/s(s^2+3s+25)$ (d) $G(s)= 6(s+7)/s^2(s^3+2s^2+4s+7)$

#5. For the transfer function of Problem#4(c), analytically determine the value of the phase- crossover frequency ω_π if $K = 50$.Hence find the GM. Use the asymptotic bode magnitude plot to find the gain-crossover frequency ω_1 and hence analytically determine the PM.

#6. Sketch the asymptotic bode plots (magnitude & phase) for the following OLTFs : (a) $GH(s)= 12.5(s+6)/(s+10)(s+16)$ (b) $G(s)= 16/s(4s+15)$

#7. From the asymptotic plots of the transfer functions of #6, find the error coefficients for the two systems.

#8 From the asymptotic plots of the transfer function of #6(b), find the PM & GM

#9. Investigate the stability of the systems with the following OLTFs using the Nyquist Stability Criterion. Check your result using a general root locus for each of the systems:

(a) $GH(s) = K/(s+2)(s+4)$ (b) $G(s)= K/(s^3+2s^2+3s+4)$

(c) $G(s) = K/(s+4)(s-5)$ (d) $GH(s)= K/s(s^2+2s+3)$

#10. A unity feedback control system is operating with a closed-loop magnitude ratio of 14 dB and phase shift of $-160°$ at a certain frequency Find the corresponding value of the OLTF $G(s)$ and hence determine the GM and PM of the system.

#11. An open-loop-unstable system has the OLTF $GH(s) = K/(s+b)(s-4)$.Show graphically (by using root locus) that the values of K and b can be chosen to stabilize the closed loop system. Determine values for K and b which will result in a system having a 2% settling time of 1.6 sec and an overshoot of 15% and state the resulting steadystate error performance.

T_s

#12. A UFS has the OLTF $G(s) = K/(s+1)(s+3)(s+9)$. For $K = 80$, the roots of the CE are at $-1.4 \pm j\,2.9$, -10.2 .Give the approximate performance specs of the closed loop system. why?.

#13. The steadystate error performance of a system may be improved by applying either cascade PI compensation or phase-lag compensation. PI compensation improves e_{ss} by increasing the system Type Number , whereas lag compensation does the same by increasing the error constant without appreciably affecting the dynamic performance. Design a cascade PI compensator for the system of Problem #12, to reduce the step error to zero with the the restriction that the maximum angle added by the compensator should not exceed $2°$. [**Please see Q #5 in '95 & '97 Exams for examples of phase-lag compensation of a similar system.** The dynamic performance specifications of a system may be improved by applying either cascade PD compensation or phase- lead compensation PD compensation introduces a zero to the left of the dominant poles, bending the root locus to the left.Lead compensation does the same by introducing a pole-zero pair.**Please see Q #5 in '96 & '98 Exams for examples of PD and phase-lead compensation, rrespectively**]

#14. A UFS has the OLTF $G(s)= K/s(s+10)(s+15)$. Determine K if the dominant poles must have a damping ratio of 0.63. What is the resulting value of T_s ? If a compensator with the transfer function $H(s)= (s+13)$ is introduced in the feedback path and the forward path gain K is maintained, re-draw the root locus and approximately locate the positions of the complex roots of the CE.

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<<<< Answers will be available at the Copy Centre
from around the 15th April 1999 >>>>

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W F99

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COURSE: **FUNDAMENTALS OF CONTROL SYSTEMS**	NUMBER: **ENGR 372/4**	SECTION: **U,X,W**

EXAMINATION: **FINAL EXAM**	DATE: **APRIL 29 , 1999**	TIME: **9: 30-12: 30**	# OF PAGES: **3**

INSTRUCTOR: **C.Rajalingham, J.V.Svoboda, N.Suresh**

MATERIALS ALLOWED: ☒ NO ☐ YES (PLEASE SPECIFY)

CALCULATORS ALLOWED: ☐ NO ☒ YES

ONLY non-programmable calculators will be allowed.

