MULTIPLE CHOICE

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\hat{\mathbf{i}} + A\sin\theta\hat{\mathbf{j}}$.

1. If $\vec{\mathbf{A}} = [15, 80°]$ and $\vec{\mathbf{B}} = 12\hat{\mathbf{i}} - 16\hat{\mathbf{j}}$, what is the magnitude of $\vec{\mathbf{A}} - \vec{\mathbf{B}}$?
   a. 15
   b. 35
   c. 32
   d. 5.0
   e. 23

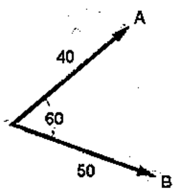
$\vec{A} - \vec{B} = 15\cos 80\,\hat{i} + 15\sin 80\,\hat{j} - 12\hat{i} + 16\hat{j}$

$= (15\cos 80 - 12)\hat{i} + (15\sin 80 + 16)\hat{j}$

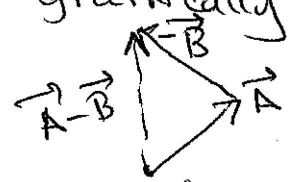
$|\vec{A} - \vec{B}| = \sqrt{(15\cos 80 - 12)^2 + (15\sin 80 + 16)^2}$

ANS: C PTS: 2 DIF: Average

2. Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are shown. What is the magnitude of a vector $\vec{\mathbf{C}}$ if $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$?



graphically: to find $\vec{A} - \vec{B}$: draw vector $-\vec{B}$ from the head of vector $\vec{A}$



Cosine method: $C = \sqrt{A^2 + B^2 - 2AB\cos 60}$

Notice in class we have $C = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ for $\vec{A} + \vec{B}$

   a. 46
   b. 10
   c. 30
   d. 78
   e. 90

ANS: A PTS: 2 DIF: Average

3. If $\vec{\mathbf{A}} = 12\hat{\mathbf{i}} - 16\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -24\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$, what is the magnitude of the vector $\vec{\mathbf{C}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}}$?
   a. 42
   b. 22
   c. 64
   d. 90
   e. 13

$\vec{C} = 2(12\hat{i} - 16\hat{j}) - (-24\hat{i} + 10\hat{j}) = 24\hat{i} - 32\hat{j} + 24\hat{i} - 10\hat{j}$

$= 48\hat{i} - 42\hat{j}$

$|\vec{C}| = \sqrt{(48)^2 + (-42)^2}$

ANS: C PTS: 2 DIF: Average

4. If $\vec{\mathbf{A}} = 12\hat{\mathbf{i}} - 16\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -24\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$, what is the direction of the vector $\vec{\mathbf{C}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}}$?
   a. −49°
   b. −41°
   c. −90°
   d. +49°
   e. +21°

$\vec{C} = 2\vec{A} - \vec{B} = 24\hat{i} - 32\hat{j} + 24\hat{i} - 10\hat{j}$

$= 48\hat{i} - 42\hat{j}$, $\tan\delta = \frac{-42}{48}$, $\delta = \tan^{-1}\left(\frac{-42}{48}\right)$

$= 41°$

ANS: B PTS: 2 DIF: Average

5. If $\vec{\mathbf{C}} = [10\text{ m}, 30°]$ and $\vec{\mathbf{D}} = [25\text{ m}, 130°]$, what is the magnitude of the sum of these two vectors?

$\vec{C} = 10\cos 30°\,\hat{i} + 10\sin 30\,\hat{j}$, $\vec{D} = 25\cos 130\,\hat{i} + 25\sin 130\,\hat{j}$

$\vec{C} + \vec{D} = (10\cos 30° + 25\cos 130)\hat{i} + (10\sin 30° + 25\sin 130°)\hat{j}$

$$|\vec{C}+\vec{D}| = \sqrt{(10\cos 30+25\cos 130)^2+(10\sin 30+25\sin 130)^2}$$

a. 20 m
b. 35 m
c. 15 m
d. 25 m
e. 50 m

ANS: D PTS: 2 DIF: Average

6. If $\vec{C}$ = [10 m, 30°] and $\vec{D}$ = [25 m, 130°], what is the direction of the sum of these two vectors?
a. 17°
b. 73°
c. 107°
d. 163°
e. 100°

from problem #5

$$\delta = \tan^{-1}\left(\frac{10\sin 30+25\sin 130}{10\cos 30+25\cos 130}\right)$$

ANS: C PTS: 2 DIF: Average

7. A vector, $\vec{B}$, when added to the vector $\vec{C} = 3\hat{i} + 4\hat{j}$ yields a resultant vector which is in the positive y direction and has a magnitude equal to that of $\vec{C}$. What is the magnitude of $\vec{B}$?
a. 3.2
b. 6.3
c. 9.5
d. 18
e. 5

$\vec{B}+\vec{C} = |\vec{C}|\hat{j}$, $\vec{B}+3\hat{i}+4\hat{j} = \sqrt{3^2+4^2}\,\hat{j}$

$\vec{B} + 3\hat{i}+4\hat{j} = 5\hat{j}$, $\vec{B} = -3\hat{i}+\hat{j}$

$|\vec{B}| = \sqrt{9+1} = \sqrt{10}$

ANS: A PTS: 2 DIF: Average

8. If vector $\vec{B}$ is added to vector $\vec{A}$, the result is $6\hat{i} + \hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4\hat{i} + 7\hat{j}$. What is the magnitude of $\vec{A}$?
a. 5.1
b. 4.1
c. 5.4
d. 5.8
e. 8.2

