Sample Formula Sheet

Trigonometry

$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$; $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

Calculus

$\frac{d}{d\theta}\sin\theta = \cos\theta$; $\frac{d}{d\theta}\cos\theta = -\sin\theta$

$\int x^n dx = \frac{x^{n+1}}{n+1} + constant$

Vectors

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\mathbf{u}_A = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$

$\mathbf{r}_{B/A} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$

$\mathbf{A}.\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$= (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$

Statics

Friction

$F_s = \mu_s N$ - static friction (impending motion)

$F_k = \mu_k N$ - kinetic friction

$T_2 = T_1 e^{\mu_s\beta}$ - belt friction

Bearing friction

$F = \mu_s dN = \mu_s p dA = \mu_s p(r d\theta dr)$

$M - \int_A r dF = 0$

Virtual work

$\delta U = 0$

$U = -Wh$ - for weight

$U = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$ - for spring

Potential energy

$V_g = Wy$ - gravity

$V_e = \frac{1}{2}ks^2$ - spring

$\frac{dV}{dq} = 0$, $\frac{d^2V}{dq^2} > 0$ - stable equilibrium

$\frac{dV}{dq} = 0$, $\frac{d^2V}{dq^2} < 0$ - unstable equilibrium

$\frac{dV}{dq} = 0$, $\frac{d^2V}{dq^2} = \ldots = 0$ - neutral equilibrium

Dynamics

Kinematics

$v = v_0 + a_c t$

$s = s_0 + v_o t + \frac{1}{2}a_c t^2$

$v^2 = v_0^2 + 2a_c(s - s_0)$

$\omega = \omega_0 + \alpha_c t$

$\theta = \theta_0 + \omega_o t + \frac{1}{2}\alpha_c t^2$

$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r}_{B/A}$

$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}\times\mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$

Mass moment of inertia

$I = \int_m r^2 dm$

$I = I_G + md^2$ - parallel axis theorem

$I_G = \frac{1}{12}ml^2$ - slender bar (rod)

$I_G = \frac{2}{5}mR^2$ - sphere

$I_G = \frac{1}{2}mR^2$ - cylinder

$I_G = mR^2$ - ring

Equations of motion

$\Sigma F_x = m(a_G)_x$

$\Sigma F_y = m(a_G)_y$

$\Sigma M_G = I_G\alpha$

Work and energy

$T = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\omega^2$

$T_1 + (\Sigma U_{1-2})_{noncons} = T_2$ - nonconservative forces

$T_1 + V_1 = T_2 + V_2$ - conservative forces

Impulse and momentum

$m(v_{Gx})_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_{Gx})_2$

$m(v_{Gy})_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_{Gy})_2$

$I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G dt = I_G\omega_2$