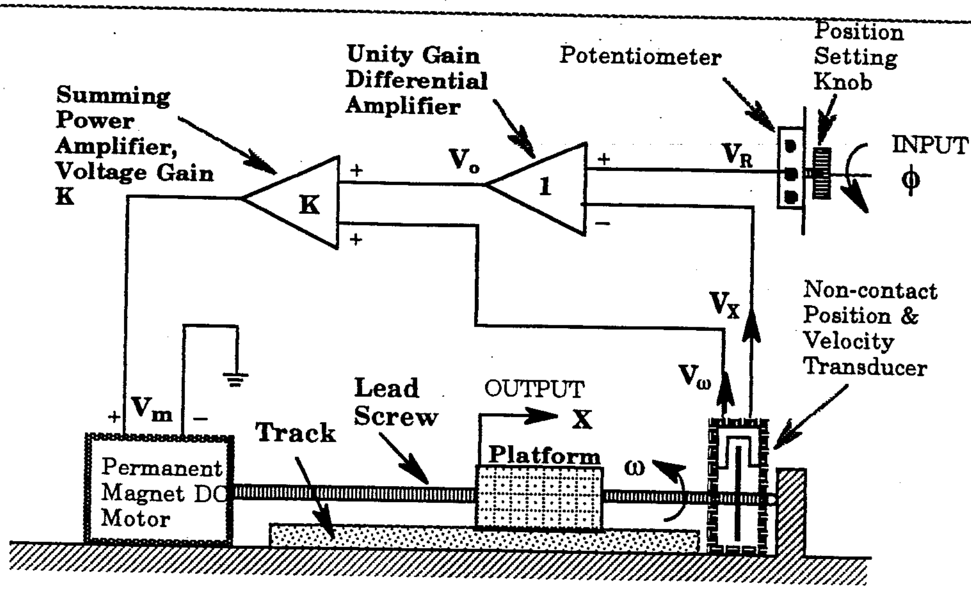
SPECIAL INSTRUCTIONS:

Attempt all problems . All solution steps must be shown clearly. Identify your final answers clearly.

PROBLEM # 1 [10 Points]

(A) The functional diagram of a linear position control system is shown below.The platform position X along the linear track is attained by the rotation of the lead-screw .This rotation (angular velocity ω) is provided by the permanent-magnet DC motor. A sensor module attached to the lead-screw at the far end produces voltages V_x and V_ω, proportional to X and ω , respectively.The sensor outputs are used to provide positional and velocity feedback. Op-amp circuits are used for the error-detector and for the power-amplifier needed to drive the motor, and the control signal is a voltage obtained from a rotary input potentiometer. The relationships between all the system components are given in functional form below.

Amplifier Equations	Transducer Equations	Motor & Leadscrew Transfer Functions
$V_o(s) = V_R(s) - V_X(s)$ $V_m(s) = K\,[\,V_o(s) + V_\omega(s)\,]$	$V_R(s) = K_1\phi(s)$ $V_X(s) = K_2X(s)$ $V_\omega(s) = -K_3\,\omega(s)$	$G_m(s) = \dfrac{\omega(s)}{V_m} = \dfrac{K_m}{1+\frac{s}{60}}$ $G_s(s) = \dfrac{X(s)}{\omega(s)} = \dfrac{K_s}{s}$
$K = 10$ $\quad K_1 = K_2 = 1$	$K_3 = 0.8$ $\quad K_s = 100$	$K_m = 0.2$



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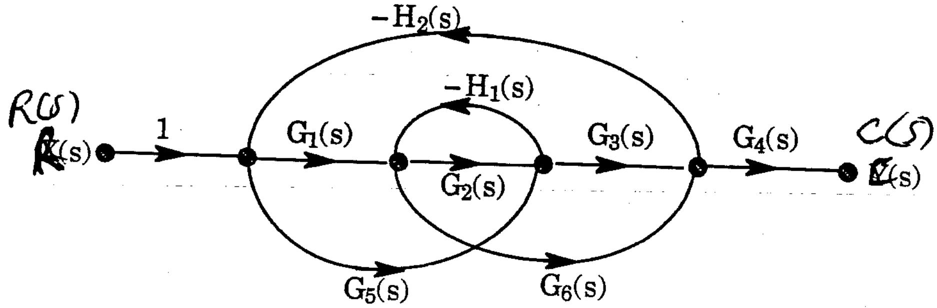
4 (i) Draw a block diagram of the position control system, showing all variables and transfer functions clearly

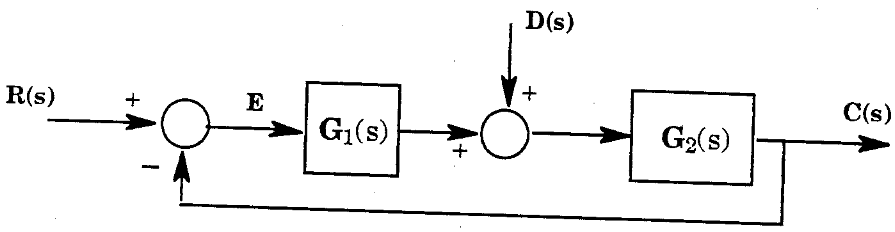
2 (ii) Obtain the transfer function $T(s) = X(s) / \phi(s)$ of the system.

4 **(B)** Determine the transfer function $T(s) = C(s) / R(s)$ of the system whose signal flow graph is shown below using Mason's gain rule.



