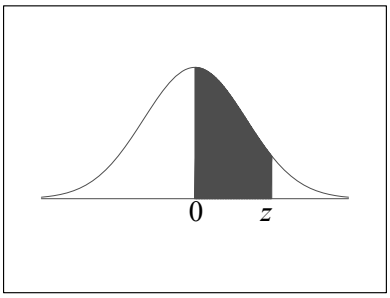
COMM 215
Midterm Tutorial

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Gilles Cazelais. Typeset with LaTeX on April 20, 2006.

Basic Counting Principles

Multiplication Principle
Consider a multistep process in which

Step 1 has n_1 possible outcomes,

Step 2 has n_2 possible outcomes,

...

Step r has n_r possible outcomes.

Then, the entire process has $n_1 \times n_2 \times \cdots \times n_r$ possible outcomes.

Inclusion-Exclusion Principle
If A and B are two finite sets, then

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

A **permutation of n distinct items taken r at a time** is an *ordered* list of r distinct items chosen from a set of n distinct items. The number of such permutations is given by

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}.$$

Observe that $P(n, n) = n!$.

A **combination of n distinct items taken r at a time** is an *unordered* set of r distinct items chosen from a set of n distinct items. The number of such combinations is given by

$$C(n, r) = \frac{n!}{r!(n-r)!}.$$

Another notation for $C(n, r)$ is $\binom{n}{r}$.

Problems

1. (a) How many bit strings of length 8 are possible?
   (b) How many of these start with a 1 or end with 00?
   Answers: (a) 256 (b) 160

2. How many functions are there from a set with 6 elements to a set with 4 elements.
   Answer: 4096

3. How many one-to-one functions are there from a set with 6 elements to a set with
   (a) 4 elements (b) 6 elements (c) 10 elements.
   Answers: (a) 0 (b) 720 (c) 151 200

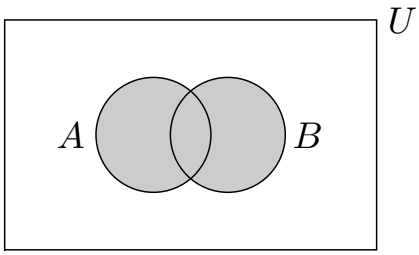
4. Each user on a computer system has a password, which is six to eight characters long, where each character is a letter (case sensitive) or a digit. If each password must contain at least one digit and at least one letter, how many possible passwords are there?
   Answer: $167\,410\,838\,583\,040 \approx 1.67 \times 10^{14}$

5. (a) How many different 7-place license plates are possible if the first 3 places are for capital letters and the other 4 for digits?
   (b) What if no letter and digit can be repeated in a single license plate?
   (c) What if adjacent letters and digits have to be different?
   Answers: (a) 175 760 000 (b) 78 624 000 (c) 118 462 500

6. The 10 letters ABCDEFGHIJ are used to form strings of length 7 (order matters).
   (a) How many possible strings are there if we do not allow repetition?

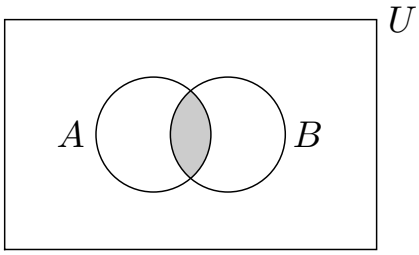
Basic Set Operations

The **union** of sets A and B is denoted $A \cup B$. It corresponds to the set of all elements that are in A or in B.



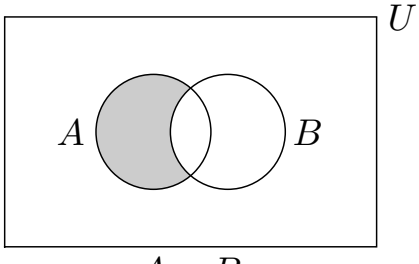
$A \cup B$

The **intersection** of sets A and B is denoted $A \cap B$. It corresponds to the set of all elements that are both in A and in B.



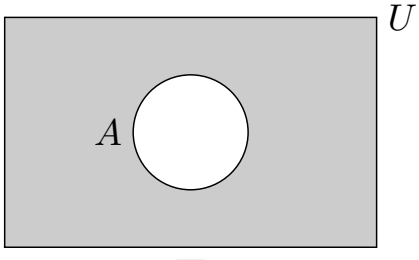
$A \cap B$

The **difference** of sets A and B is denoted $A - B$. It corresponds to the set of all elements that are in A but not in B.



