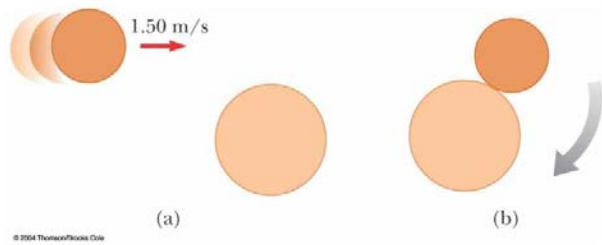
Physics 121.6 2007/2008

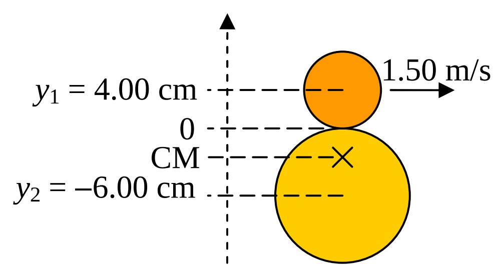
Assignment 11- Solutions

1. **Chapter 11, Problem 36.** A puck of mass 80.0 g and radius 4.00 cm slides along an air table at a speed of 1.50 m/s as shown in Figure P11.36a. It makes a glancing collision with a second puck of radius 6.00 cm and mass 120 g (initially at rest) such that their rims just touch. Because their rims are coated with instant-acting glue, the pucks stick together and spin after the collision (Fig. P11.36b).
(a) What is the angular momentum of the system relative to the center of mass?
(b) What is the angular speed about the center of mass?



Solution:

(a) At the instant the rims the situation looks like…



So

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{(80\text{ g})(4.00\text{ cm}) + (120\text{ g})(-6.00\text{ cm})}{(80\text{ g} + 120\text{ g})} = -2.00\text{ cm}$$

Therefore, just before they stick, the angular momentum relative to the centre of mass is

$$L = r_1 m_1 v_1 = (6.00\times10^{-2}\text{ m})(80\times10^{-3}\text{ kg})(1.50\text{ m/s}) = 7.20\times10^{-3}\text{ kg}\cdot\text{m}^2\text{/s}$$

(b) After they stick together the angular momentum about the CM will be the same. Since $L = I\omega$, to find ω we must find I relative to the CM. For a disk through its centre the moment of inertia is $\frac{1}{2}MR^2$ so we can use the parallel axis theorem to find the moment of inertia relative to the CM. Therefore total moment of inertia is

$$I = \tfrac{1}{2}m_1 r_1^{\,2} + m_1 d_1^{\,2} + \tfrac{1}{2}m_2 r_2^{\,2} + m_2 d_2^{\,2}$$
$$= \tfrac{1}{2}(80\times10^{-3}\text{ kg})(4.00\times10^{-2}\text{ m})^2 + (80\times10^{-3}\text{ kg})(6.00\times10^{-2}\text{ m})^2 + \tfrac{1}{2}(120\times10^{-3}\text{ kg})(6.00\times10^{-2}\text{ m})^2 + (120\times10^{-3}\text{ kg})(4.00\times10^{-2}\text{ m})^2$$
$$= 7.60\times10^{-4}\text{ kg}\cdot\text{m}^2$$

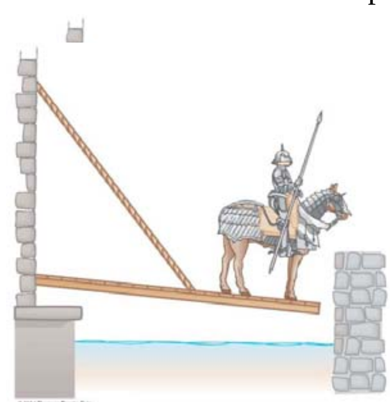
Therefore $\omega = \dfrac{L}{I} = \dfrac{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2\text{/s}}{7.60\times10^{-4}\text{ kg}\cdot\text{m}^2} = 9.47\text{ rad/s}$

2. To change the direction of the axis of rotation of a spinning object, one must
- (A) apply a torque about the axis of rotation.
- (B) change the moment of inertia about the axis of rotation
- (C) apply a torque about an axis that is not the axis of rotation.
- (D) Any of (A), (B) or (C).
- (E) None of (A), (B) or (C) will work.

Solution:

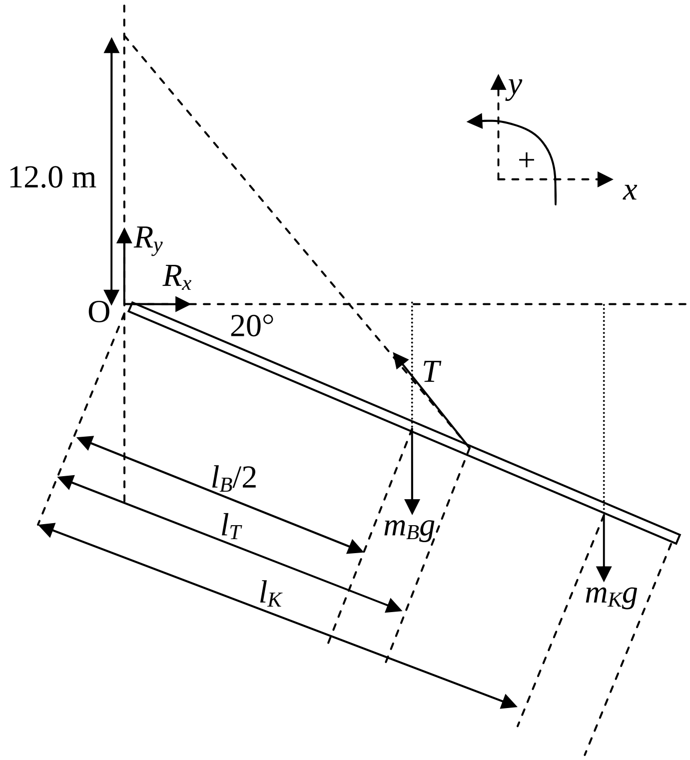
Since the torque is $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$, in a time interval Δt the change in angular momentum is $\Delta\mathbf{L} = \boldsymbol{\tau}\Delta t$. Thus the change in angular momentum is in the same direction as the torque. Thus if the direction of **L** is to change, the torque vector cannot be in the same direction as **L**. Thus the torque must be applied about an axis that does not coincide with the direction of **L**. Changing the moment of inertia does not change **L** at all.
Therefore the answer is **C.**

3. Chapter 12, Problem 16. Sir Lost-a-Lot dons his armour and sets out from the castle on his trusty steed in his quest to improve communication between damsels and dragons (Fig. P12.16). Unfortunately his squire lowered the drawbridge too far and finally stopped it 20.0° below the horizontal. Lost-a-Lot and his horse stop when their combined center of mass is 1.00 m from the end of the bridge. [i.e. assume his weight acts on the bridge at 1.00 m from the end.] The uniform bridge is 8.00 m long and has mass 2 000 kg. The lift cable is attached to the bridge 5.00 m from the hinge at the castle end, and to a point on the castle wall 12.0 m above the bridge. Lost-a-Lot's mass combined with his armour and steed is 1 000 kg. Determine (a) the tension in the cable and the (b) horizontal and (c) vertical force components acting on the bridge at the hinge.



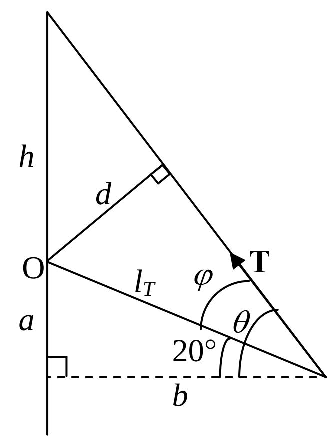
Solution:

FBD of forces on bridge:



Length of bridge $l_B = 8.00\,\text{m}$,
$l_T = 5.00\,\text{m}$, $l_K = 7.00\,\text{m}$.
Mass of bridge = m_B = 2000 kg.
Mass of knight, horse and armour = m_K = 1000 kg.

In order find the components of T, and the torque due to T, we need to find the angle the tension force makes with the horizontal (θ) and with the bridge (φ).



From the right angle triangles in the diagram at right:

$$a = l_T \sin 20° = (5.00\,\text{m})\sin(20°) = 1.71\,\text{m}$$

$$b = l_T \cos 20° = (5.00\,\text{m})\cos(20°) = 4.70\,\text{m}$$

$$\tan\theta = \frac{a+h}{b} = \frac{1.71\,\text{m} + 12.0\,\text{m}}{4.70\,\text{m}} \Rightarrow \theta = 71.1°$$

$$\Rightarrow \varphi = \theta - 20° = 51.1°$$

The moment arm for **T** is

$$d = l_T \sin\varphi = (5.00\,\text{m})\sin(51.1°) = 3.89\,\text{m}$$

$$\sum F_x = R_x - T\cos\theta = 0 \;\ldots ①$$

$$\sum F_y = R_y + T\sin\theta - m_B g - m_K g = 0 \;\ldots ②$$

Take torques about hinge, O:

$$\sum \tau = Td - m_B g(l_B/2)\cos 20^{\circ} - m_K g l_K \cos 20° = 0 \;\ldots ③$$

(a) From ③

$$T = \frac{m_B g\left(l_B/2\right)\cos 20^\circ + m_K g l_K \cos 20^\circ}{d}$$

$$= \frac{(2000\ \text{kg})(9.80\ \text{m/s}^2)(4.00\ \text{m})\cos(20.0^\circ) + (1000\ \text{kg})(9.80\ \text{m/s}^2)(7.00\ \text{m})\cos(20.0^\circ)}{3.89\ \text{m}}$$

$$= 3.55\times10^4\ \text{N}$$

(b) From ①

$$R_x = T\cos\theta = (3.55\times10^4\ \text{N})\cos(71.1^\circ) = 1.15\times10^4\ \text{N}$$

(c) From ②

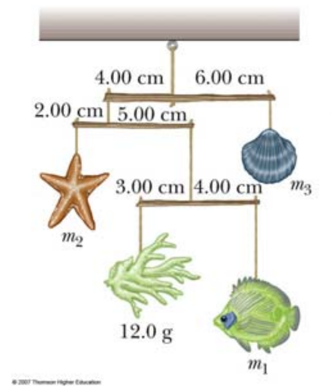
$$R_y = m_B g + m_K g - T\sin\theta$$

$$= (2000\ \text{kg})(9.80\ \text{m/s}^2) + (1000\ \text{kg})(9.80\ \text{m/s}^2) - (3.55\times10^4\ \text{N})\sin(71.1^\circ)$$

$$= -4.19\times10^3\ \text{N}$$

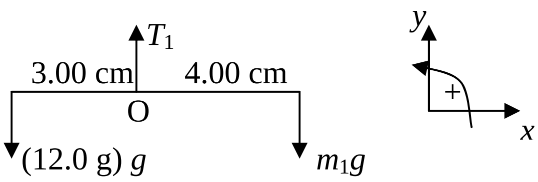
Thus this component points downward.