PROBLEM # 2 [10 Points]

The block diagram of a unity-feedback control system is shown below.



$$G_1(s) = \frac{2}{s+\alpha}, \quad \alpha = 1, \quad G_2(s) = \frac{2}{s}$$

3 (i) Assuming a unit step input $\mathbf{r(t) = u(t)}$ and disturbance $\mathbf{d(t) = 0}$, determine the following: natural resonant frequency ω_n, the damping ratio ζ, the percent overshoot (%OS), and the settling time T_s.

2 (ii) Assuming an input $\mathbf{r(t) = (1 + 4t)u(t)}$ and disturbance $\mathbf{d(t) = 0}$, determine the steadystate error e_{ss}.

(iii) Assuming a unit step input $\mathbf{r(t) = u(t)}$ and a unit step disturbance input $\mathbf{d(t) = u(t)}$ determine the following:

(a) the value of the output $C(s)$ for the closed loop system, and

(b) the value of the output $C(s)$ for the open loop system (ie with the feedback interrupted)

3 (iv) Determine the sensitivity of $\mathbf{T(s) = C(s) / R(s)}$ with respect to α, ie S_α^T

PROBLEM # 3 [12 Points]

(A) The open loop transfer function of a unity-feedback control system is given by:

$$G(s) = \frac{K}{s(s^2 + 6s + 13)}$$

4 (i) Using the Routh-Hurwitz stability criterion, determine the range of **K** for which the system will be stable.

2 (ii) Determine the frequency of oscillation when the system is marginally stable.

4 **(B)** A unity-feedback control system has the open loop transfer function : $G(s) = \dfrac{K(s+4)}{(s^2 + 2s + 2)}$

4 (i) Draw the root locus of the closed-loop system, showing all relevant steps and details.

2 (ii) Show that the points $s = -\frac{3}{2} \pm j\frac{\sqrt{15}}{2}$ lie on the locus and determine the gain at these points.

W F98 29

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COURSE: FUNDAMENTALS OF CONTROL SYSTEMS	NUMBER: ENGR 372/4	SECTION: U,W,X
EXAMINATION: FINAL EXAM	DATE: APRIL 24 , 1998	TIME: 9:30-12:30 — # OF PAGES: 3
INSTRUCTOR: A.K.Elhakeem, J.V.Svoboda, & N.Suresh		

MATERIALS ALLOWED: ☐ NO ☐ YES (PLEASE SPECIFY)

CALCULATORS ALLOWED: ☐ NO ☐ YES

ONLY non-programmable calculators will be allowed.

SPECIAL INSTRUCTIONS:

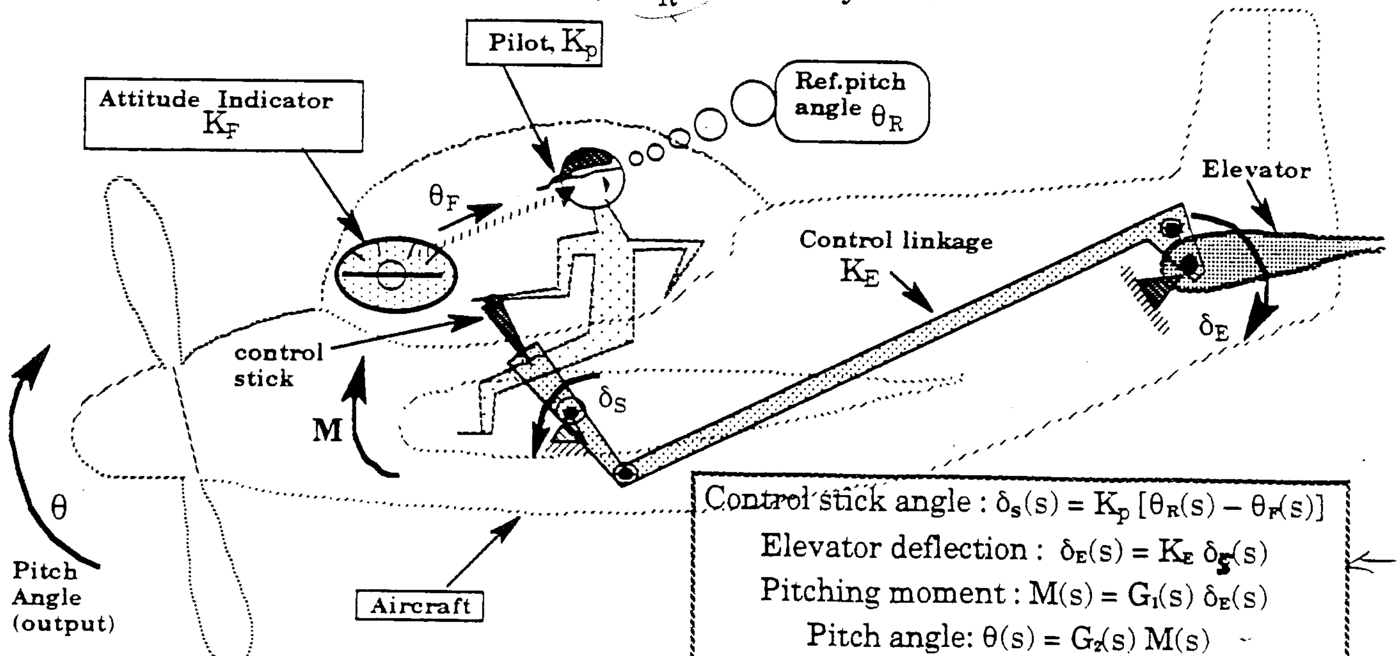
Attempt all problems . All solution steps must be shown clearly. Identify your final answers clearly.