$\vec{A}+\vec{B} = 6\hat{i}+\hat{j}$

$\vec{A}-\vec{B} = -4\hat{i}+7\hat{j}$

$2\vec{A} = 2\hat{i}+8\hat{j} \rightarrow \vec{A} = \hat{i}+4\hat{j}$

$|\vec{A}| = \sqrt{1+16} = \sqrt{17}$

ANS: B PTS: 2 DIF: Average

9. If $\vec{C}$ = [2.5 cm, 80°], i.e., the magnitude and direction of $\vec{C}$ are 2.5 cm and 80°, $\vec{D}$ = [3.5 cm, 120°], and $\vec{E} = \vec{D} - 2\vec{C}$, what is the direction of $\vec{E}$ (to the nearest degree)?
a. 247°
b. 235°
c. 243°
d. 216°
e. 144°

$\vec{E} = 3.5\cos 120\,\hat{i} + 3.5\sin 120\,\hat{j} - 2(2.5\cos 80\,\hat{i} + 2.5\sin 80\,\hat{j})$

$= (3.5\cos 120 - 5\cos 80)\,\hat{i} + (3.5\sin 120 - 5\sin 80)\,\hat{j}$

$\tan\delta = \frac{E_y}{E_x} = \frac{3.5\sin 120 - 5\sin 80}{3.5\cos 120 - 5\cos 80} = \frac{3.03-4.92}{-1.75-0.86} = \frac{-1.89}{-2.61}$

ANS: D PTS: 3 DIF: Challenging

$\delta = 36°$ with $-ve$ x-axis

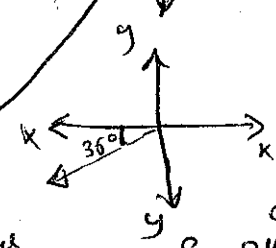
10. If vector $\vec{C}$ is added to vector $\vec{B}$, the result is $-9\hat{i} - 8\hat{j}$. If $\vec{B}$ is subtracted from $\vec{C}$, the result is $5\hat{i} + 4\hat{j}$. What is the direction of $\vec{B}$ (to the nearest degree)?
a. 225°
b. 221°
c. 230°

$\vec{C}+\vec{B} = -9\hat{i}-8\hat{j}$

$\vec{C}-\vec{B} = 5\hat{i}+4\hat{j}$ subtracting

$2\vec{B} = -14\hat{i}-12\hat{j}$

$\vec{B} = -7\hat{i}-6\hat{j}$, $\tan\delta = \frac{-6}{-7}$, $\delta = 41°$



then $\alpha = 180+\delta = 216°$

d. 236°
e. 206°

with −ve x-axis
$\theta = \delta + 180 = 221^\circ$

ANS: B PTS: 2 DIF: Average

11. A vector $\vec{\mathbf{A}}$ is added to $\vec{\mathbf{B}} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$. The resultant vector is in the positive x direction and has a magnitude equal to $\vec{\mathbf{A}}$. What is the magnitude of $\vec{\mathbf{A}}$?
a. 11
b. 5.1
c. 7.1
d. 8.3
e. 12.2

$\vec{A} + \vec{B} = \vec{A} + 6\hat{i} - 8\hat{j} = (A_x + 6)\hat{i} + (A_y - 8)\hat{j} = \sqrt{A_x^2 + A_y^2}\,\hat{i}$

$A_y - 8 = 0$, $A_x + 6 = \sqrt{A_x^2 + A_y^2}$

$A_y = 8$, $A_x = 2.3$, $A = \sqrt{(2.3)^2 + (8)^2} = 8.3$

ANS: D PTS: 3 DIF: Challenging

12. A vector $\vec{\mathbf{A}}$ is added to $\vec{\mathbf{B}} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$. The resultant vector is in the positive x direction and has a magnitude equal to that of $\vec{\mathbf{A}}$. What is the direction of $\vec{\mathbf{A}}$? $\vec{A} = A_x\hat{i} + A_y\hat{j}$
a. 74°
b. 100°
c. −81°
d. −62°
e. 106°

$\vec{A} + \vec{B} = (A_x + 6)\hat{i} + (A_y - 8)\hat{j} = \sqrt{A_x^2 + A_y^2}\,\hat{i}$

$A_x + 6 = \sqrt{A_x^2 + A_y^2} \rightarrow (1)$ $A_y - 8 = 0$, $A_y = 8$

then from (1) $A_x^2 + 12A_x + 36 = A_x^2 + 64 \rightarrow A_x = 2.3$

$\tan\delta = \frac{8}{2.3}$, $\delta = 74^\circ$

ANS: A PTS: 3 DIF: Challenging

13. If two collinear vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added, the resultant has a magnitude equal to 4.0. If $\vec{\mathbf{B}}$ is subtracted from $\vec{\mathbf{A}}$, the resultant has a magnitude equal to 8.0. What is the magnitude of $\vec{\mathbf{B}}$?
a. 2.0
b. 3.0
c. 4.0
d. 5.0
e. 6.0

