Assignment 3:
First Law of Thermodynamics, Kinetic Theory of Gases, Equipartition Theorem

Assigned: Oct 2 14:30 *Due: Oct 9 14:00*

1 A 4 liter sample of a diatomic gas with γ =1.4 confined to a cylinder, is carried through a closed cycle. The gas is initially at 1.00atm. and 300K. First, its pressure is tripled under constant volume. Then it expands adiabatically to its original pressure. Finally the gas is compressed isobarically to its original volume.

a) draw pV diagram of this cycle
b) determine the volume of the end of the adiabatic expansion
c) find the temperature of the gas at the start of the adiabatic expansion
d) find the temperature at the end of the cycle
e) what was the net work done on the gas for this cycle

(a) See the diagram at the right.

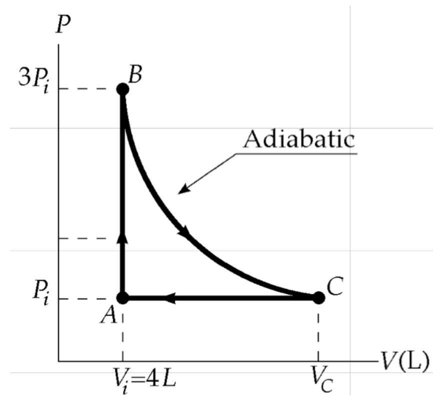


FIG. P17.43

(b) $P_B V_B^{\gamma} = P_C V_C^{\gamma}$

$3P_i V_i^{\gamma} = P_i V_C^{\gamma}$

$V_C = \left(3^{1/\gamma}\right)V_i = \left(3^{5/7}\right)V_i = 2.19V_i$

$V_C = 2.19(4.00\text{ L}) = \boxed{8.77\text{ L}}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = 3T_i = 3(300\text{ K}) = \boxed{900\text{ K}}$

(d) After one whole cycle, $T_A = T_i = \boxed{300\text{ K}}$

(e) In AB, $Q_{AB} = nC_V\Delta V = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$, $Q_{BC} = 0$ as this process is adiabatic

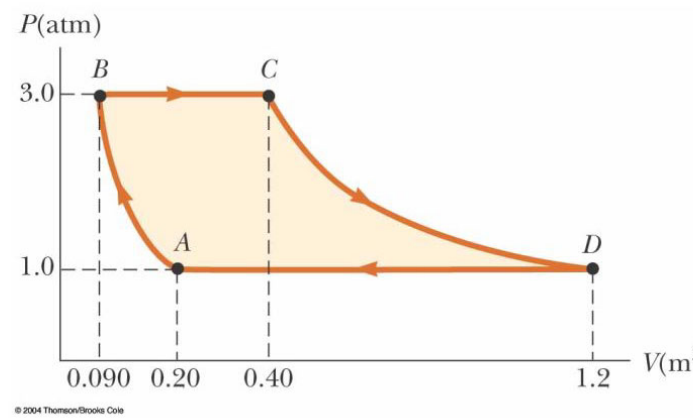
$P_C V_C = nRT_C = P_i(2.19V_i) = (2.19)nRT_i$ so $T_C = 2.19T_i$,

$$Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = (-4.17)nRT_i$$

For the whole cycle

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = (0.829)nRT_i$$
$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$
$$W_{ABCA} = -Q_{ABCA} = -(0.829)nRT_i = -(0.829)P_iV_i$$
$$W_{ABCA} = -(0.829)\left(1.013\times10^5\text{ Pa}\right)\left(4.00\times10^{-3}\text{ m}^3\right) = \boxed{-336\text{ J}}$$

2. A sample of an ideal gas goes through the process shown. From A to B, the process is adiabatic; from B to C, it is isobaric with 100 kJ of energy entering the system by heat. From C to D, the process is isothermal; from D to A, it is isobaric with 150 kJ of energy leaving the system by heat. Determine the difference in internal energy $E_{\text{int},B} - E_{\text{int},A}$.



$$W_{BC} = -P_B\left(V_C - V_B\right) = -3.00 \text{ atm}\left(0.400 - 0.090\,0\right) \text{ m}^3$$
$$= -94.2 \text{ kJ}$$
$$\Delta E_{\text{int}} = Q + W$$
$$E_{\text{int, C}} - E_{\text{int, B}} = \left(100 - 94.2\right) \text{ kJ}$$
$$E_{\text{int, C}} - E_{\text{int, B}} = 5.79 \text{ kJ}$$