PROBLEM # 1 [10 Points]

(A) A greatly simplified schematic describing the manual control of the pitch attitude of a small aircraft is shown below. The pilot , modelled as a zero- order system with gain K_p, visually compares the desired pitch angle θ_R with the indicated pitch angle θ_F and manually varies the control-stick angle δ_S in the direction required to reduce the attitude error. The control linkage , represented by gain K_E, transforms the stick angle into the elevator- deflection-angle δ_E thus generating a pitching moment M which causes a change in the actual pitch angle θ of the aircraft . The angle θ is measured by the attitude indicator (modelled as a zero-order instrument with gain K_F). The dynamics of the aircraft are modelled by the transfer functions $G_1(s)= M(s) / \delta_E(s)$ and $G_2(s)= \theta(s) / M(s)$. The relationships given above are also reproduced in functional form in the box below the figure.

5 (i) Draw a block diagram or signal flow graph of the pitch attitude control system

2 (ii) Obtain the transfer function $T(s) = \theta(s) / \theta_R(s)$ of the system.



Control stick angle : $\delta_s(s) = K_p\,[\theta_R(s) - \theta_F(s)]$

Elevator deflection : $\delta_E(s) = K_E\,\delta_S(s)$

Pitching moment : $M(s) = G_1(s)\,\delta_E(s)$

Pitch angle: $\theta(s) = G_2(s)\,M(s)$

Indicated pitch angle : $\theta_F(s) = K_F\,\theta(s)$

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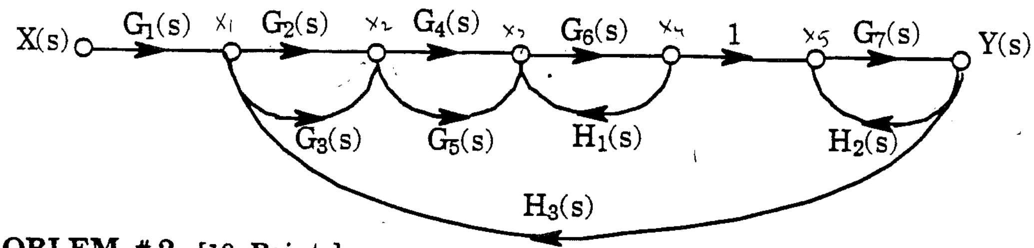
(B) For the system whose signal flow graph is shown below, determine:

2 (i) the corresponding block diagram

3 (ii) the closed loop transfer function $T(s) = Y(s)/X(s)$



PROBLEM # 2 [10 Points]

(A) A certain second-order control system is to be designed with the following performance specifications: The percent overshoot must be between **15%** and **25%** and the settling time $\mathbf{T_s}$ must be less than **0.8** sec.

2 (i) Identify the s-plane region where the closed-loop system poles must be located in order to realize the given specifications.

2 (ii) Obtain the transfer function $T(s)$ for the system if the root locus gain **K** = **1**, the percent overshoot = **20%** and $\mathbf{T_s = 0.6}$ sec.

(B) A unity feedback control system has the open-loop transfer function $\mathbf{G(s) = K/(s+\tau)}$.

3 (i) Find the sensitivity of the open loop and closed loop systems to **K**, ie S_K^G and S_K^T

3 (ii) For $\tau = 2$, find the value of **K** required if the closed loop system must have a steady state error of **0.05** to a unit step input.

PROBLEM # 3 [13 Points]

(A) The open loop transfer function of a control system is given by $\mathbf{GH(s) = K/s(s+1)(s+3)}$

4 (i) Draw the root locus diagram for the system, *showing all relevant construction steps and reference points clearly.*

3 (ii) Find the gain **K** if the closed loop poles are located at $\mathbf{s = -3.7}$ and $\mathbf{-0.15 \pm j1.37}$. Also determine the damping ratio ζ of the dominant complex pole pair at this gain.