$\vec{C} = \vec{A} + \vec{B}$, $|\vec{C}| = \sqrt{A^2 + B^2 + 2AB\cos 0} = \sqrt{(A+B)^2} = A + B = 4$ (1)

$\vec{D} = \vec{A} - \vec{B}$, $|\vec{D}| = \sqrt{A^2 + B^2 - 2AB\cos 0} = \sqrt{(A-B)^2} = A - B = 8$ (2)

from (1) & (2) $|B| = 2.0$

ANS: A PTS: 1 DIF: Easy

14. If two collinear vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added, the resultant has a magnitude equal to 4.0. If $\vec{\mathbf{B}}$ is subtracted from $\vec{\mathbf{A}}$, the resultant has a magnitude equal to 8.0. What is the magnitude of $\vec{\mathbf{A}}$?
a. 2.0
b. 3.0
c. 4.0
d. 5.0
e. 6.0

from problem # 13, Eqs (1) & (2) $|\vec{A}| = 6$

ANS: E PTS: 1 DIF: Easy

15. When vector $\vec{\mathbf{A}}$ is added to vector $\vec{\mathbf{B}}$, which has a magnitude of 5.0, the vector representing their sum is perpendicular to $\vec{\mathbf{A}}$ and has a magnitude that is twice that of $\vec{\mathbf{A}}$. What is the magnitude of $\vec{\mathbf{A}}$?
a. 2.2
b. 2.5
c. 4.5
d. 5.0
e. 7.0

$|\vec{A} + \vec{B}| = \sqrt{A^2 + 25 + 2(5)A\cos\theta} = 2A$

$A^2 + 25 + 10A\cos\theta = 4A^2 \rightarrow (1)$

$\tan\delta = \frac{1}{0} = \frac{B\sin\theta}{A + B\cos\theta}$, $B\cos\theta = -\frac{A}{5} \rightarrow (2)$

$A^2 + 25 + 10(A)\left(-\frac{A}{5}\right) = 4A^2 \rightarrow A = \sqrt{5} = 2.2$

ANS: A PTS: 2 DIF: Average

16. Starting from one oasis, a camel walks 25 km in a direction 30° south of west and then walks 30 km toward the north to a second oasis. What distance separates the two oases?
    a. 15 km
    b. 48 km
    c. 28 km
    d. 53 km
    e. 55 km

$$\tan \delta = \frac{A \sin 120}{B + A \cos 120}$$

δ with $B = 51$

ANS: C PTS: 2 DIF: Average

17. Starting from one oasis, a camel walks 25 km in a direction 30° south of west and then walks 30 km toward the north to a second oasis. What is the direction from the first oasis to the second oasis?
    a. 21° N of W
    b. 39° W of N
    c. 69° N of W
    d. 51° W of N
    e. 42° W of N

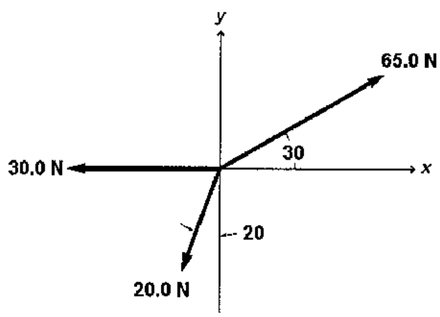
120° 60° 30°

$$\text{distance} = \sqrt{A^2+B^2+2AB\cos\theta}$$
$$= \sqrt{900+625+2(30)(25)\cos 120}$$
$$= 28 \text{ km}$$

ANS: D PTS: 3 DIF: Challenging

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\,\hat{\mathbf{i}} + A\sin\theta\,\hat{\mathbf{j}}$.

Exhibit 3-1
The three forces shown act on a particle.



Use this exhibit to answer the following question(s).

18. Refer to Exhibit 3-1. What is the magnitude of the resultant of these three forces?
    a. 27.0 N
    b. 33.2 N
    c. 36.3 N
    d. 23.8 N
    e. 105 N

$F_x = 65\cos 30 - 30 - 20\sin 20 = 19.45\text{N}$

$F_y = 65\sin 30 - 20\cos 20 = 13.70\text{N}$

$F = \sqrt{F_x^2 + F_y^2} = \cancel{18.xx}\ 23.8$

ANS: D PTS: 2 DIF: Average

19. Refer to Exhibit 3-1. What is the direction of the resultant of these three forces?
    a. 35°
    b. 45°
    c. 65°

$\tan\delta = F_y / F_x \rightarrow \delta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= 35°$, F_y & F_x both +ve

1st Quadrant

d. 55°
e. 85°

ANS: A PTS: 3 DIF: Challenging

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\hat{\mathbf{i}} + A\sin\theta\hat{\mathbf{j}}$.