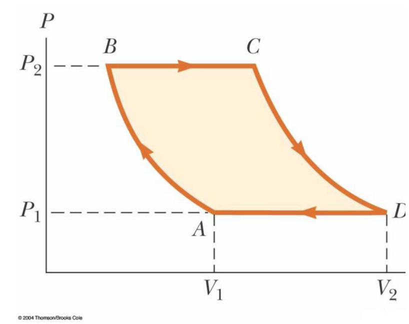
Since T is constant,

$$E_{\text{int, D}} - E_{\text{int, C}} = 0$$
$$W_{DA} = -P_D\left(V_A - V_D\right) = -1.00 \text{ atm}\left(0.200 - 1.20\right) \text{ m}^3$$
$$= +101 \text{ kJ}$$
$$E_{\text{int, A}} - E_{\text{int, D}} = -150 \text{ kJ} + \left(+101 \text{ kJ}\right) = -48.7 \text{ kJ}$$

since change of internal energy for the whole cycle is 0, we can write:

$$E_{\text{int, B}} - E_{\text{int, A}} = -\left[\left(E_{\text{int, C}} - E_{\text{int, B}}\right) + \left(E_{\text{int, D}} - E_{\text{int, C}}\right) + \left(E_{\text{int, A}} - E_{\text{int, D}}\right)\right] = -(5.79+0-48.7)\text{kJ} = 42.91\text{kJ}$$

3 An ideal gas is carried through a thermodynamic cycle consisting of two isobaric and two isothermal processes as shown. Show that the net work done on the gas in the entire cycle is given by



$$W_{\text{net}} = -P_1\left(V_2 - V_1\right)\ln\frac{P_2}{P_1}$$

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$W = -nRT_A \ln\frac{V_B}{V_A} - p_B(V_C - V_B) - p_D(V_A - V_D) - nRT_C \ln\frac{V_D}{V_C} \Rightarrow W = -nRT_A \ln\frac{\frac{nRT_B}{p_B}}{\frac{nRT_A}{p_A}} - nRT_C \ln\frac{\frac{nRT_D}{p_D}}{\frac{nRT_C}{p_C}} - p_B(V_C - V_B) -$$

$$W = -nRT_A \ln\frac{p_A}{p_B} - p_B(V_C - V_B) - p_D(V_A - V_D) - nRT_C \ln\frac{p_C}{p_D} = -nRT_1 \ln\frac{p_1}{p_2} - p_2(V_C - V_B) - p_1(V_1 - V_2) - nRT_2 \ln\frac{p_2}{p_1}$$

$$W = -nRT_1 \ln\frac{p_1}{p_2} + nRT_2 \ln\frac{p_1}{p_2} - p_2(V_C - V_B) - p_1(V_1 - V_2) = (nRT_2 - nRT_1)\ln\frac{p_1}{p_2} - p_2(V_C - V_B) - p_1(V_1 - V_2)$$

$$W = (p_1V_2 - p_1V_1)\ln\frac{p_1}{p_2} - p_2(V_C - V_B) - p_1(V_1 - V_2) = (p_1V_2 - p_1V_1)\ln\frac{p_1}{p_2} - (nRT_2 - nRT_1) - (nRT_1 - nRT_2)$$

Thus: $W = (p_1V_2 - p_1V_1)\ln\dfrac{p_1}{p_2}$ Q. E. D.

4 An ideal gas initially at 300 K undergoes an isobaric expansion at 2.50 kPa. If the volume increases from 1.00 m^3 to 3.00 m^3 and 12.5 kJ is transferred to the gas by heat,

a) what is the the change in its internal energy?

b) what is the final temperature

$$\Delta E_{\text{int}} = W + Q = -p\Delta V + Q = -2500(3-1)J + 12500J = 7500J$$

$$\Delta E_{\text{int}} = nC_V\Delta T = (\frac{p_1V_1}{T_1R})C_V\Delta T \Rightarrow 7500J = \frac{(2500)(1)}{300R}\frac{3}{2}R(T_f - 300K) \Rightarrow 7500 = \frac{(2500)(1)}{300}\frac{3}{2}(T_f - 300) \Rightarrow 1 = \frac{(T_f - 3}{30(}$$

$$T_f = 600K$$

5 Fill the missing entries in this table

Molecules	Degrees of freedom	Average Energy of 1 molecule (k)	Cv (R)	Cp(R)	Gamma
A	6	3kT	3R	4R	4/3
B	7	7/2kT	7/2R	9/2 R	9/7
C	9	9/2kT	9/2R	11/2R	11/9
D	10	5kT	5R	6R	6/5
E	2	kT	R	2R	2

6 Using the approach demonstrated during the lecture show that for pV^γ =const. for adiabatic gas process (Present your derivation on the opposite site of this page

<u>CHECK LECTURE NOTES FOR THIS DERIVATION!!!</u>