(B) The open loop transfer function of a feedback control system is $\mathbf{GH(s) = \dfrac{K\,(s+4)}{s\,(s+1)\,(s+2)\,(s+3)}}$

4 (i) Use the Routh–Hurwitz criterion to find the range of positive **K** for which the closed loop system will be stable.

2 (ii) Find the frequency ω_{osc} at which the closed loop system will oscillate if the maximum value of **K** is used.

PROBLEM # 4 [12 Points]

(A) Experimental frequency response testing of the open loop transfer function $G(j\omega)$ of a unity feedback system yielded the magnitude and phase angle values given in the table below.It is also known that $G(j\omega)$ does not have any poles in the right-half of the s-plane

4 (i) Sketch the polar plot of $G(j\omega)$ and determine whether the closed loop system is stable using the Nyquist criterion.

3 (ii) Determine approximate values of the gain margin (in dB) and the phase margin (in degees) using the data given in the table.

ω rad/s	|G(jω)| NOT in dB	∠G(jω)
0.12	9.64	− 97°
0.33	3.22	− 110°
0.65	1.45	− 128°
0.98	1.02	− 143°
1.76	0.78	− 156°
2.49	0.66	− 179°
6.78	0.32	− 227°
10	0.01	− 268°

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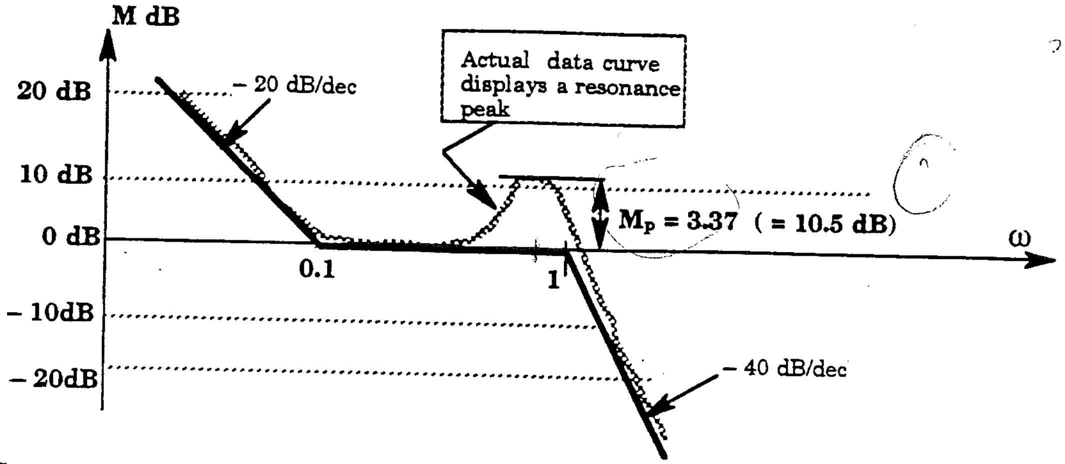
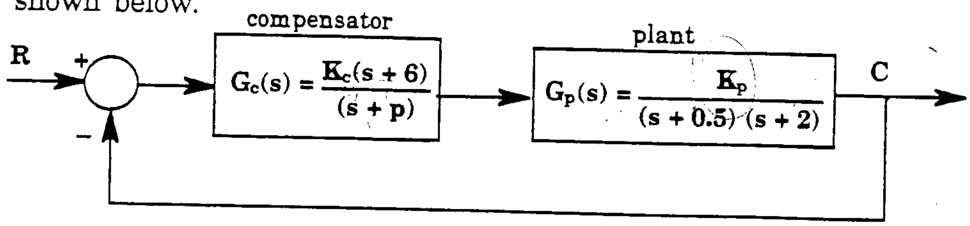
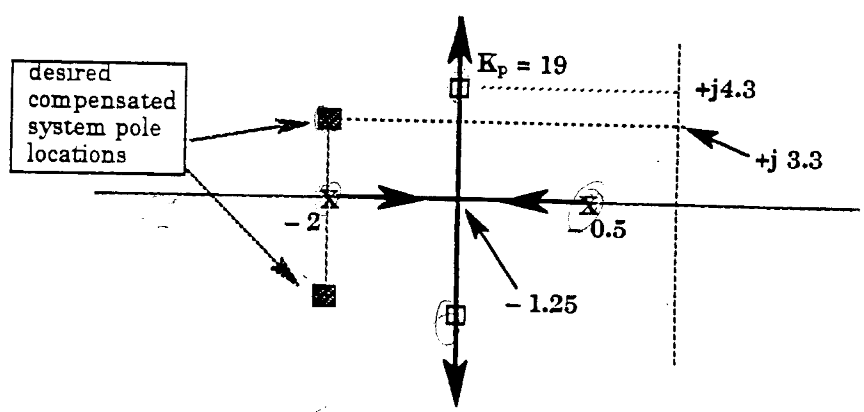
5 **(B)** Shown below is the asymptotic Bode magnitude frequency response plot of a unity-feedback open loop transfer function **G(s)** which was obtained from experimental data. Determine the transfer function **G(s)**.
{ Note: Please consider the relation $M_p = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$,which gives the magnitude of the resonance peak M_p, in your modelling of **G(s)** }