4. Chapter 12, Problem 8. A mobile is constructed of light rods, light strings, and beach souvenirs as shown in Figure P12.8. Determine the masses of the objects (a) m_1, (b) m_2, and (c) m_3.



Solution:

(a) Consider forces on lowest bar:



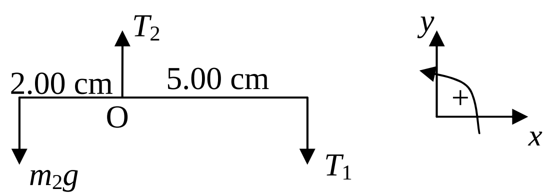
$$\sum F_y = T_1 - (12.0\text{ g})g - m_1 g = 0 \ \ldots \ ①$$

$$\sum \tau_O = (12.0\text{ g})g(3.00\text{ cm}) - m_1 g(4.00\text{ cm}) = 0 \ \ldots \ ②$$

From ② $m_1 = \dfrac{(12.0\text{ g})(3.00\text{ cm})}{(4.00\text{ cm})} = 9.00\text{ g}$

(b) From ① $T_1 = (12.0\text{ g})g + m_1 g = (12.0\text{ g} + 9.00\text{ g})g = (21.0\text{ g})g$

Consider forces on second lowest bar:



$$\sum F_y = T_2 - m_2 g - T_1 = 0 \ \ldots \ ③$$

$$\sum \tau_O = m_2 g(2.00\text{ cm}) - T_1(5.00\text{ cm}) = 0 \ \ldots \ ④$$

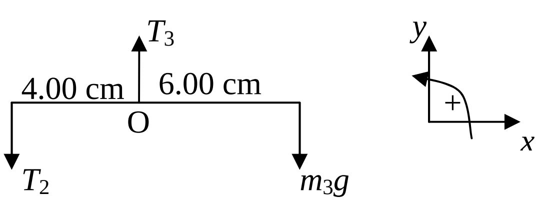
From ④

$$m_2 g(2.00\text{ cm}) = (21.0\text{ g})g(5.00\text{ cm})$$

$$\Rightarrow \mathrm{m}_2 = \frac{(21.0\text{ g})(5.00\text{ cm})}{(2.00\text{ cm})} = 52.5\text{ g}$$

(c) From ③ $T_2 = m_2 g + T_1 = (52.5\text{ g})g + (21.0\text{ g})g = (73.5\text{ g})g$

Consider forces on top bar:



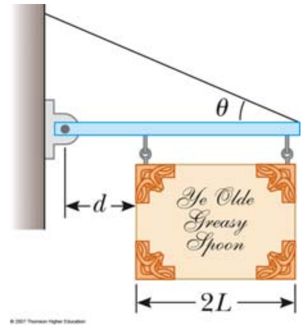
$$\sum \tau_O = T_2(4.00\text{ cm}) - m_3 g(6.00\text{ cm}) = 0 \ \ldots \ ⑤$$

From ⑤

$$(73.5\text{ g})g(4.00\text{ cm}) = m_3 g(6.00\text{ cm})$$

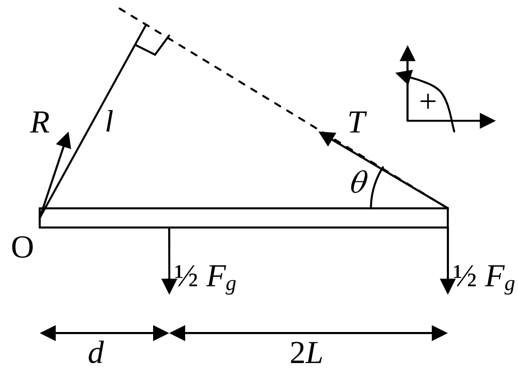
$$\Rightarrow m_3 = \frac{(73.5\text{ g})(4.00\text{ cm})}{(6.00\text{ cm})} = 49.0\text{ g}$$

5. **Chapter 12, Problem 39.** A uniform sign of weight F_g and width $2L$ hangs from a light horizontal beam hinged at the wall and supported by a cable (Fig. P12.39). Determine (a) the tension in the cable and (b) the components of the reaction force exerted by the wall on the beam, in terms of F_g, d, L, and θ.



Solution:

FBD of forces on beam.
Note that it is a "light" beam so we ignore its weight.
Also the sign is supported by the two attachment points so half its weight is supported by each. (Alternatively we could consider its whole weight to be supported in its middle.)
We do not know the direction of the reaction force of magnitude R.



$\sum F_x = R_x - T\cos\theta = 0$ … ①

$\sum F_y = R_y + T\sin\theta - \frac{1}{2}F_g - \frac{1}{2}F_g = 0$ … ②

$\sum \tau_O = Tl - \frac{1}{2}F_g d - \frac{1}{2}F_g(d+2L) = 0$ … ③ where l is the lever arm for the tension force.

