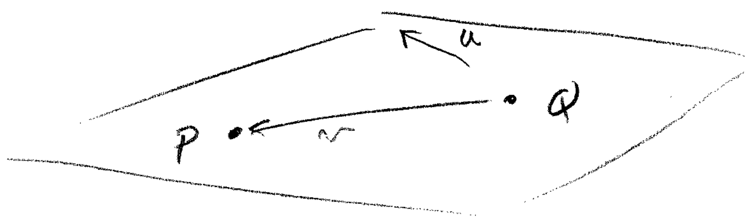
1. An equation of the plane parallel to the vector $\overset{u}{(1,1,-2)}$ and which passes through the points $\underset{P}{(1,0,3)}$ and $\underset{Q}{(0,1,11)}$ is:



A. $5x - 11y + 2z = 11$
B. $7x - 9y + 2z = 13$
C. $5x - 7y - z = 2$
D. $5x - 3y + z = 8$
E. $x + 18y + 9z = 8$
F. $9x - 6y + 5z = 8$

If $v = P - Q = (1,-1,-8)$, then a normal to the plane is

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -1 & -8 \end{vmatrix} = (-10, -(-6), -2) = (-10, 6, -2)$$

The vector $(5,-3,1)$ is also a normal. Hence D is correct. (You may check that it passes through P.)

2. Parametric equations of the line containing (2, 0, 1) and which is parallel to the two planes $x - y + 3z = 0$ and $3x - 5y + 4z = 1$ are:

A. $x = 2 + 11t,\ y = 5t,\ z = 1 + 2t,\ t \in \mathbf{R}$
B. $x = -2 + 5t,\ y = -5t,\ z = 1 - 10t,\ t \in \mathbf{R}$
C. $x = -2t,\ y = 0,\ z = t,\ t \in \mathbf{R}$
D. $x = -2 + 11t,\ y = -3t,\ z = 1 + 2t,\ t \in \mathbf{R}$
E. $x = 2t,\ y = 0,\ z = t,\ t \in \mathbf{R}$
F. $x = 2 + 11t,\ y = 5t,\ z = 1 - 2t,\ t \in \mathbf{R}$

A direction vector for this line is

$$d = (1,-1,3) \times (3,-5,4) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 3 & -5 & 4 \end{vmatrix}$$

$= (11, -(-5), -2)$. Hence F is correct

3. If $u = (1,\ 3,\ -2)$, $v = (0,\ 2,\ -1)$, $w = (1,\ -1,\ 2)$ then the cosine of the angle between $(v \times w)$ and $(u \times v)$ is:

A. $\frac{2}{21}$

B. $-\frac{1}{21}$

C. $\frac{\sqrt{2}}{\sqrt{21}}$

D. $-\frac{1}{\sqrt{7}}$

E. $-\frac{1}{\sqrt{21}}$

F. $\frac{2}{\sqrt{7}}$

$$v \times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = (3, -1, -2)$$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 0 & 2 & -1 \end{vmatrix} = (1, -(-1), 2) = (1, 1, 2)$$

Hence $\frac{(v \times w) \cdot (u \times v)}{\|v \times w\| \ \|u \times v\|} = \frac{-2}{\sqrt{9+1+4}\ \sqrt{1+1+4}}$

$$= \frac{-2}{\sqrt{14}\ \sqrt{6}} = \frac{-1}{\sqrt{21}}$$

4. If $\mathbf{u} = (1,\ 0,\ 1)$ and $\mathbf{v} = (-3,\ 4,\ 10)$, the orthogonal projection of $\mathbf{v}$ along $\mathbf{u}$ is:

$$\mathrm{Proj}_u v = \frac{u \cdot v}{\|u\|^2} \cdot u$$

A. $(7, 0, 7)$

B. $(-7, 0, -7)$

C. $(-\frac{7}{2},\ 0,\ -\frac{7}{2})$

D. $(\frac{7}{2},\ 0,\ \frac{7}{2})$

E. $(\frac{11}{2},\ 0,\ \frac{11}{2})$

F. $(-\frac{11}{2},\ 0,\ -\frac{11}{2})$

$$= \frac{7}{2} \cdot (1, 0, 1)$$

5. The volume of the parallelepiped with edges given by the vectors $u = (1,\ 1,\ 1)$, $v = (1,\ 3,\ 2)$ and $w = (1,\ 1,\ 3)$ is:

A. 2

B. $\frac{\sqrt{2}}{2}$

C. $1/\sqrt{2}$

D. $1\sqrt{2}$

E. 4

F. $4\sqrt{2}$

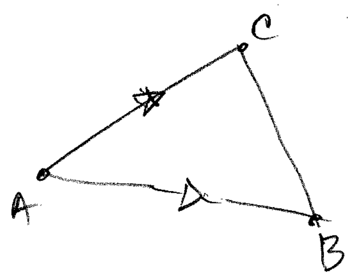
$$\text{Vol} = |u \times v \cdot w|$$

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 3 & 2 \end{vmatrix} = (-1, -1, 2)$$

$$\therefore |u \times v \cdot w| = |4| = 4$$

6. Find the area of the triangle with vertices $A = (0,\ 6,\ 1)$, $B = (2,\ 1,\ 5)$ and $C = (2,\ 5,\ 1)$.

A. 1
B. 2
C. 3
D. 4
E. 5
F. 6



$$\text{area} = \frac{1}{2}\|(B-A) \times (C-A)\|$$

$$B-A = (2, -5, 4)\ ,\quad C-A = (2, -1, 0)\ ;\quad v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 4 \\ 2 & -1 & 0 \end{vmatrix}$$

$$= (4, +8, 8);\ \|v\| = 4\,\|(1, 2, 2)\| = 12 \qquad \therefore\ \text{area} = 6$$

P Q

7. Let L be the line passing through $(1,\ 1,\ 0)$ and $(3,\ 5,\ 2)$. The point of intersection of L with the plane $x + y - z = 1$ is:

$Q - P = (2, 4, 2)$

A. $(\frac{1}{2},\ \frac{1}{2},\ 0)$

B. $(\frac{1}{2},\ 0,\ -\frac{1}{2})$

C. $(1,\ 0,\ 0)$

D. $(0,\ \frac{1}{2},\ -\frac{1}{2})$

E. $(0,\ 1,\ 0)$

F. $(-1,\ 0,\ -1)$

$\therefore$ L has parametric eqns

$x = 1 + t$

$y = 1 + 2t$

$z = t$

This intersects $x + y - z = 1$ when

$(1+t) + (1+2t) - t = 1$, i.e. $2 + 2t = 1$ or $t = -\frac{1}{2}$.

Hence $(x, y, z) = (\frac{1}{2}, 0, \frac{-1}{2})$

8. Find the intersection of the lines $x = 2 + 2s,\ y = 2 - s,\ z = 2 - 2s$ and $x = 4 + 5t,\ y = 3 - t,\ z = 4 - 2t$.

A. $(6,\ 7,\ -4)$

B. $(-4,\ 8,\ 3)$

C. $(4,\ 4,\ 4)$

D. $(2,\ 0,\ -2)$

E. $\frac{1}{3}(14,\ -23,\ 13)$

F. $\frac{1}{3}(-8,\ 13,\ 20)$

We solve

$2 + 2s = 4 + 5t$ (1)

$2 - s = 3 - t$ (2)

& $2 - 2s = 4 - 2t$ (3)

(1) + (3) yields $4 = 8 + 3t$ or $t = -\frac{4}{3}$

Hence $x = 4 - \frac{20}{3} = -8/3$

$y = 3 + 4/3 = \frac{13}{3}$

$z = 4 + 8/3 = \frac{20}{3}$

9. Find a scalar equation for the plane with vector parametric equation

$$v = (0, 2, -2) + s(1, -1, 2) + t(4, -6, 3);\ s, t \in \mathbf{R}.$$

A normal is $n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 4 & -6 & 3 \end{vmatrix}$

A. $4x - 9y + 36z = 18$

B. $9x + 5y - 2z = 14$

C. $7x - 8y + 5z = 6$

D. $9x - 11y + 18z = -40$

E. $9x - 2y + 2z = 0$

F. $3x + 2y - z = 0$

$= (9, -(-5), -2).$

Hence B is correct.