PROBLEM # 5 [10 Points]

A second-order control system and its uncompensated root locus are shown below. Without compensation [$G_c(s) = 1$], and for $K_p = 19$, the closed loop poles are located at $-1.25 \pm j4.3$ as shown which result in undesirable performance. It is proposed to improve performance by using a cascade phase-lead compensator, with the transfer function $G_c(s) = K_c\,(s+6)/(s+p)$
The new specifications require the dominant second order poles to be located at $s = -2 \pm j\,3.3$ which are also shown below.





6 (i) Determine the the compensator gain K_c and the compensator pole location **p**
[Note that the compensator zero has been pre-determined at $s = -6$. Also note that the root locus gain of the compensated system at the desired pole locations will be equal to K_cK_p , ie = $19K_c$]

4 (ii) Compare the uncompensated and compensated system steady state error to a unit step input.

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COURSE	NUMBER	SECTION
FUNDAMENTALS OF CONTROL SYSTEMS	**ENGR 372/4**	**U,W,X**

EXAMINATION	DATE	TIME	# OF PAGES
FINAL EXAM	**May 5th 1997**	**9:30 - 12:30**	**4**

INSTRUCTOR
K.Khorasani , J.V.Svoboda , N.Suresh

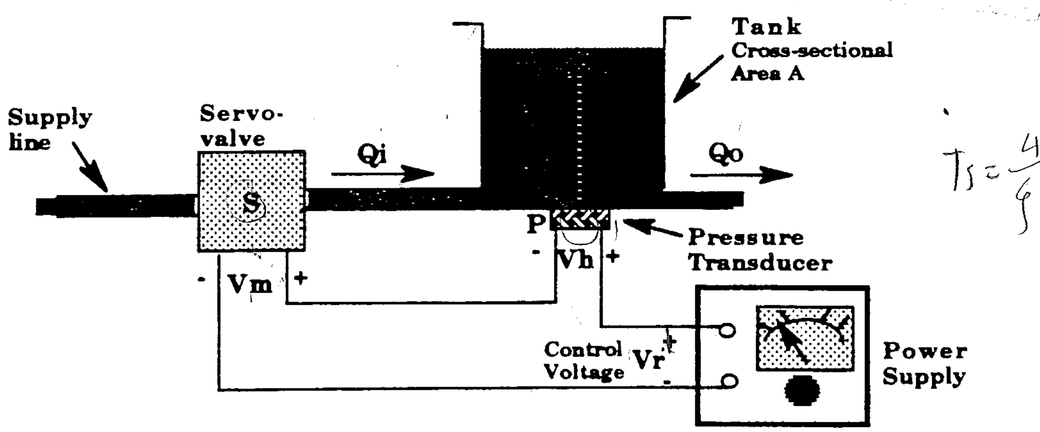
MATERIALS ALLOWED: ☒ NO ☐ YES (PLEASE SPECIFY)

CALCULATORS ALLOWED: ☐ NO ☒ YES (Standard Issue Only)

SPECIAL INSTRUCTIONS:
Attempt all problems. All solution steps are to be shown clearly
Identify your final answers clearly

Problem # 1 [11 Points]

(A) A simplified schematic of a liquid- level control system is shown below.The cylindrical open-topped tank has an inlet and an outlet. Its internal cross sectional area is **A** ,m^2 .A pressure transducer P, mounted at the bottom of the tank , functions as the level sensor with a sensitivity K_h ,volts/m. and negligible response time. The transducer output voltage V_h is connected in series with a control input voltage V_r such that the difference $V_m = V_r - V_h$ appears as the input to a servo-valve S which controls the inlet flow Q_i The transfer function of **S** is $G_s(s) = Q_i(s) / V_m(s) = K_q / 1 + s\tau)$,m^3/sec per volt. The inlet-outlet differential flow results in the level change ie in the time-domain, the level **h** is governed by the eqn. $Q_i(t) - Q_o(t) = d(Ah)/dt$