From the triangle: $l = (d+2L)\sin\theta$

(a) From ③

$T(d+2L)\sin\theta = \frac{1}{2}F_g(2d+2L) = F_g(d+L)$

$$\Rightarrow T = \frac{F_g(d+L)}{(d+2L)\sin\theta}$$

(b) From ①

$$R_x = T\cos\theta = \frac{F_g(d+L)\cos\theta}{(d+2L)\sin\theta} = \frac{F_g(d+L)}{(d+2L)\tan\theta}$$

and from ②

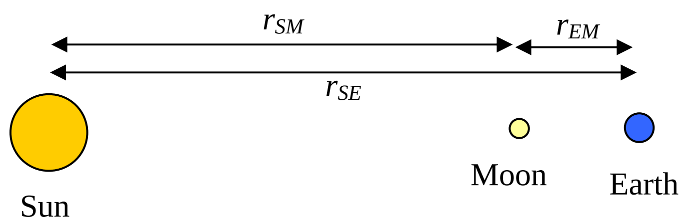
$$R_y = F_g - T\sin\theta = F_g - \frac{F_g(d+L)}{(d+2L)\sin\theta}\sin\theta$$

$$= F_g\left(1 - \frac{d+L}{d+2L}\right) = F_g\left(\frac{d+2L-(d+L)}{d+2L}\right)$$

$$= \frac{F_g L}{d+2L}$$

6. **Chapter 13, Problem 6.** During a solar eclipse, the Moon, Earth, and Sun all lie on the same line, with the Moon between the Earth and the Sun. (a) What force is exerted by the Sun on the Moon? (b) What force is exerted by the Earth on the Moon? (c) What force is exerted by the Sun on the Earth? (d) Compare the answers from parts (a) and (b). Why doesn't the Sun capture the Moon away from the Earth?

Solution:



From textbook:
$r_{SE} = 1.496 \times 10^{11}$ m,
$r_{EM} = 3.84 \times 10^{8}$ m.
$M_S = 1.991 \times 10^{30}$ kg,
$M_E = 5.98 \times 10^{24}$ kg,
$M_M = 7.36 \times 10^{22}$ kg.
So $r_{SM} = r_{SE} - r_{EM} = 1.496 \times 10^{11}\ \text{m} - 3.84 \times 10^{8}\ \text{m} = 1.492 \times 10^{11}$ m.

(a) Force exerted by Sun on Moon:

$$F_{SM} = \frac{GM_S M_M}{r_{SM}^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(7.36\times10^{22}\ \text{kg})}{\left(1.492\times10^{11}\ \text{m}\right)^2} = 4.39\times10^{20}\ \text{N}$$

(b) Force exerted by Earth on Moon:

$$F_{EM} = \frac{GM_E M_M}{r_{EM}^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(7.36\times10^{22}\ \text{kg})}{\left(3.84\times10^{8}\ \text{m}\right)^2} = 1.99\times10^{20}\ \text{N}$$

(c) Force exerted by Sun on Earth:

$$F_{SE} = \frac{GM_S M_E}{r_{SE}^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})}{\left(1.496\times10^{11}\ \text{m}\right)^2} = 3.55\times10^{22}\ \text{N}$$

(d) The force of the Sun on the Moon is greater than the force of the Earth on the Moon. The Sun has captured the whole Earth-Moon system. Both the Earth and the Moon orbit the Sun. In fact the orbit of the Moon is concave towards the Sun at all points with a little wobble due to the Earth presence (and the same could be said for the Earth).

7. Chapter 13, Problem 11. The free-fall acceleration on the surface of the Moon is about one-sixth that on the surface of the Earth. If the radius of the Moon is about 0.250 R_E, find the ratio of their average densities, $\rho_{\text{Moon}}/\rho_{\text{Earth}}$.

Solution:

For the Earth: $\rho_{\text{Earth}} = \frac{M_E}{V_E} = \frac{M_E}{\frac{4}{3}\pi R_E^{\ 3}}$

And $g_E = \frac{GM_E}{R_E^{\ 2}} \Rightarrow M_E = \frac{g_E R_E^{\ 2}}{G}$

So $\rho_{\text{Earth}} = \frac{M_E}{\frac{4}{3}\pi R_E^{\ 3}} = \frac{3 g_E R_E^{\ 2}}{4\pi R_E^{\ 3} G} = \frac{3}{4\pi G}\frac{g_E}{R_E}$

Similarly for the Moon: $\rho_{\text{Moon}} = \frac{3}{4\pi G}\frac{g_M}{R_M}$

We know: $g_M = g_E / 6$ and $R_M = 0.250 R_E = R_E / 4$

So:

$$\frac{\rho_{\text{Moon}}}{\rho_{\text{Earth}}} = \frac{\left(g_M / R_M\right)}{\left(g_E / R_E\right)} = \frac{g_M R_E}{g_E R_M} = \frac{(g_E/6) R_E}{g_E (R_E/4)} = \frac{2}{3} = 0.667$$

8. The magnitude of the acceleration of a meteor at a height above the Earth's surface equal to the radius of the Earth is

(A) about 2.5 m/s^2.
(B) about 4.9 m/s^2.
(C) 9.8 m/s^2.
(D) is greater than 9.8 m/s^2.
(E) a value that depends on how the meteor is moving.

