MATH 208—APRIL 2008

Solutions

Question 1

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{115200 - 224000}{16 - 0} = -6800$$

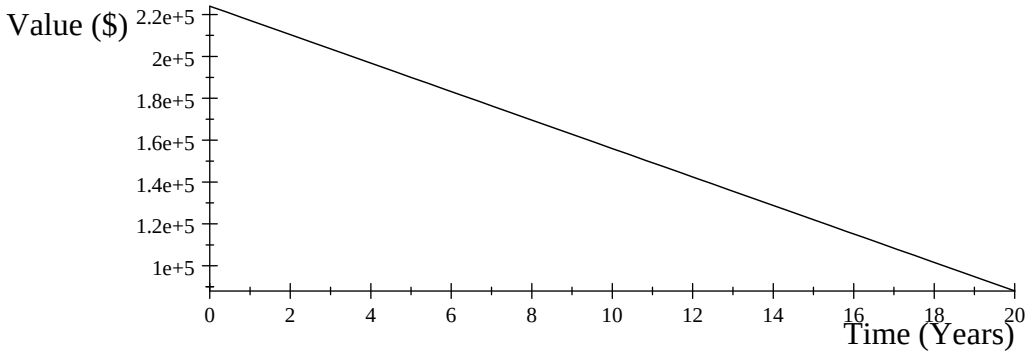
$$y = m(x - x_1) + y_1 = -6800x + 224000$$

A $V(t) = -6800t + 224000$

B $V(10) = -6800 \times 10 + 244000 = \$176\,000$

C $100000 = -6800t + 224000$, Solution is : $\left\{t = \frac{310}{17}\right\}$

D



Question 2

A)

$$4^{x-x^2} = ((\frac{1}{2})^{-2})^{x-x^2} = (\frac{1}{2})^{-2(x-x^2)}$$

$$(\frac{1}{2})^{-2(x-x^2)} = (\frac{1}{2})^{1}$$

$-2(x - x^2) = 1$, Solution is : $\{x = -.\,366\,03\}, \{x = 1.\,366\}$

B)

$$(25)^{2x} = ((5)^2)^{2x} = 5^{4x}$$

$$5^{4x} = 5^{x^2-12}$$

$4x = x^2 - 12$, Solution is : $\{x = -2.\,0\}, \{x = 6.\,0\}$

C)

$$\log_{10} x = \frac{3}{2}\log_{10} 4 - \frac{2}{3}\log_{10} 8 + \log_{10} 2 = .602\,06$$

$$\log_{10} x = .602\,06$$

$$x = 10^{.602\,06} = 4.0$$

D)

$$\log_a x + \log_a (x+2) = \log_a (x+4)$$

$x(x+2) = (x+4)$, Solution is : $\{x = -2.561\,6\}, \{x = 1.561\,6\}$

E)

$$\ln x + \ln(x-3) = \ln 10$$

$x(x-3) = 10$, Solution is : $\{x = -2.0\}, \{x = 5.0\}$

Question 3

A)

$$a_1 = 3, n = 24, a_{24} = 187$$

Arithmetic:

$$S_n = \frac{n(a_1 + a_n)}{2} = \frac{24(3+187)}{2} = 2280$$

$$\sum_{1}^{24}(8x-5) = 2280$$

B)

$$a_1 = \frac{1}{27}, r = 3, n = 20$$

Geometric:

$$S_n = a_1 \frac{r^n - 1}{r-1} = \frac{1}{27}\frac{3^{20}-1}{3-1} = 6.457 \times 10^7$$

$$\sum_{1}^{20}(3^{x-4}) = \frac{1743\,392\,200}{27} = 6.457 \times 10^7$$

Question 4

$t = 5$ years, $FV = \$800000$, $r = 0.066$, $m = 12$, $i = \frac{r}{m} = \frac{0.066}{12} = .00\,55$, $n = mt = 60$

A)

$$PMT = FV\frac{i}{(1+i)^n - 1} = 800000\frac{0.0055}{(1+0.0055)^{60} - 1} = 11290.0$$

B)

$$PMT = 11290, t = 4 years, m = 12, i = 0.0055, n = 48$$

Find value after 4 years:

$$FV = PMT\frac{(1+i)^n - 1}{i} = 11290\frac{(1+0.0055)^{48} - 1}{0.0055} = 6.1825 \times 10^5$$

$$800000 - 6.1825 \times 10^5 - 11290 \times 12 = \$46270.0$$

Question 5

A)

$$PV = \$50000, \quad r = 0.072, \quad m = 12, \quad t = 20, \quad n = mt = 240, \quad i = \frac{r}{m} = \frac{0.072}{12} = .006$$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 50000\frac{0.006}{1-(1+0.006)^{-240}} = \$393.67$$

Amount paid to the bank: $393.67\$ \times 240 = 94481\$$.
Interest/money made by bank: $94481\$ - 50000\$ = 44481\$$.

