Concordia University
STAT 250/2AA
Fall 2010

Midterm Exam
19 October 2010
18:00 – 20:00

This is a closed-book exam. No aids are allowed except an approved calculator. **You have at most 2 hours to finish this exam.** *Cheating will be reported to the administration.* Make sure that you print your name *and* ID number clearly on the exam.

Problem	Points	Your Mark
1	20	
2	10	
3	13	
4	12	
5	8	
6	17	
Total	80	

Last Name, First Name

Student ID Number

1. Let Y_1, Y_2 be random variables with joint density function

$$\begin{aligned} f(y_1, y_2) &= y_1 e^{-y_1 y_2} \text{ for } 0 \le y_1 \le 1, y_2 \ge 0 \\ &= 0 \text{ otherwise} \end{aligned}$$

Calculate:

(a) (8 pts) Find the joint density function of the random variables $U_1 = Y_1$ and $U_2 = Y_1 Y_2$.

(b) (4 pts) Find the marginal density of U_2 and identify whether it is a well known type of random variable.

(c) (4 pts) Find $E(U_1)$ and $V(U_1)$

(d) (4 pts) Are U_1 and U_2 independent? You must prove your answer.

2. (10 pts) Let Y_1 and Y_2 have a joint density that is uniform over $(y_1, y_2) : y_1 \in (0, 1), y_2 \in (0, 2)$. Find the density of the variable $U = 2Y_1 + Y_2$.

3. Let there be independent trials of independent experiments all with the same probability of success p that is unknown. Suppose that this unknown probability of success $p = X$ is a random variable with density

$$\begin{aligned} f(p) &= 3p^2 \ \text{ for } \ 0 \leq p \leq 1 \\ &= 0 \ \text{ otherwise} \end{aligned}$$

. Let N be an the number of the trial on which the first success occurs.

(a) (5 pts) Find $E(N)$.

(b) (8 pts) Find $V(N)$.

4. Let $Y_1, Y_2, \ldots, Y_n$ be independent random variables with the same density $f(y) = \lambda e^{-\lambda y}, y > 0$, and let $U = Y_1 + Y_2 + \cdots + Y_n$.

   (a) (8 pts) What random variable is U. You must justify your answer with a proof.

   (b) (4 pts) What are the values for $E(U)$ and $V(U)$?

5. (8 pts) Let X, Y be independent random variables with Uniform distribution on $[0, 1]$. Let $U = \min(X, Y)$. Find the density of the random variable U and calculate $E(U)$.

6. A sample of prices for the same item over 100 different stores has been collected. The mean price of this item is unknown. The standard deviation of the price is known to be equal to 10.

   (a) (8 pts) What is the approximate probability that the sample mean of the prices over these 100 stores will be within 1.5 of the true mean price?

   (b) (7 pts) If we want the sample mean to be within 1.5 of the true mean price with approximate probability 0.95, how many stores should we sample the prices from?

   (c) (2 pts) Why did we say that the probabilities above are 'approximate' (we did not say anything about the distribution of the prices in the population)?