20. If vector $\vec{\mathbf{C}}$ is added to vector $\vec{\mathbf{D}}$, the result is a third vector that is perpendicular to $\vec{\mathbf{D}}$ and has a magnitude equal to $3\vec{\mathbf{D}}$. What is the ratio of the magnitude of $\vec{\mathbf{C}}$ to that of $\vec{\mathbf{D}}$?
a. 1.8
b. 2.2
c. 3.2
d. 1.3
e. 1.6

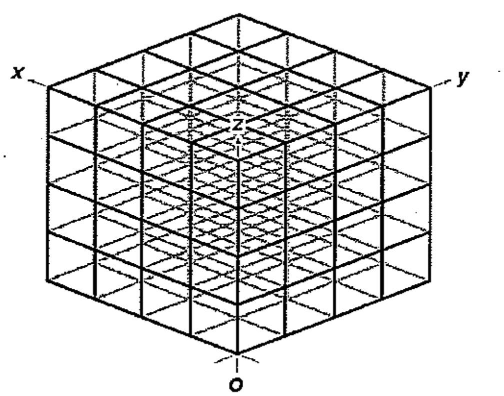
$C^2+D^2+2CD\cos\theta=9D^2 \rightarrow (1)$ $\quad \frac{1}{0} = \frac{C\sin\theta}{D+C\cos\theta}$, $\cos\theta = \frac{-D}{C} \rightarrow (2)$

then $C^2+D^2+2\cancel{C}D\left(\frac{-D}{\cancel{C}}\right)=9D^2 \rightarrow \frac{C^2}{D^2}=10$, $\frac{C}{D}=\sqrt{10}$

ANS: C PTS: 2 DIF: Average

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\hat{\mathbf{i}} + A\sin\theta\hat{\mathbf{j}}$.

Exhibit 3-2
A child starts at one corner of a cubical jungle gym in a playground and climbs up to the diagonally opposite corner. The original corner is the coordinate origin, and the *x*, *y* and *z* axes are oriented along the jungle gym edges. The length of each side is 2 m.



Use this exhibit to answer the following question(s).

21. Refer to Exhibit 3-2. The child's displacement is:
a. $2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
b. $2.8\hat{\mathbf{i}} + 2.8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
c. $2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2.8\hat{\mathbf{k}}$
d. $2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3.5\hat{\mathbf{k}}$
e. $3.5\hat{\mathbf{i}} + 3.5\hat{\mathbf{j}} + 3.5\hat{\mathbf{k}}$

$\vec{d} = (2-0)\hat{i} + (2-0)\hat{j} + (2-0)\hat{k}$
$= 2\hat{i} + 2\hat{j} + 2\hat{k}$

ANS: A PTS: 1 DIF: Easy

22. Refer to Exhibit 3-2. What is the child's distance from her starting position?
a. 2.8 m

in $xy = \sqrt{2^2+2^2} = 2\sqrt{2}$

in $xyz = \sqrt{(2\sqrt{2})^2 + 2^2} = \sqrt{12} = 3.5$ m

b. 3.5 m
c. 6.0 m
d. 6.9 m
e. 12.0 m

ANS: B PTS: 2 DIF: Average

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\hat{\mathbf{i}} + A\sin\theta\hat{\mathbf{j}}$.

23. The displacement of the tip of the 10 cm long minute hand of a clock between 12:15 A.M. and 12:45 P.M. is:
a. 10 cm, 90°
b. 10 cm, 180°
c. 10 cm, 4 500°
d. 20 cm, 180°
e. 20 cm, 540°

ANS: D PTS: 1 DIF: Easy

24. A student decides to spend spring break by driving 50 miles due east, then 50 miles 30 degrees south of east, then 50 miles 30 degrees south of that direction, and to continue to drive 50 miles deviating by 30 degrees each time until he returns to his original position. How far will he drive, and how many vectors must he sum to calculate his displacement?
a. 0, 0
b. 0, 8
c. 0, 12
d. 400 mi, 8
e. 600 mi, 12

ANS: E PTS: 2 DIF: Average

25. Given that $\vec{\mathbf{A}} + 2\vec{\mathbf{B}} = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}}$ and $\underbrace{2\vec{\mathbf{A}} - \vec{\mathbf{B}}}$ $= x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}}$, what is $\vec{\mathbf{A}}$? Express $\vec{B}$ in terms of $\vec{A}$ from any eq. and subs. in the other eq.
a. $\vec{\mathbf{A}} = \frac{1}{5}(x_1 + 2x_2)\hat{\mathbf{i}} + \frac{1}{5}(y_1 + 2y_2)\hat{\mathbf{j}}$
b. $\vec{\mathbf{A}} = \frac{1}{5}(x_1 - 2x_2)\hat{\mathbf{i}} + \frac{1}{5}(y_1 - 2y_2)\hat{\mathbf{j}}$
c. $\vec{\mathbf{A}} = \frac{1}{5}(x_1 + 4x_2)\hat{\mathbf{i}} + \frac{1}{5}(y_1 + 2y_2)\hat{\mathbf{j}}$
d. $\vec{\mathbf{A}} = \frac{1}{5}(x_1 + 4x_2)\hat{\mathbf{i}} + \frac{1}{5}(y_1 + 4y_2)\hat{\mathbf{j}}$
e. $\vec{\mathbf{A}} = \frac{1}{5}(x_1 + 4x_2)\hat{\mathbf{i}} + \frac{1}{5}(y_1 - 4y_2)\hat{\mathbf{j}}$

$\vec{B} = 2\vec{A} - x_2\hat{i} - y_2\hat{j}$

in (1)