$T_s = \frac{4}{\zeta \omega_n}$

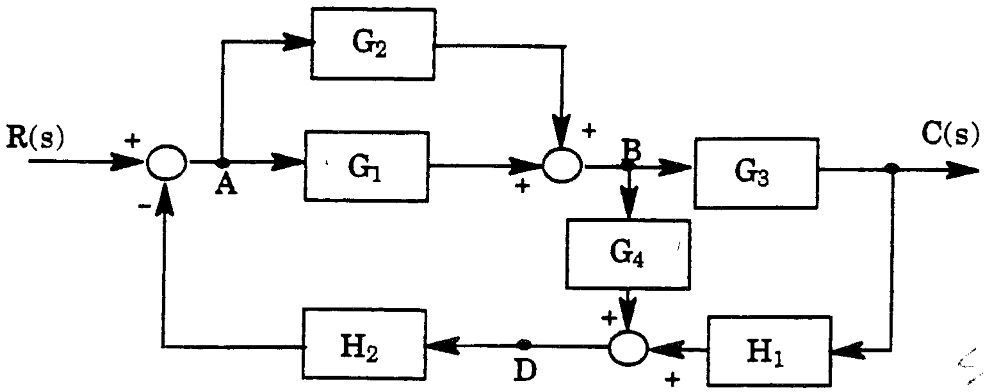
(a) Draw a block diagram of the system in the s -domain showing all transfer functions and variables.[Hint: $V_r(s)$= system input, $h(s)$= system output, and $Q_o(s)$ = a disturbance input]

(b) Using clearly written equations, show that the transfer function of the closed loop system is $T(s) = h(s) / V_r(s) = K/(s^2 + as + b)$, where $K = K_q / A\tau$, $a = 1/\tau$ and $b = KK_h$

(c) If τ = 625 milliseconds, $A = 0.2\ m^2$, K_h= 3.2 volts/m and K_q= 0.064 m^3 /volt-sec , determine the damping ratio ζ and the 2% settling time to a step change in the control voltage setting.

Continued on Page 2

(B) Directly convert the block diagram shown into a signal flow graph and obtain its transfer function **T(s) =C(s) /R(s)** using Mason's rule.



Problem # 2 [11 Points]

(A) A unity feedback control system has the open-loop transfer function :

$$G(s) = \frac{K(2s+1)}{s(4s+1)(s+1)^2}$$

An input **r(t) = 1 + 5t** is applied to the system at time t=0. Determine the minimum value of **K** required to obtain a steady-state error **ess ≤ 0.1**

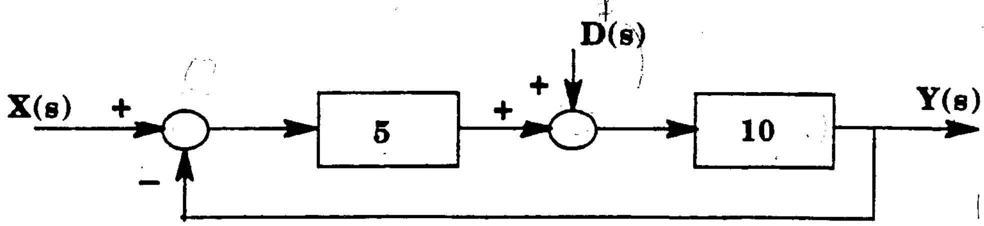
(B) A closed-loop system is described by the following transfer function :

$$T(s) = \frac{C(s)}{R(s)} = \frac{K}{(s^2+3s+3)(s+20)}$$

(i) Obtain a 2nd order approximation of the system and justify the approximation

(ii) For the approximated system from (i), find the damping ratio ζ, the undamped natural frequency ω_n, and the settling time T_s

(C) The closed-loop system shown below is affected by a step disturbance **D(s) = 20 / s**



$\frac{10}{1+50}$

Determine the steady-state effect of **D(s)** on the output **Y(s)**

(i) for the closed-loop system shown

(ii) for the open-loop system ie assuming that the feedback loop is interrupted

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Problem # 3 [11 Points]

(A) For a **unity feedback** system with the open-loop transfer function :

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

(i) Use the Routh-Hurwitz Criterion to find the range of **K** for which the closed-loop system will be stable

(ii) Find the value of **K** at which the system will oscillate, and also the frequency of oscillation ω .