$A - B$

Let U be the universal set. The **complement** of set A is denoted $\overline{A}$. It corresponds to the set of all elements that are in U but not in A, in fact we have $\overline{A} = U - A$.



$\overline{A}$

Wes J. Nawfal M.Sc.
Winter 2015

1. Using the data below, complete the table, create a histogram, and draw an ogive graph.

Category	Frequency			
1-5	5			
6-10	2			
11-15	5			
16-20	1			
21-25	4			
26-30	1			

2. Create a Stem & Leave Diagram

1, 10, 50, 51, 66, 69, 76, 77, 79, 82, 83, 85, 87, 88, 88, 89, 92, 92, 96, 96, 98, 108

3. Using the data above create a Box Plot

4. Three events occur with probabilities of $P(E1) = 0.35$, $P(E2) = 0.25$, $P(E3) = 0.40$. Other probabilities are: $P(B \mid E1) = 0.25$, $P(B \mid E2) = 0.15$, $P(B \mid E3) = 0.60$.

    a. Compute $P(E1 \mid B)$
    b. Compute $P(E2 \mid B)$
    c. Compute $P(E3 \mid B)$.

5. Until the summer of 2008, the real estate market in Fresno, California, had been booming, with prices skyrocketing. Recently, a study showed the sales patterns in Fresno for single-family homes. One chart presented in the commission's report is reproduced here. It shows the number of homes sold by price range and number of days the home was on the market.

	Days on the Market		
Price Range ($000)	**1-7**	**8-30**	**Over 30**
Under $200	125	15	30
$200-$500	200	150	100
$501-1000	400	525	175
Over $1000	125	140	35

    a. Using the relative frequency approach to probability assessment, what is the probability that a house will be on the market more than 7 days?
    b. Is the event *1–7 days on the market* independent of the price $200–$500?
    c. Suppose a home has just sold in Fresno and was on the market less than 8 days, what is the most likely price range for that home?

6. A company wants to know how important higher education is when selecting a new CEO. Prior studies have shown that 60% of all CEOs will be successful, 85% of successful CEOs have a university degree, while 70% of those who don't have a University degree turn out to be unsuccessful CEOs
    a. What is the probability of a CEO having a university degree
    b. What is the probability of a CEO not having a university degree and being successful
    c. What is the probability of a CEO having a university degree or being successful

7. Radio Shack stocks four alarm clock radios. If it has fewer than four clock radios available at the end of a week, the store restocks the item to bring the in-stock level up to four. If weekly demand is greater than the four units in stock, the store loses the sale. The radio sells for $25 and costs the store $15. The Radio Shack manager estimates that the probability distribution of weekly demand for the radio is as follows:

x (weekly demand)	P(x)
0	0.05
1	0.05
2	0.10
3	0.20
4	0.40
5	0.10
6	0.05
7	0.05

    a. What is the expected weekly demand for the alarm clock radio?
    b. What is the probability that weekly demand will be greater than the number of available radios?
    c. What is the expected weekly profit from the sale of the alarm clock radio? (Remember: There are only four clock radios available in any week to meet demand.)
    d. On average, how much profit is lost each week because the radio is not available when demanded?

8. Use the binomial formula to calculate the following probabilities for an experiment in which $n = 5$ and $p = 0.4$:
    a. the probability that x is at most 1
    b. the probability that x is at least 4
    c. the probability that x is less than 1

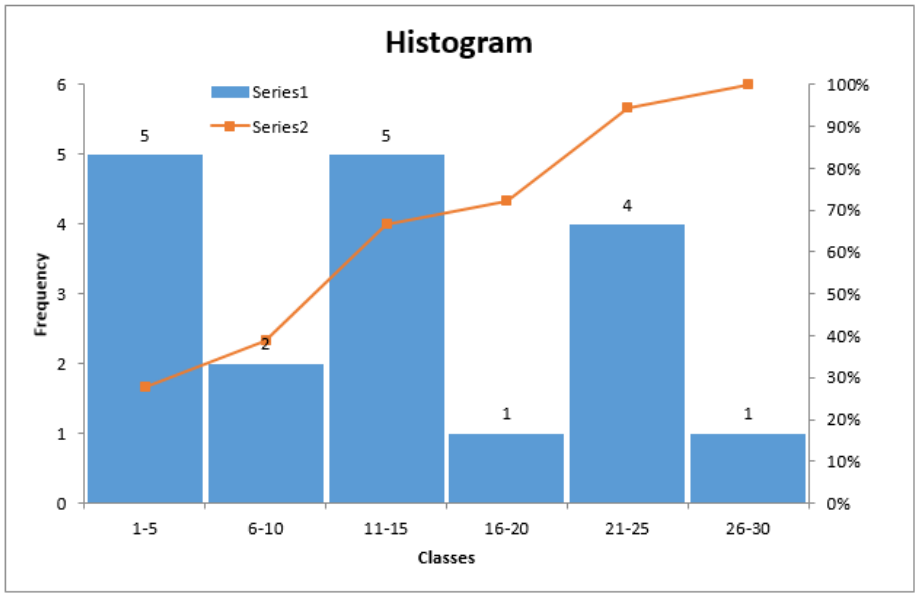
9. A manufacturing firm produces a product that has a ceramic coating. The coating is baked on to the product, and the baking process is known to produce 15% defective items (for example, cracked or chipped finishes). Every hour, 20 products from the thousands that are baked hourly are sampled from the ceramic coating process and inspected. a.
    a. What is the probability that 5 defective items will be found in the next sample of 20?
    b. On average, how many defective items would be expected to occur in each sample of 20?
    c. How likely is it that 15 or more non-defective (good) items would occur in a sample due to chance alone?