Solution:

$g = \frac{GM_E}{R_E^{\ 2}}$ at Earths surface.

And so at the position of the meteor: $g_M = \frac{GM_E}{R_M^{\ 2}} = \frac{GM_E}{\left(2R_E\right)^2} = \frac{1}{4}\frac{GM_E}{R_E^{\ 2}} = \frac{g}{4}$

So $g_M \approx \frac{9.8\,\text{m/s}^2}{4} \approx 2.5\,\text{m/s}^2$

Answer **A**.

9. **Chapter 13, Problem 18.** Neutron stars are extremely dense objects that are formed from the remnants of supernova explosions. Many rotate very rapidly. Suppose that the mass of a certain spherical neutron star is twice the mass of the Sun and its radius is 10.0 km. Determine the greatest possible angular speed it can have so that the matter at the surface of the star on its equator is just held in orbit by the gravitational force.

Solution:

For the matter near the surface to be in orbit: $F_g = ma_c$.Thus:

$$\frac{GMm}{r^2} = ma_c = mr\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{GM}{r^3}}$$

Now $M = 2M_{Sun}$, $M_{Sun} = 1.99 \times 10^{30}$ kg, $r = 10.0$ km

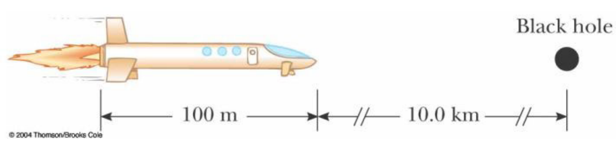
$$\omega = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)2(1.99\times10^{30}\ \text{kg})}{(1.00\times10^4\ \text{m})^3}}$$

$$= 1.63\times10^4\ \text{rad/s}$$

$$= 2.59\times10^3\ \text{revolutions/s}$$

If the neutron star rotated faster than this, the matter near its surface would be flung off! Many neutron stars (pulsars) have been found with very high revolution rates. These observations place limits on the size of these objects.

10. **Chapter 13, Problem 22.** A spacecraft in the shape of a long cylinder has a length of 100 m and its mass with occupants is 1 000 kg. It has strayed too close to a black hole having a mass 100 times that of the Sun (Fig. P13.22). The nose of the spacecraft points toward the black hole, and the distance between the nose and the center of the black hole is 10.0 km. (a) Determine the total force on the spacecraft. (b) What is the difference in the gravitational fields acting on the occupants in the nose of the ship and on those in the rear of the ship, farthest from the black hole? This difference in accelerations grows rapidly as the ship approaches the black hole. It puts the body of the ship under extreme tension and eventually tears it apart.



Solution:

(a) The distance of the centre of mass of the spacecraft from the black hole is r = 10.0 km + 50 m = 1.005 × 10^4 m. $M = 100M_{Sun}$.

Total force on Spacecraft is:

$$F_g = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100)(1.99\times10^{30}\ \text{kg})(1000\ \text{kg})}{(1.005\times10^4\ \text{m})^2} = 1.31\times10^{17}\ \text{N}$$

$g_{front} = \frac{GM}{\left(r_{front}\right)^2}$ and $g_{back} = \frac{GM}{\left(r_{back}\right)^2}$, r_{front} = 1.000 × 10^4 m, r_{back} = 1.010 × 10^4 m.

$$\Delta g = g_{front} - g_{back} = GM\left(\frac{1}{\left(r_{front}\right)^2} - \frac{1}{\left(r_{back}\right)^2}\right) = GM\left(\frac{\left(r_{back}\right)^2 - \left(r_{front}\right)^2}{\left(r_{front}r_{back}\right)^2}\right)$$

$$\Rightarrow \Delta g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100)(1.99\times10^{30}\ \text{kg})\left(\frac{\left(1.01\times10^4\ \text{m}\right)^2 - \left(1.00\times10^4\ \text{m}\right)^2}{\left(\left(1.01\times10^4\ \text{m}\right)\left(1.00\times10^4\right)\right)^2}\right)$$

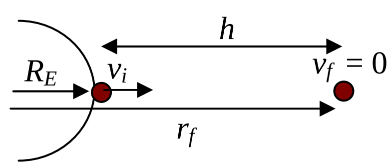
$$\Rightarrow \Delta g = 2.62\times10^{12}\ \text{N/kg} = 2.62\times10^{12}\ \text{m/s}^2$$

A very large difference in the acceleration of the front of the spacecraft relative to the back! Thus the spaceship will be torn apart.

11. **Chapter 13, Problem 26.** At the Earth's surface a projectile is launched straight up at a speed of 10.0 km/s. To what height will it rise? Ignore air resistance and the rotation of the Earth.

Solution:

Conservation of Energy:



$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \frac{1}{2}mv_i^2 - \frac{GM_Em}{R_E} = \frac{1}{2}mv_f^2 - \frac{GM_Em}{r_f}$$

$$\Rightarrow \frac{1}{2}v_i^2 - \frac{GM_E}{R_E} = -\frac{GM_E}{r_f}$$

$$\Rightarrow \frac{1}{r_f} = \frac{1}{R_E} - \frac{v_i^2}{2GM_E}$$

$$r_f = \left(\frac{1}{(6.37\times10^6\ \text{m})} - \frac{(10.0\times10^3\ \text{m/s})^2}{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}\right)^{-1}$$

$$= 3.16\times10^7\ \text{m}$$

So height above Earth is

$h = r_f - R_E$ = 3.16 × 10^7 m – 6.37 × 10^6 m = 2.52 × 10^7 m.