(B) A **unity feedback** system has the open-loop transfer function :

$$G(s) = \frac{K(s+2)}{(s^2 + 2s + 1)}$$

(i) Draw the root locus of the closed-loop system showing all relevant steps clearly

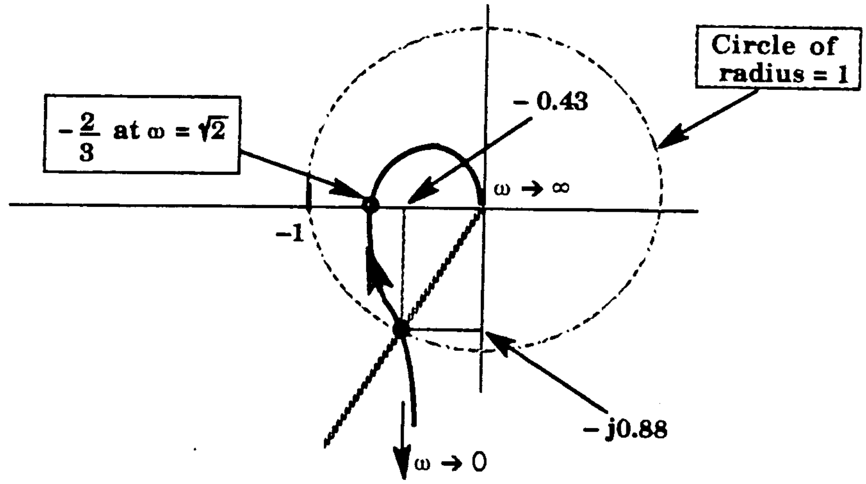
(ii) Using the angle criterion, show that the point $s = -1.5 + j\,1.32$ lies on the root locus.

(iii) Using the magnitude criterion, determine the gain **K** at $s = -1.5 + j\,1.32$

(iv) Determine undamped natural frequency ω_n with the gain **K** obtained in (iii) above.

Problem # 4 [11 Points]

(A) The polar plot of the open-loop transfer function **GH(s)= K / s(s+1) (s+2)** of a feedback control system is shown below, for a certain value of **K** :



(i) Determine the gain margin (GM in dB) and the phase margin (PM in degrees) using the data given on the plot.

(ii) Determine the value of **K** used in the plot

(iii) Using the Nyquist Criterion ,show (by mapping the relevant Nyquist contour on to *the* **GH** -plane) why the closed-loop system will be stable for $K < 6$.

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(B) (i) Obtain the asymptotic Bode **magnitude** plot for the transfer function:

$$G(s) = \frac{16(s^2+0.2s+4)}{s^2\,(s^2+0.04s+16)}$$

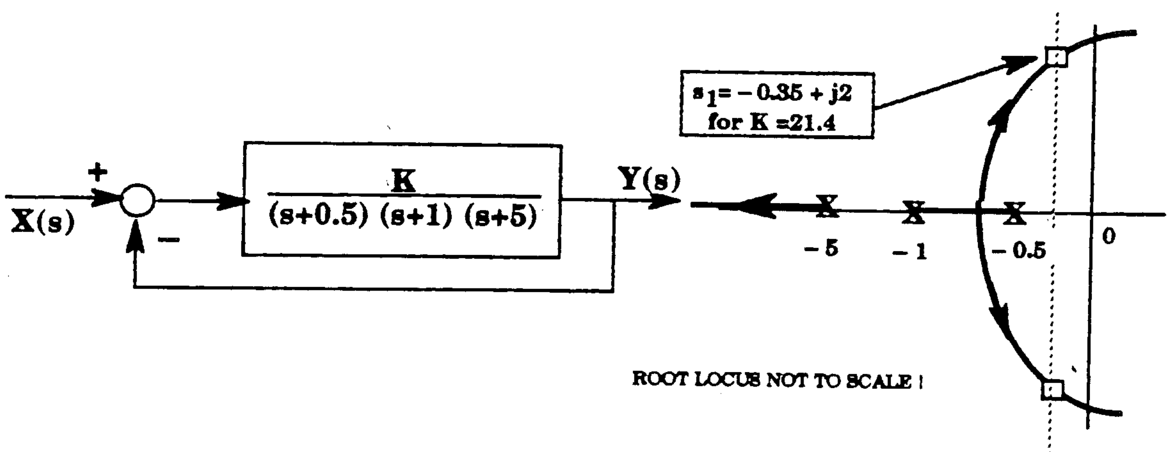
(ii) Determine the resonance peak **Mp (dB)** which occurs near $\omega = 4$ radians/sec.

Problem # 5 [11 Points]

Shown below are the block diagram and root locus of a unity feedback system.The desired location of the dominant closed-loop poles $s_{1,2} = -0.35 \pm j\,2$ is achieved by a gain adjustment of $K = 21.4$. It is proposed to use a phase-lag cascade compensator with the following transfer function,

$$G_c(s) = \frac{(s+0.1)}{(s+p_c)}$$

in order to reduce the unit-step steady-state error e_{ssu} of the uncompensated system by a factor of 5.



(A) Determine the unit-step steady-state error e_{ssu} of the uncompensated system

(B) Determine the value required for the compensator pole p_c to obtain the desired reduction in steady-state error

(C) Draw the block diagram of the compensated system

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