10. For a standardized normal distribution, calculate the following probabilities:
    a. $P(z < 1.5)$
    b. $P(z \geq 0.85)$
    c. $P(-1.28 < z < 1.75)$

11. The average sales price of tablets was $386 during the first quarter, a 21% drop compared to the same period last year, according to a report from IMS Research. The price decline is a result of intense competition in the tablet PC market, and Apple is paving the way, according to IMS. Assume that the standard deviation of prices was $25
    a. If a local Future Shop outlet sold 80 tablets at or below $314, how many tablets where sold last year?
    b. Compute the third quartile of the tablet prices sold by this Future Shop outlet
    c. What is the probability of selling a tablet that's priced more than the price of the average price of a tablet + tax? (assume the tax is 15%)

1.

Category	Frequency	Cumulative Freq.	Relative Freq.	Cumulative Rel. Freq
1-5	5	5	27.78 %	27.78 %
6-10	2	7	11.11 %	38.89 %
11-15	5	12	27.78 %	66.67 %
16-20	1	13	5.56 %	72.23 %
21-25	4	17	22.22 %	94.45 %
26-30	1	18	5.56 %	100.00 %
	18		**100.00 %**	



2.

Stem Unit = 10	Leaf Unit = 1						
0	1						
1	0						
2							
3							
4							
5	0	1					
6	6	9					
7	6	7	9				
8	2	3	5	7	8	8	9
9	2	2	6	6	8		
10	8						

using the book's rounding method

3. $n = 22$

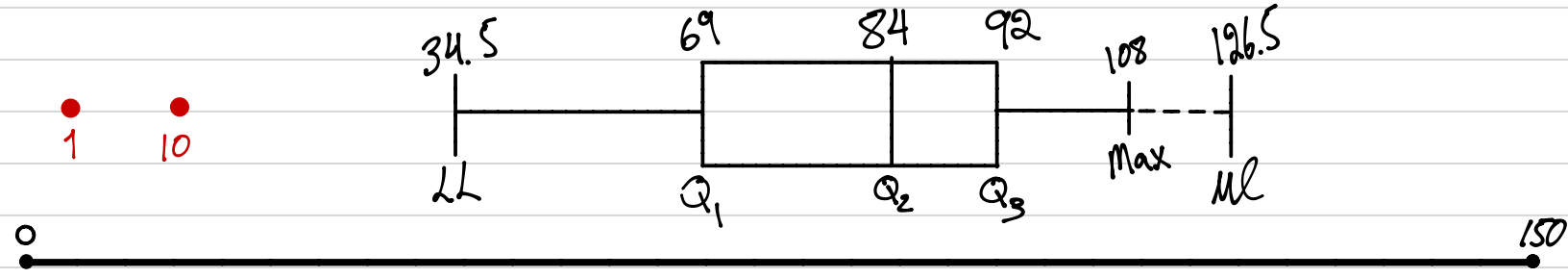
Q_1 $0.25(22) = 5.5 \rightarrow 6^{th} = 69$

Q_2 $0.50(22) = 11 \rightarrow \frac{11^{th} + 12^{th}}{2} = \frac{83+85}{2} = 84$

Q_3 $0.75(22) = 16.5 \rightarrow 17^{th} \rightarrow 92$

ML $Q_3 + 1.5(IQR) \rightarrow 92 + 1.5(92-69) = 126.5$

LL $Q_1 - 1.5(IQR) \rightarrow 69 - 1.5(92-69) = 34.5$



4.

a. $P(E_1|B) = \frac{P(E_1 \cap B)}{P(B)}$

since we don't have 2 of the 3 numbers needed, we must deduce them!

