Midterm 1 (B) Solution

MAT1300D, Fall 2015

I. Multiple-choice Questions ($2 \times 6 = 12$ marks)

ADBCAD

1. If $f(x) = \dfrac{2x-3}{x-4}$, then $f^{-1}(1) =$

(A) −1; (B) 9; (C) −7; (D) 5; (E) 1 / 3.

Answer. (A) $\dfrac{2x-3}{x-4} = 1$, $2x - 3 = x - 4$, $x = -1$.

2. Suppose a function $y = f(x)$ defined by

$$f(x) = \begin{cases} ax+1, & x < 2 \\ 5+2ax, & x \geq 2 \end{cases}$$

is continuous for all values of x. Then $a =$

(A) 1; (B) −1; (C) 2; (D) −2; (E) 0.

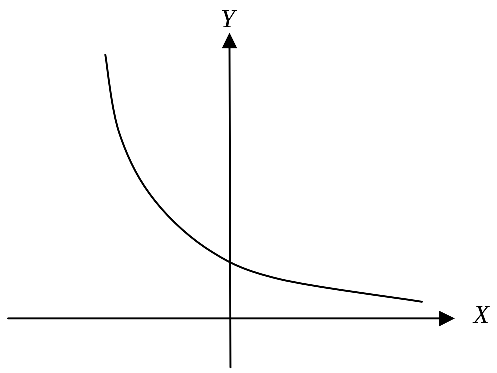
Answer. (D) When $x = 2$, we must have $ax + 1 = 5 + 2ax$, or $2x + 1 = 5 + 4a$. $2x = -4$, $x = -2$.

3. If $\log_a x = 2$, then $\log_{1/a} x =$

(A) 1; (B) −2; (C) 2; (D) $\dfrac{1}{2}$; (E) $-\dfrac{1}{2}$.

Answer. (B) Since $x = a^2 = \left(\dfrac{1}{a}\right)^{-2}$. $\log_{1/a} x = -2$.

4. Which one of the following functions can have the graph in the figure below?



(A) $y = 2^x$; (B) $y = \log_2 x$; (C) $y = (1/2)^x$; (D) $y = \log_{1/2} x$; (E) $y = 1/x$.

Answer. (C)

5. The derivative of the function $y = \dfrac{3+\ln x}{x^2+1}$ at $x = 1$ is

(A) -1; (B) e; (C) 0; (D) $-1/e^2$; (E) $-1/2$.

Solution. (A) $y' = \dfrac{(1/x)(x^2+1)-(3+\ln x)(2x)}{(x^2+1)^2} = \dfrac{x^2+1-2x^2(3+\ln x)}{x(x^2+1)^2}$. When $x = 1$, $y' = -1$.

6. At which value of x, the graph of the function $y = (x-3)e^{2x+3}$ has a horizontal tangent line?

(A) 0; (B) 3 / 2; (C) 1; (D) 5 / 2; (E) 7 / 2.

Answer. (D) $y' = e^{2x+3} + 2(x-2)e^{2x+3} = e^{2x+3}(2x-3)$. Let $y' = 0$. $2-5x = 0$, $x = 5/2$.

II. Long-answer Questions (8 marks)

1. (2 marks) Assume that the demand function $p = mx + b$ of a kind of umbrella is linear, where x is the sales and p is the unit price. At the price \$5 each, the company can sell 500 umbrellas every day. When the price is reduced to \$4.5 each, the sales are increased to 525 every day. Find the demand function of this product.

Solution. This graph of this function goes through two points (300, 5) and (4.5, 350). The slope of the line is $m = \dfrac{4.5-5}{525-500} = -0.02$. Then $p = -0.02x + b$. Since $5 = -0.02\times 500 + b$, $b = 15$. The demand function is $p = -0.02x + 15$.

2. (3 marks) Use the definition of the derivative to find the derivative of function $y = \dfrac{1}{2x+3}$.

Solution. $$y' = \lim_{h\to 0}\frac{1}{h}\left(\frac{1}{2(x+h)+3}-\frac{1}{2x+3}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{(2x+3)-(2x+2h+3)}{(2x+3)(2x+2h+3)}\right)$$
$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{-2h}{(2x+3)(2x+2h+3)}\right) = \lim_{h\to 0}\frac{-2}{(2x+3)(2x+2h+3)} = -\frac{2}{(2x+3)^2}.$$

3. (3 marks) Suppose a function $y = f(x)$ is defined implicitly by the equation $2xy + 3x^2 - y^3 + 1 = 0$. Find the derivative $f'(x)$ at the point $x = 1$, $y = 2$.

Solution. Take the derivative on both sides with respect to x: $2(y + xy') + 6x - 3y^2y' = 0$. When $x = 1$, $y = 2$, $4 + 2y' + 6 - 12y' = 0$. $10y' = 10$, $y' = 1$.