P

10. The distance from the point (5, 0, 0) to the plane $2x - y + 8z = -3$ is:

A. $\dfrac{13}{\sqrt{69}}$

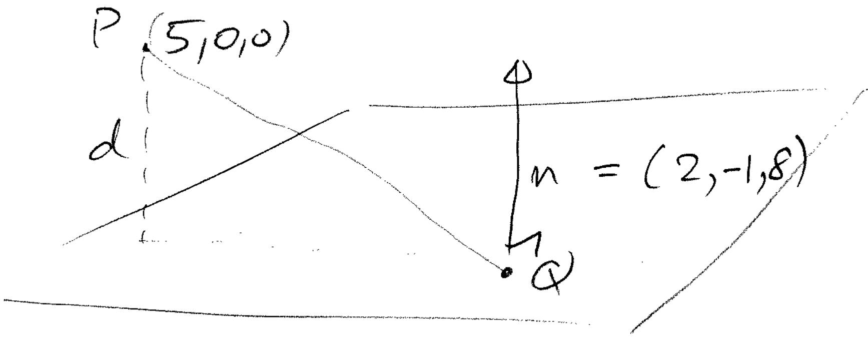
B. $\dfrac{19}{\sqrt{69}}$

C. $\dfrac{15}{\sqrt{69}}$

D. 0

E. $\dfrac{13}{69}$

F. $\dfrac{19}{69}$



Let $Q = (0, 3, 0)$, which belongs to the plane.

Then $d = \| \text{proj}_n (P-Q) \|$; $P - Q = (5, -3, 0)$

$$\| \text{proj}_n (P-Q) \| = \frac{|n \cdot (P-Q)|}{\|n\|} = \frac{13}{\sqrt{4+1+64}} = \frac{13}{\sqrt{69}}$$

11. Evaluate $\mathrm{Im}(z)$ if

$$z = \frac{1}{(-1+i)(2-2i)}.$$

A. $\frac{1}{2}$
B. $-\frac{1}{2}$
C. $-\frac{1}{5}$
D. $\frac{1}{4}$
E. $-\frac{1}{4}$
F. 1

$$z = \frac{-1-i}{2} \cdot \frac{2+2i}{8} = \frac{-4i}{16} = -\frac{1}{4} i$$

12. Find the polar form of:

$$\frac{z}{w} = \frac{1+i\sqrt{3}}{-\sqrt{2}+i\sqrt{2}}$$

$|1+i\sqrt{3}| = 2, \ \theta = \mathrm{Arg}(z)$

$\cos\theta = 1/2 \quad \sin\theta = \sqrt{3}/2$

$\therefore \theta = \pi/3$

A. $2(\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12}))$
B. $\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12})$
C. $\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})$
D. $\cos(-\frac{5\pi}{12}) + i\sin(-\frac{5\pi}{12})$
E. $2(\cos(-\frac{5\pi}{12}) + i\sin(-\frac{5\pi}{12}))$
F. $2(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12}))$

$|-\sqrt{2}+i\sqrt{2}| = \sqrt{2}\,|-1+i|$
$= 2$

$\varphi = \mathrm{Arg}(w) \quad \cos\varphi = \frac{-\sqrt{2}}{2}$

$\sin\varphi = \frac{\sqrt{2}}{2}$

$\therefore \varphi = 3\pi/4$

$$\therefore \frac{z}{w} = \frac{2e^{i\pi/3}}{2e^{i3\pi/4}} = e^{i\pi(1/3 - 3/4)} = e^{-i5\pi/12}$$