$\vec{A} + 2(2\vec{A} - x_2\hat{i} - y_2\hat{j}) = x_1\hat{i} + y_1\hat{j}$

$5\vec{A} = x_1\hat{i} + y_1\hat{j} + 2x_2\hat{i} + 2y_2\hat{j}$

$\vec{A} = \frac{1}{5}(x_1 + 2x_2)\hat{i} + \frac{1}{5}(y_1 + 2y_2)\hat{j}$

ANS: A PTS: 2 DIF: Average

26. Given that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}}$ and $\vec{\mathbf{A}} - \vec{\mathbf{B}} = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}}$, what is $\vec{\mathbf{A}}$?
a. $\vec{\mathbf{A}} = \frac{1}{2}(x_1 - x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 - y_2)\hat{\mathbf{j}}$
b. $\vec{\mathbf{A}} = \frac{1}{2}(x_1 + x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 - y_2)\hat{\mathbf{j}}$

$\vec{A} + \vec{B} = x_1\hat{i} + y_1\hat{j}$

$+ \quad \vec{A} - \vec{B} = x_2\hat{i} + y_2\hat{j}$

$2\vec{A} = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j}$

$\vec{A} = \frac{1}{2}(x_1 + x_2)\hat{i} + \frac{1}{2}(y_1 + y_2)\hat{j}$

c. $\vec{\mathbf{A}} = \frac{1}{2}(x_1 - x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 + y_2)\hat{\mathbf{j}}$

d. $\vec{\mathbf{A}} = \frac{1}{2}(x_1 + x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 + y_2)\hat{\mathbf{j}}$

e. $\vec{\mathbf{A}} = \frac{1}{2}(x_1 + x_2)\hat{\mathbf{i}}$

ANS: D PTS: 2 DIF: Average

27. Given that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}}$ and $\vec{\mathbf{A}} - \vec{\mathbf{B}} = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}}$, what is $\vec{\mathbf{B}}$?

a. $\vec{\mathbf{B}} = \frac{1}{2}(x_1 - x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 - y_2)\hat{\mathbf{j}}$

b. $\vec{\mathbf{B}} = \frac{1}{2}(x_1 + x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 - y_2)\hat{\mathbf{j}}$

c. $\vec{\mathbf{B}} = \frac{1}{2}(x_1 - x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 + y_2)\hat{\mathbf{j}}$

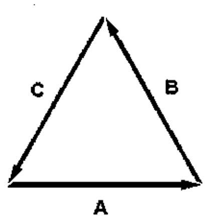
d. $\vec{\mathbf{B}} = \frac{1}{2}(x_1 + x_2)\hat{\mathbf{i}} + \frac{1}{2}(y_1 + y_2)\hat{\mathbf{j}}$

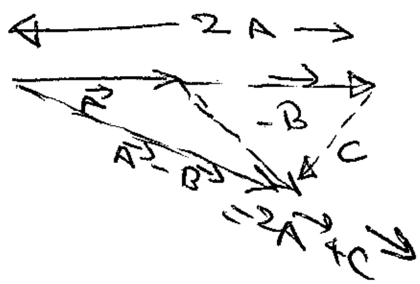
e. $\vec{\mathbf{B}} = \frac{1}{2}(y_1 - y_2)\hat{\mathbf{j}}$

$$\vec{A} + \vec{B} = x_1\hat{i} + y_1\hat{j}$$
$$-\ \vec{A} - \vec{B} = x_2\hat{i} + y_2\hat{j}$$
$$2\vec{B} = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$$
$$\vec{B} = \tfrac{1}{2}(x_1 - x_2)\hat{i} + \tfrac{1}{2}(y_1 - y_2)\hat{j}$$

ANS: A PTS: 2 DIF: Average

28. The diagram below shows 3 vectors which sum to zero, all of equal length. Which statement below is true?





a. $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{A}} - \vec{\mathbf{C}}$

b. $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} - \vec{\mathbf{C}}$

c. $\vec{\mathbf{A}} - \vec{\mathbf{B}} = 2\vec{\mathbf{A}} - \vec{\mathbf{C}}$

d. $\vec{\mathbf{A}} - \vec{\mathbf{B}} = 2\vec{\mathbf{A}} + \vec{\mathbf{C}}$

e. $2\vec{\mathbf{A}} + 2\vec{\mathbf{B}} = 2\vec{\mathbf{C}}$

ANS: D PTS: 1 DIF: Easy

29. Which statement is true about the unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$?

a. Their directions are defined by a left-handed coordinate system.
b. The angle between any two is 90 degrees.
c. Each has a length of 1 m.
d. If $\hat{\mathbf{i}}$ is directed east and $\hat{\mathbf{j}}$ is directed south, $\hat{\mathbf{k}}$ points up out of the surface.
e. All of the above.

ANS: B PTS: 1 DIF: Easy

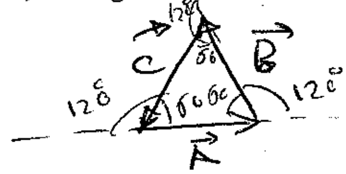
30. Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have equal magnitudes. Which statement is always true?

a. $\vec{\mathbf{A}} + \vec{\mathbf{B}} = 0.$

b. $\vec{\mathbf{A}} - \vec{\mathbf{B}} = 0$.
c. $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{A}} + \vec{\mathbf{B}}$.
d. $\vec{\mathbf{B}} - \vec{\mathbf{A}}$ is perpendicular to $\vec{\mathbf{A}} - \vec{\mathbf{B}}$.
e. The magnitude of $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ equals the magnitude of $\vec{\mathbf{A}} + \vec{\mathbf{B}}$.