B)

$$PMT = \$493.67, \qquad m = 12, \qquad i = 0.06,$$
$$t = ?, \qquad n = mt = 12t, \quad PV = \$50000 \; PV = PMT\frac{1-(1+i)^{-n}}{i}$$

$50000 = 493.67\frac{1-(1+0.006)^{-12t}}{0.006}$, Solution is : $\{t = 13.035\}$
$13.035 \times 12 = 156.42 \Rightarrow 157$ paymentS
Amount paid to bank: $PMT \times n = 493.67\$ \times 156.42 = \$77220.0$
Interest/amount made by bank: $\$77220 - \$50000 = \$27220.0$
Difference between (A) amount saved by PMT increase :$\$44481 - \$27220 = \$17261$

Question 6

$\begin{bmatrix} 5 & -3 & 2 & 13 \\ 2 & -1 & -3 & 1 \\ 4 & -2 & 4 & 12 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$x_1 = 1, x_2 = -2, x_3 = 1$

Question 7

x_1 = Coal Output ($)
x_2 = Oil Output ($)
x_3 = Transportation Output ($)

Inputs.

Coal: $0.20x_1, 0x_1, 0.40x_1$
Oil: $0.0x_2, 0.1x_2, 0.20x_2$
Transportation: $0.40x_3, 0.20x_3, 0.20x_3$

Costs/Technology Matix

Coal: $0.20x_1 + 0x_2 + 0.40x_3$
Oil: $0x_1 + 0.1x_2 + 0.20x_3$
Trans: $0.40x_1 + 0.20x_2 + 0.20x_3$

A)

$$M = \begin{bmatrix} 0.20 & 0 & 0.40 \\ 0 & 0.1 & 0.20 \\ 0.40 & 0.20 & 0.20 \end{bmatrix}$$

B)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} 0.20 & 0 & 0.40 \\ 0 & 0.1 & 0.20 \\ 0.40 & 0.20 & 0.20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 10 \\ 20 \end{bmatrix}$$

C)

$$\begin{bmatrix} 0.80x_1 - 0x_2 - 0.40x_3 \\ 0x_1 + 0.9x_2 - 0.20x_3 \\ -0.40x_1 - 0.20x_2 + 0.80x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 10 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} 0.80 & 0 & -0.40 & 30 \\ 0 & 0.9 & -0.2 & 10 \\ -0.40 & -0.20 & +0.80 & 20 \end{bmatrix}, \text{ row echelon form: } \begin{bmatrix} 1 & 0 & 0 & 71.0 \\ 0 & 1 & 0 & 26.0 \\ 0 & 0 & 1 & 67.0 \end{bmatrix}$$

$$x_1 = 71B\$, x_2 = 26B\$, x_3 = 67B\$$$

Alternate Method

$I - M$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.20 & 0 & 0.40 \\ 0 & 0.1 & 0.20 \\ 0.40 & 0.20 & 0.20 \end{bmatrix} = \begin{bmatrix} .8 & 0 & -.4 \\ 0 & .9 & -.2 \\ -.4 & -.2 & .8 \end{bmatrix}$$

$(I - M)^{-1}$

$$\begin{bmatrix} .8 & 0 & -.4 \\ 0 & .9 & -.2 \\ -.4 & -.2 & .8 \end{bmatrix}, \text{inverse: } \begin{bmatrix} 1.7 & .2 & .9 \\ .2 & 1.2 & .4 \\ .9 & .4 & 1.8 \end{bmatrix}$$