12. Chapter 13, Problem 38. A satellite moves around the Earth in a circular orbit of radius r. (a) What is the speed v_0 of the satellite? Suddenly, an explosion breaks the satellite into two pieces, with masses m and $4m$. Immediately after the explosion the smaller piece of mass m is stationary with respect to the Earth and falls directly toward the Earth. (b) What is the speed v_i of the larger piece immediately after the explosion? (c) Because of the increase in its speed, this larger piece now moves in a new elliptical orbit. Find its distance away from the center of the Earth when it reaches the other end of the ellipse.

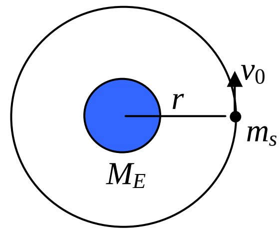
Solution:

(a) Circular orbit: Mass of satellite is $m_s = 5m$.

$$F_g = m_s a_c$$

$$\Rightarrow \frac{GM_E m_s}{r^2} = m_s \frac{v_0^{\ 2}}{r}$$

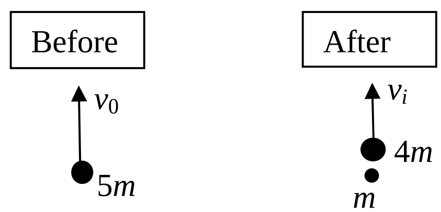
$$\Rightarrow v_0 = \sqrt{\frac{GM_E}{r}}$$



(b) Conservation of momentum:

$$5mv_0 = m(0) + 4mv_i$$

$$\Rightarrow v_i = \tfrac{5}{4} v_0$$



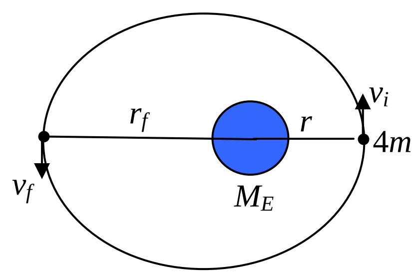
(c) The initial position will be where the satellite piece is closest to Earth. Using conservation of angular momentum:

$$r4mv_i = r_f 4mv_f \Rightarrow r_f v_f = rv_i \ \ldots ①$$

Conservation of Energy: $K_i + U_i = K_f + U_f$

$$\tfrac{1}{2}(4m)v_i^{\ 2} - \frac{GM_E 4m}{r} = \tfrac{1}{2}(4m)v_f^{\ 2} - \frac{GM_E 4m}{r_f}$$

$$\Rightarrow v_i^{\ 2} - \frac{2GM_E}{r} = v_f^{\ 2} - \frac{2GM_E}{r_f} \ \ldots ②$$



From ① $v_f = \dfrac{r}{r_f} v_i$

$$\Rightarrow v_i^{\ 2} - \frac{2GM_E}{r} = \frac{r^2}{r_f^{\ 2}} v_i^{\ 2} - \frac{2GM_E}{r_f}$$

Also from (b) and (a): $v_i^{\ 2} = \frac{25}{16} v_0^{\ 2} = \frac{25}{16} \frac{GM_E}{r}$, so:

$$\Rightarrow \frac{25}{16} \frac{GM_E}{r} - \frac{2GM_E}{r} = \frac{r^2}{r_f^{\ 2}} \frac{25}{16} \frac{GM_E}{r} - \frac{2GM_E}{r_f}$$

$$\Rightarrow \frac{25}{16} - 2 = \frac{25}{16} \frac{r^2}{r_f^{\ 2}} - 2\frac{r}{r_f}$$

$$\Rightarrow \frac{25}{16} \frac{r^2}{r_f^{\ 2}} - 2\frac{r}{r_f} + \frac{7}{16} = 0$$

$$\Rightarrow 7r_f^{\ 2} - 32rr_f + 25r^2 = 0$$

$$\Rightarrow r_f = \frac{-(-32r) \pm \sqrt{(-32r)^2 - 4(7)(25r^2)}}{2(7)} = \frac{32r \pm \sqrt{324r^2}}{14} = \frac{32 \pm 18}{14} r$$

Thus: $r_f = r$ or $\frac{50}{14} r$, the first result is the initial value so the final value is $r_f = \frac{25}{7} r$.

13. If *Martian Orbiter I* is sailing around the planet in a circle which has an orbital radius nine times that of *Martian Orbiter II*, who's speed is v_2, what is the speed of *Martian Orbiter I*?