ANS: C PTS: 3 DIF: Challenging

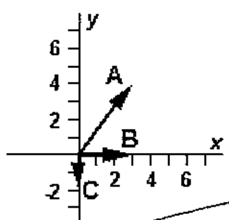
31. When three vectors, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are placed head to tail, the vector sum $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$. If the vectors all have the same magnitude, the angle between the directions of any two adjacent vectors is
a. 30°
b. 60°
c. 90°
d. 120°
e. 150°

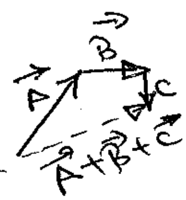


ANS: D PTS: 1 DIF: Easy

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\,\hat{\mathbf{i}} + A\sin\theta\,\hat{\mathbf{j}}$.

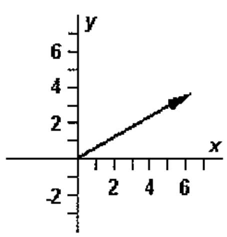
Exhibit 3-3
The vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ are shown below.

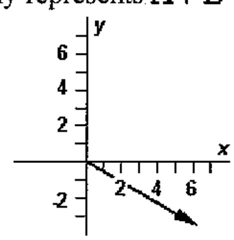


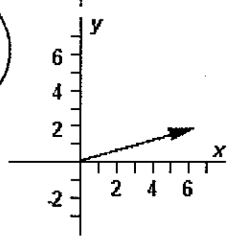


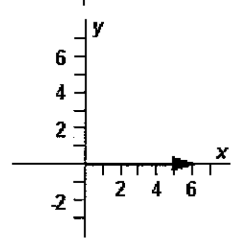
Use this exhibit to answer the following question(s).

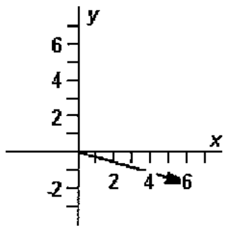
32. Refer to Exhibit 3-3. Which diagram below correctly represents $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$?

a.


d.


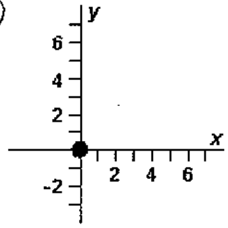
b.


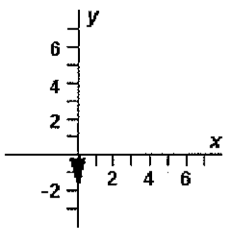
e.


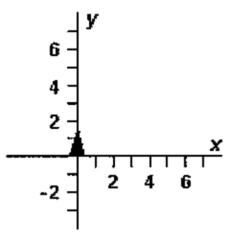
c. 

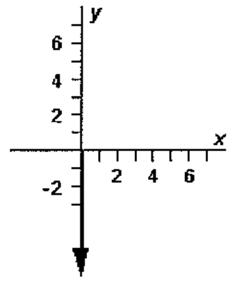
ANS: B PTS: 2 DIF: Average

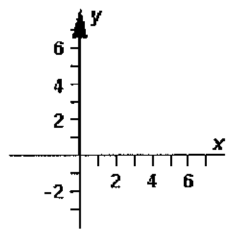
33. Refer to Exhibit 3-3. Which diagram below correctly represents $\vec{\mathbf{A}} - \vec{\mathbf{B}} + 2\vec{\mathbf{C}}$?

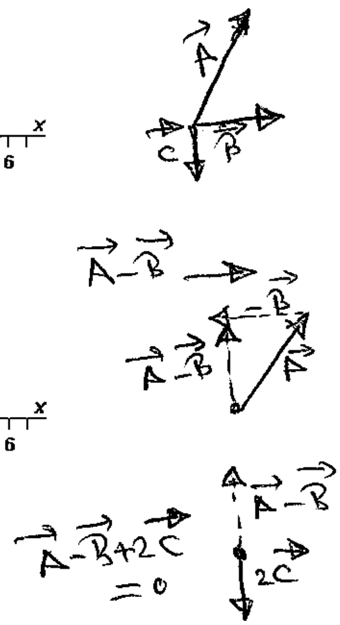
a. 

b. 

c. 

d. 

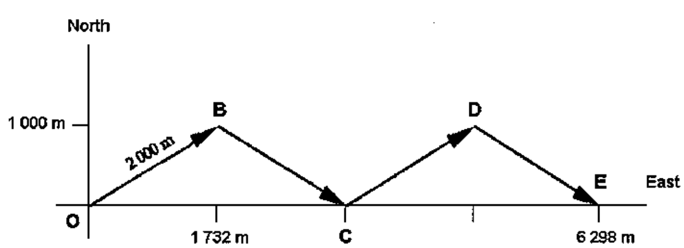
e. 



ANS: A PTS: 2 DIF: Average

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\hat{\mathbf{i}} + A\sin\theta\hat{\mathbf{j}}$.