$$(I-M)^{-1}D = X$$

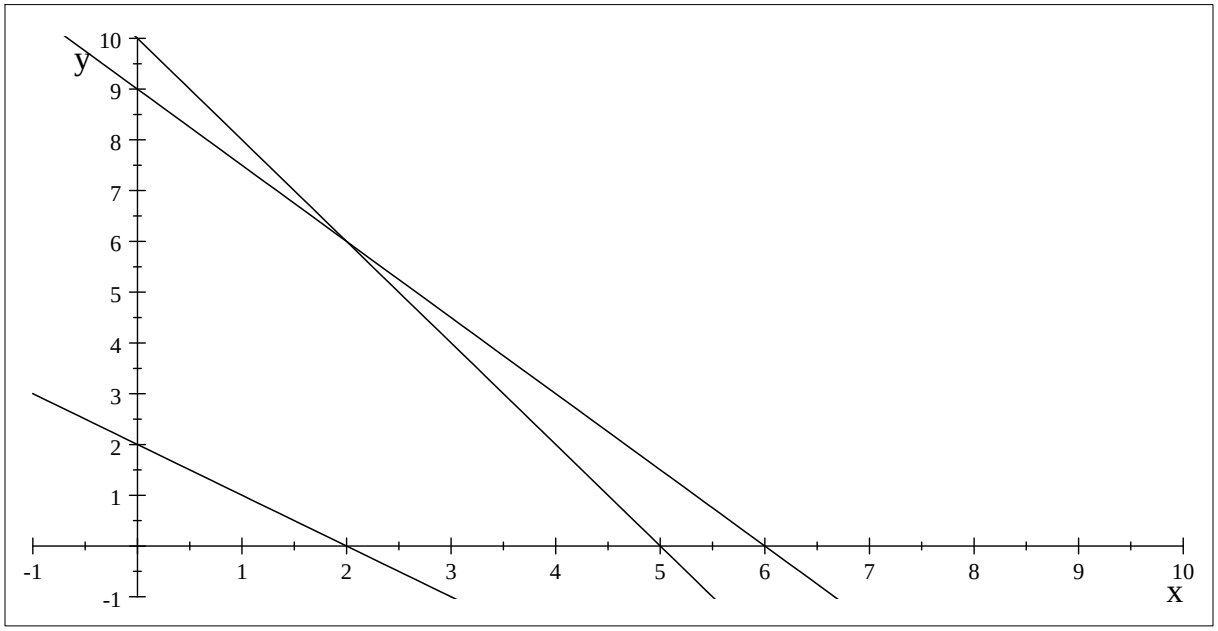
$$\begin{bmatrix} 1.7 & .2 & .9 \\ .2 & 1.2 & .4 \\ .9 & .4 & 1.8 \end{bmatrix}\begin{bmatrix} 30 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 71.0 \\ 26.0 \\ 67.0 \end{bmatrix}$$

$$x_1 = 71B\$, x_2 = 26B\$, x_3 = 67B\$$$

Question 8

(1) $2x+2y \geq 4$ (2) $6x+4y \leq 36$ (3) $2x+y \leq 10$ (4) $x \geq 0$ (5) $y \geq 0$

$y = -x+2$ $y = \frac{-3}{2}x+9$ $y = -2x+10$ $x = 0$ $y = 0$



Feasability region bounded by all three lines and the axes.

Points of Intersection (x,y)	$P(x,y) = 30x+10y$
$(2,0)$	60
$(0,2)$	20
(5.0)	150
$(0,9)$	90
$(2,6)$	140

$Max: (5,0), Min: (0,2)$

Question 9

Group: 9 People in total

Barbara:1

John: 1

Others: 7

A) ${}_9C_4 = \binom{9}{4} = 126$

B) ${}_2C_2 \times {}_7C_2 = \binom{2}{2} \times \binom{7}{2} = 21$

C) ${}_1C_1 \times {}_7C_3 + {}_1C_1 \times {}_7C_3 = \binom{1}{1} \times \binom{7}{3} + \binom{1}{1} \times \binom{7}{3} = 70$

Question 10

Group: 32 animals

	Monkey	Chimp	Dogs	Total/Gender
Male	3	6	2	11
Female	7	6	8	21
Total/Animal	10	12	10	

A) $P_+ = \frac{\text{Positive Outcomes}}{\text{Possible Outcomes}} = \frac{6+6+2+8}{32} = 0.6875$

B) $P_+ = \frac{\text{Positive Outcomes}}{\text{Possible Outcomes}} = \frac{7+6+6+8}{32} = 0.84375$

C) $P_+ = \frac{\text{Positive Outcomes}}{\text{Possible Outcomes}} = \frac{3+6+2+6+8}{32} = .78125$

OR $P_- = \frac{7}{32} = .21875$

$P_+ = 1 - P_- = 1 - .21875 = .78125$