(A) $\frac{1}{9} v_2$
(B) $\frac{1}{3} v_2$
(C) v_2
(D) $3v_2$
(E) $81v_2$

Solution:

For a circular orbit for *Martian Orbiter I*:

$\frac{GM_M m_1}{r_1^2} = m_1 a_c = m_1 \frac{v_1^2}{r_1}$ where v_1 is its speed, m_1 is its mass and r_1 is its orbit radius.

$$\Rightarrow v_1 = \sqrt{\frac{GM_M}{r_1}}$$

Similarly for *Martian Orbiter II*:

$$\Rightarrow v_2 = \sqrt{\frac{GM_M}{r_2}}$$

Therefore,

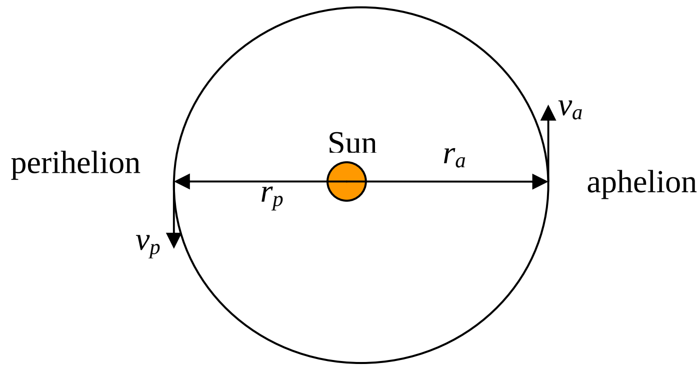
$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$ and since $r_1 = 9r_2$,

$$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{9r_2}} = \frac{1}{3} \Rightarrow v_1 = \frac{1}{3} v_2$$

Answer **B**.

14. Chapter 13, Problem 52. The maximum distance from the Earth to the Sun (at aphelion) is 1.521×10^{11} m, and the distance of closest approach (at perihelion) is 1.471×10^{11} m. The Earth's orbital speed at perihelion is 3.027×10^4 m/s. Determine (a) the Earth's orbital speed at aphelion [do not use energy conservation to do this part], (b) the kinetic and potential energies of the Earth-Sun system at perihelion, and (c) the kinetic and potential energies at aphelion. Is the total energy of the system constant? (Ignore the effect of the Moon and other planets.)

Solution:



(a) Conservation of Angular Momentum

Let m = mass of Earth = 5.98×10^{24} kg

$$L_a = L_p$$

$$\Rightarrow r_a m v_a = r_p m v_p$$

$$\Rightarrow v_a = \frac{r_p}{r_a} v_p$$

$$v_a = \frac{(1.471\times10^{11}\text{ m})(3.027\times10^4\text{ m/s})}{(1.521\times10^{11}\text{ m})}$$

$$= 2.927\times10^4\text{ m/s}$$

(b) At perihelion:

$$K_p = \tfrac{1}{2} m v_p^2 = \tfrac{1}{2}(5.98\times10^{24}\text{ kg})(3.027\times10^4\text{ kg})^2 = 2.74\times10^{33}\text{ J}$$

$$U_p = -\frac{GM_S m}{r_p} = -\frac{(6.67\times10^{-11}\text{ N.m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})(5.98\times10^{24}\text{ kg})}{(1.471\times10^{11}\text{ m})} = -5.40\times10^{33}\text{ J}$$

(c) At aphelion:

$$K_a = \tfrac{1}{2} m v_a^2 = \tfrac{1}{2}(5.98\times10^{24}\text{ kg})(2.927\times10^4\text{ kg})^2 = 2.56\times10^{33}\text{ J}$$

$$U_a = -\frac{GM_S m}{r_a} = -\frac{(6.67\times10^{-11}\text{ N.m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})(5.98\times10^{24}\text{ kg})}{(1.521\times10^{11}\text{ m})} = -5.22\times10^{33}\text{ J}$$

Total energy at perihelion: $E_p = K_p + U_p = -2.66\times10^{55}$ J

Total energy at aphelion: $E_a = K_a + U_a = -2.66\times10^{55}$ J

They agree, so we see that the total mechanical energy is indeed conserved as we expect since there are no external forces doing work on the Sun-Earth system.

Note: We have ignored the kinetic energy of the Sun in this calculation. The Sun does have some motion as the Sun-Earth system actually rotates about the centre of mass of the Sun-Earth system. But, since the mass of the Sun is so much larger than the mass of the Earth, its speed is small and so its kinetic energy is negligible.

15. **Chapter 14, Problem 4.** What is the total mass of the Earth's atmosphere? (The radius of the Earth is 6.37×10^6 m, and atmospheric pressure at the Earth's surface is 1.013×10^5 Pa.) [This is an estimate only. You can ignore the mountains and treat the Earth as a sphere. The atmosphere is not very thick compared to the radius of the Earth so you may assume that the acceleration due to gravity is approximately constant, at 9.8 m/s^2, throughout the height of the atmosphere.]

Solution:

We assume that the pressure at the Earth's surface is the total weight of the atmosphere spread over the surface area of the Earth. i.e.

$P_0 = \frac{mg}{A}$, where m is to total mass of the atmosphere, and assuming g does not change much with height. If the radius of the Earth is r, $A = 4\pi r^2$. So

$$P_0 = \frac{mg}{4\pi r^2}$$

$$\Rightarrow m = \frac{4\pi r^2 P_0}{g} = \frac{4\pi(6.37\times10^6 \text{ m})^2(1.013\times10^5 \text{ Pa})}{(9.80 \text{ m/s}^2)} = 5.3\times10^{18} \text{ kg}$$

16. **Chapter 14, Problem 14.** Figure P14.14 shows Superman attempting to drink water through a very long straw. With his great strength he achieves maximum possible suction. The walls of the tubular straw do not collapse. (a) Find the maximum height through which he can lift the water. (b) **What If?** Still thirsty, the Man of Steel repeats his attempt on the Moon, which has no atmosphere. Find the difference between the water levels inside and outside the straw.