Exhibit 3-4
The diagram below shows the path taken by a sailboat tacking sideways because it cannot sail directly into the wind.



Use this exhibit to answer the following question(s).

34. Refer to Exhibit 3-4. The total distance it travels is

distance = actual path = 2000 m (4) = 8000 m

a. 1 000 m.
b. 1 732 m.
c. 2 000 m.
d. 6 298 m.
e. 8 000 m.

ANS: E PTS: 1 DIF: Easy

35. Refer to Exhibit 3-4. The total displacement of the sailboat, the vector sum of its displacements **OB**, **BC**, **CD** and **DE**, is
a. 1 732 m, East.
b. 2 000 m, Northeast.
c. 6 298 m, East.
d. 8 000 m, Southeast.
e. 8 000 m, East.

displacement $= x_f - x_i$
$= 6298\text{m} - 0$
$= 6298\text{m}$

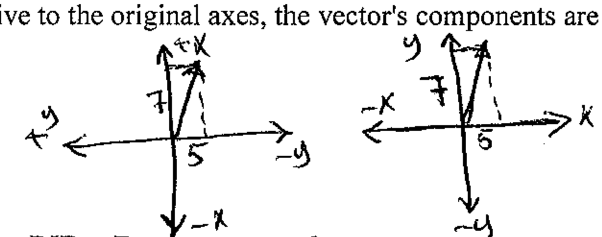
ANS: C PTS: 1 DIF: Easy

Instructions: On occasion, the notation $\vec{\mathbf{A}} = [A, \theta]$ will be a shorthand notation for $\vec{\mathbf{A}} = A\cos\theta\,\hat{\mathbf{i}} + A\sin\theta\,\hat{\mathbf{j}}$.

36. Dana says any vector $\vec{\mathbf{R}}$ can be represented as the sum of two vectors: $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. Ardis says any vector $\vec{\mathbf{R}}$ can be represented as the difference of two vectors: $\vec{\mathbf{R}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$. Which one, if either, is correct?
a. They are both wrong: every vector is unique.
b. Dana is correct: Any vector can be represented as a sum of components and not as a difference.
c. Ardis is correct: Any vector can be represented as a difference of vector components and not as a sum.
d. They are both correct: A difference of vectors is a sum $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$.
e. They are both wrong: Vectors can be moved as long as they keep the same magnitude and direction.

ANS: D PTS: 2 DIF: Average

37. The vector $\vec{\mathbf{A}}$ has components +5 and +7 along the x and y axes respectively. Along a set of axes rotated 90 degrees counterclockwise relative to the original axes, the vector's components are
a. −7; −5.
b. 7; −5.
c. −7; 5.
d. 7; 5.
e. 7; 0.



ANS: B PTS: 1 DIF: Easy

Rotation by 9° Counterclockwise

38. Anthony has added the vectors listed below and gotten the result $\vec{\mathbf{R}} = 9\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$. What errors has he made?

$\vec{\mathbf{A}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$
$\vec{\mathbf{B}} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$
$\vec{\mathbf{C}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\vec{A} + \vec{B} + \vec{C} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

a. He lost the minus sign in vector $\vec{\mathbf{B}}$.
b. He read the $2\hat{\mathbf{k}}$ in $\vec{\mathbf{C}}$ as $3\hat{\mathbf{k}}$.
c. He lost the minus sign in vector $\vec{\mathbf{A}}$.
d. All of the above are correct.
e. Only (a) and (b) above are correct.

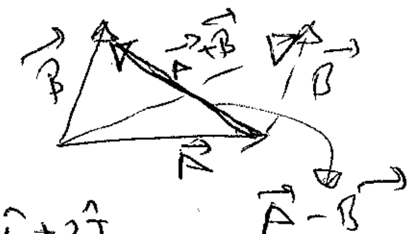
ANS: E PTS: 2 DIF: Average

39. Given the statement that $\vec{\mathbf{A}} - \vec{\mathbf{B}} = -\vec{\mathbf{A}} + \vec{\mathbf{C}}$, what can we conclude?
a. $\vec{\mathbf{C}} = \vec{\mathbf{A}}$ and $\vec{\mathbf{B}} = \vec{\mathbf{A}}$.
b. $2\vec{\mathbf{A}} = \vec{\mathbf{B}} + \vec{\mathbf{C}}$.
c. $\vec{\mathbf{C}} = -\vec{\mathbf{B}}$ and $-\vec{\mathbf{A}} = \vec{\mathbf{A}}$.
d. Any one of the answers above is correct.
e. Only (a) and (b) may be correct.

