

ECON 4020 A Mid-Term Examination

Tuesday, October 16, 2012

Prof. K. G. Armstrong

Name: _____

Instructions: Answer all questions in the space provided.

30 pts

1. Consider the production possibilities set

10 pts

$$Y = \{(y_1, y_2) : y_2 \leq e^{-y_1} - 1 \wedge y_1 \leq 0\} ,$$

where e^{\cdot} denotes the exponential function and \wedge denotes the logical operator “and.”

(a) Draw the graph of Y .

3 pts

(b) Show whether or not Y is closed, convex, and satisfies free disposal.

3 pts

(c) Characterize the set of efficient points of Y .

1 pt

(d) What returns to scale property does Y exhibit, if any?

2 pts

- (e) If $(p_1, p_2) = (e, 1)$, which production plan will a profit-maximizing firm select from Y and how much profit will it make? 1 pt

2. Consider the constrained minimization problem 10 pts

$$c(w, y) = \min_x \{wx : x \geq g(y)\} ,$$

where $g(\cdot)$ is an increasing, twice continuously differentiable function.

- (a) Explain the meaning of $g(y)$ in economic terms. 2 pts

- (b) State the necessary and sufficient conditions for $x^* > 0$ to solve the above constrained minimization problem. 2 pts

Now consider the unconstrained maximization problem

$$\pi(p, w) = \max_y \{py - c(w, y)\} .$$

- (c) In the simplest possible terms, state the necessary and sufficient conditions for $y^* > 0$ to solve the preceding unconstrained maximization problem. 2 pts

- (d) Determine the signs of the derivatives $\left. \frac{dy^*}{dp} \right|_{dw=0}$ and $\left. \frac{dy^*}{dw} \right|_{dp=0}$. 1.5 pts

(e) Show that $\pi_p(p, w) = y^*$ and $\pi_w(p, w) = -x^*$. 2.5 pts

3. Consider the production function given by 10 pts

$$y = \frac{x_1}{a_1} + \frac{x_2}{a_2}, \quad a_1 > 0, a_2 > 0.$$

(a) For y fixed, draw the graph of the input requirement set. 2.5 pts

(b) Derive the associated cost function. 2 pts

(c) For w_1 and w_2 fixed, draw the graphs of total cost, average total cost and marginal cost. 2.5 pts

- (d) State and prove the monotonicity, homogeneity, and curvature properties of the part-(b) cost function.

3 pts

End of Examination