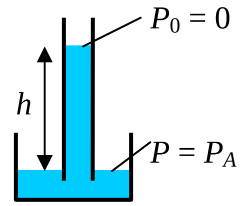
Solution:

(a) With maximum possible suction there will be a vacuum at the top of the tube. In this case the pressure at the top is zero. Pressure at the bottom is atmospheric pressure P_A.

$$P = P_0 + \rho g h$$

$$\Rightarrow P_A = 0 + \rho g h$$

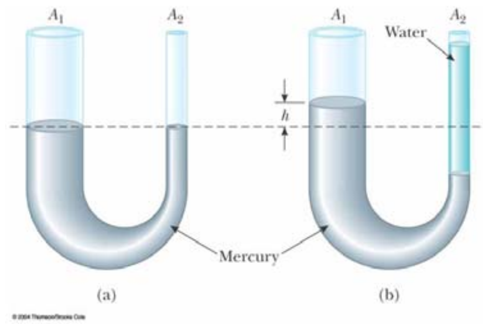
$$\Rightarrow h = \frac{P_A}{\rho g} = \frac{1.01\times10^5 \text{ Pa}}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 10.3 \text{ m}$$



(b) On the moon there is no atmosphere, so both P and P_0 are zero. Thus $h = 0$.

This is a clear illustration that on the Earth it is there air pressure that pushes the water up the straw, not the vacuum that 'pulls' it up. On the Moon there is no atmosphere to do the pushing.

17. Chapter 14, Problem 16. Mercury is poured into a U-tube as in Figure P14.18a. The left arm of the tube has cross-sectional area A_1 of 10.0 cm^2, and the right arm has a cross-sectional area A_2 of 5.00 cm^2. One hundred grams of water are then poured into the right arm as in Figure P14.18b. (a) Determine the length of the water column in the right arm of the U-tube. (b) Given that the density of mercury is 13.6 g/cm^3, what distance h does the mercury rise in the left arm?

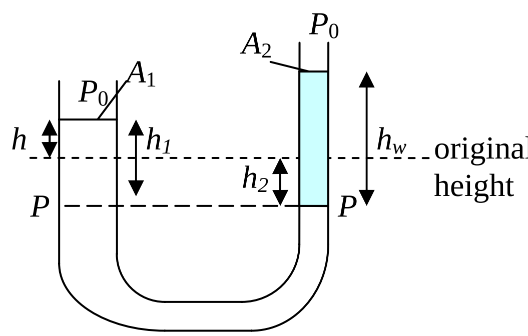


Solution:

(a) Mass of water column: $m = \rho V = \rho A_2 h_w$ where h_w = height of water column, ρ = density of water.

$$\Rightarrow h_w = \frac{m}{\rho A_2} = \frac{(100\,\text{g})}{(1.00\,\text{m/cm}^3)(5.00\,\text{cm}^2)} = 20.0\,\text{cm}$$

(b) The right hand column of mercury has been depressed by a height h_2 and the left hand column of mercury has risen by height h. A volume ($A_2 h_2$) has been displaced by water in the right hand column. This must be equal to the additional volume of mercury in the left hand column ($A_1 h$).



So $A_1 h = A_2 h_2 \ldots①$

The pressure at the top of the mercury in the right had column (P) must be equal to the pressure in the mercury at the same level in the left hand column.

In the right hand column of water:

$P = P_0 + \rho g h_w \ldots②$

In the left hand column of mercury:

$P = P_0 + \rho_{Hg} g h_1 \ \ldots③$

Noting that from ① $h_1 = h_2 + h = \frac{A_1}{A_1} h + h = \left(\frac{A_1}{A_2} + 1\right) h$

Equating ② and ③ and inserting h_1:

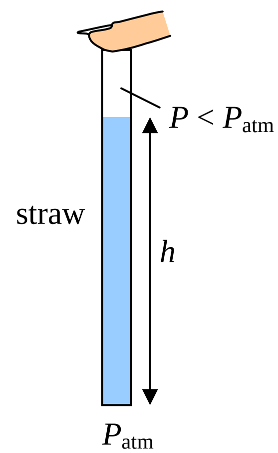
$$\rho g h_w = \rho_{Hg} g h_1 = \rho_{Hg} g \left(\frac{A_1}{A_2} + 1\right) h = \rho_{Hg} g \left(\frac{A_1 + A_2}{A_2}\right) h$$

$$\Rightarrow h = \frac{\rho h_w}{\rho_{Hg}\left(\frac{A_1}{A_2} + 1\right)} = \frac{(1.00\,\text{g/cm}^3)(20.0\,\text{cm})}{(13.6\,\text{g/cm}^3)\left(\frac{10.0\,\text{cm}^2}{5.00\,\text{cm}^2} + 1\right)} = 0.490\,\text{cm}$$

18. Suppose you stick a drinking straw into a deep cup of liquid, tightly cover the upper end with a finger, then raise the straw up and out of the liquid.
(A) The liquid will always run out of the straw.
(B) Some liquid stays in the straw, but its level initially drops.
(C) The liquid will all stay in the straw.
(D) The result depends on the type of liquid.

Solution:

Some liquid will begin to run out of the straw. As it does so volume between the top of the liquid and your finger increases, and so the pressure in that volume will decrease since no air can enter. Soon the pressure in this volume will be low enough so that the pressure difference between the bottom and the top of the liquid in the straw will be enough to support it. Try it!



Answer **B.**