ANS: D PTS: 2 DIF: Average

40. Adding vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ by the graphical method gives the same result for $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and $\vec{\mathbf{B}} + \vec{\mathbf{A}}$. If both additions are done graphically from the same origin, the resultant is the vector that goes from the tail of the first vector to the tip of the second vector, i.e, it is represented by a diagonal of the parallelogram formed by showing both additions in the same figure. Note that a parallelogram has 2 diagonals. Keara says that the sum of two vectors by the parallelogram method is $\vec{\mathbf{R}} = 5\hat{\mathbf{i}}$. Shamu says it is $\vec{\mathbf{R}} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}}$. Both used the parallelogram method, but one used the wrong diagonal. Which one of the vector pairs below contains the original two vectors?
a. $\vec{\mathbf{A}} = -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$; $\vec{\mathbf{B}} = -2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$
b. $\vec{\mathbf{A}} = +3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$; $\vec{\mathbf{B}} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$
c. $\vec{\mathbf{A}} = -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$; $\vec{\mathbf{B}} = +2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$
d. $\vec{\mathbf{A}} = +3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$; $\vec{\mathbf{B}} = +2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$
e. $\vec{\mathbf{A}} = +3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$; $\vec{\mathbf{B}} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$\vec{A} - \vec{B} = 5\hat{i}$
$A + B = \hat{i} + 4\hat{j}$
$A = 3\hat{i} + 2\hat{j}$, $\vec{B} = -2\hat{i} + 2\hat{j}$



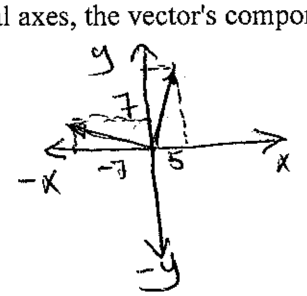
ANS: E PTS: 3 DIF: Challenging

41. Given two non-zero vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, such that $|\vec{\mathbf{A}}| = A = B = |\vec{\mathbf{B}}|$, the sum $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ satisfies
a. $0 \le |\vec{\mathbf{A}} + \vec{\mathbf{B}}| \le 2A$.
b. $0 < |\vec{\mathbf{A}} + \vec{\mathbf{B}}| < 2A$.
c. $A \le |\vec{\mathbf{A}} + \vec{\mathbf{B}}| \le 2A$.
d. $A < |\vec{\mathbf{A}} + \vec{\mathbf{B}}| < 2A$.
e. $0 \le |\vec{\mathbf{A}} + \vec{\mathbf{B}}| \le 4A$.

cosine method
$|\vec{A} + \vec{B}| = \sqrt{A^2 + A^2 + 2A^2\cos\theta} = \sqrt{2A^2(1+\cos\theta)}$
$= \sqrt{2}A\sqrt{(1+\cos\theta)}$
$-1 \le \cos\theta \le 1$
$|\vec{A} + \vec{B}| = 0$, $\cos\theta = -1$, $|\vec{A} + \vec{B}| = 2A$ $\cos\theta = 1$

ANS: A PTS: 2 DIF: Average

42. The vector $\vec{\mathbf{A}}$ has components +5 and +7 along the x and y axes respectively. If the vector is now rotated 90 degrees counterclockwise relative to the original axes, the vector's components are now
a. −7; −5.
b. 7; −5.
c. −7; 5.



d. 7; 5.
e. 7; 0.

ANS: C PTS: 1 DIF: Easy

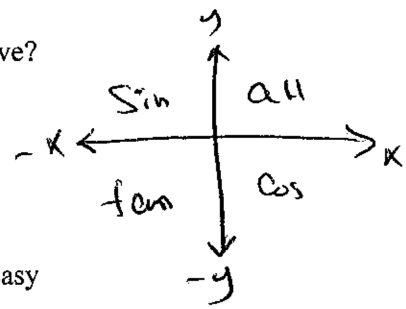
43. The rectangular coordinates of a point are (5.00, y) and the polar coordinates of this point are (r, 67.4°). What is the value of the polar coordinate r in this case?
a. 1.92
b. 4.62
c. 12.0
d. 13.0
e. More information is needed.

$r = \sqrt{x^2+y^2} = \sqrt{25+y^2}$, $\tan\delta = \frac{y}{x}$, $y = x\tan\delta = 5\tan 67.4°$

Then $r = \sqrt{25+(5\tan 67.4)^2} = 13$

ANS: D PTS: 2 DIF: Average

44. In what quadrant are both the sine and tangent negative?
a. 1st
b. 2nd
c. 3rd
d. 4th
e. This cannot happen.



ANS: D PTS: 1 DIF: Easy

PROBLEM

45. Two vectors starting at the same origin have equal and opposite x components. Is it possible for the two vectors to be perpendicular to each other? Justify your answer.

ANS:
Yes. If the y components are of the right magnitudes, the angle can be 90 degrees. (This will occur if $\theta_2 = \theta_1 + \frac{\pi}{2}$ and $A = B\tan\theta_1$.)

PTS: 3 DIF: Challenging

46. A vector starts at coordinate (3.0, 4.0) and ends at coordinate (−2.0, 16.0). What are the magnitude and direction of this vector?

$\vec{A} = (-2-3)\hat{i} + (16-4)\hat{j} = -5\hat{i} + 12\hat{j}$

$|\vec{A}| = \sqrt{25+144} = \sqrt{169} = 13.0\text{ m}$

$\tan\delta = \frac{+12}{-5}$, $\delta = \tan^{-1}\left(\frac{12}{5}\right)$, y is +ve and x is -ve it is in the 2nd quadrant

ANS:
13.0 m, 113°.

PTS: 2 DIF: Average

47. What two vectors are each the same magnitude as and perpendicular to $7\hat{\mathbf{i}} + 24\hat{\mathbf{j}}$?

ANS:
$-24\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$ and $24\hat{\mathbf{i}} - 7\hat{\mathbf{j}}$.

PTS: 3 DIF: Challenging