$P(B|E_1) = \frac{P(B \cap E_1)}{P(E_1)}$

$0.25 = \frac{P(B \cap E_1)}{0.35} \rightarrow P(B \cap E_1) = 0.875$

	B	B′	
E_1	8.75%		35
E_2	$P(E_2 \cap B)$		25
E_3	$P(E_3 \cap B)$		40
	36.5		100

Repeat the process above to collect all three (3) parts of event B

Now you can solve for a., b., & c.

a. 23.97% (8.75/36.5)

b. 10.27% (3.75/36.5)

c 65.75% (24/36.5)

5.

a. $P(X>7 \text{ days}) = P(8 \leq X \leq 30) + P(X>30 \text{ days})$ "or"

$$\frac{1170}{2020} = 57.92\%$$

b. $P(1 \leq X \leq 7) = \frac{850}{2020} = 42.10\%$

$P(\$200 < X < \$500) = \frac{450}{2020} = 22.27\%$

if $A \cap B$ are *independant* $\rightarrow P(A \cap B) = P(A) \times P(B)$

$\frac{200}{2020} \neq \frac{850}{2020} \times \frac{450}{2020}$ so the 2 categories are *dependant*

C. $P(<200 \mid <8 \text{ days}) = \frac{125}{850} = 14.71\%$

$P(200\text{-}500 \mid <8 \text{ days}) = \frac{200}{850} = 23.53\%$

$P(501\text{-}1000 \mid <8 \text{ days}) = \frac{400}{850} = 47.06\%$ ✓ most likely

$P(>1000 \mid <8 \text{ days}) = \frac{125}{850} = 14.71\%$

2 categories

Education: Degree, No Degree

Performance: Successful, Not Successful

$P(S) = 60\%$

$P(D|S) = 85\%$

$P(S'|D') = 70\%$

	S	S'	
D	51	19	70
D'	9	21	30
	60	40	100

using the compliment rule

$$P(D|S) = \frac{P(D \cap S)}{P(S)}$$

$$0.85 = \frac{P(D \cap S)}{0.60}$$

$$P(D \cap S) = 51$$

$$P(S'|D') = \frac{P(D' \cap S')}{P(D')}$$

since $P(S'|D') = 70\%$

$P(S|D') = 30\%$

of the CEOs without a degree 70% are not successful so that means that 30% are!

a. $P(D) = 70\%$

b. $P(D' \cap S) = 9\%$

c. $P(D \cup S) = P(D) + P(S) - P(D \cap S)$

$70 + 60 - 51 = 79$

$$P(S|D') = \frac{P(D' \cap S)}{P(D')}$$

$$30\% = \frac{9}{P(D')} \quad D' = 30$$

7.

a. $E(X) = \sum P(X_i)\, X_i = 0.05(0) + 0.05(1) + \cdots + 0.05(7) = 3.6$ radios

b $P(X > 3.6) = P(4) + P(5) + P(6) + P(7)$

$0.4 + 0.1 + 0.5 + 0.5 = 60\%$

c. Profit(X) = $0.05 \times \$0$ (no sale, no profit)

$x=1 \quad + 0.05 \times \$10$ ← Sales Price − Cost $\$25 - \$15$

$x=2 \quad + 0.10 \times \$20$

$x=3 \quad + 0.20 \times \$30$

$x=4 \quad + 0.40 \times \$40$

$x=5 \quad + 0.10 \times \$40$ (no more additional $ from this point

$x=6 \quad + 0.05 \times \$40$ Why? Because the store only stocks 4 radios

$x=7 \quad + 0.05 \times \$40$ any demand > 4 is lost

$\$32.50$

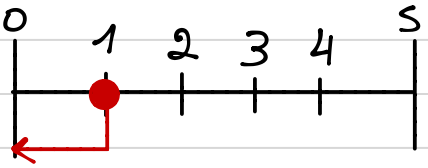
d. $(\$10 \times 3.6) - \$32.50 = \$3.50$

8

a. $P(X \leq 1) = P(1) + P(0) = 33.70\%$

$P(0) = \frac{5!}{0!(5-0)!}\, 0.4^0 \times 0.6^5$

$P(1) = \frac{5!}{1!(5-1)!}\, 0.4^1 \times 0.6^4$



b. $P(X \geq 4) = P(4) + P(5) = 7.68\% + 1.02\% = 8.70\%$

c. $P(X < 1) = P(0) = 7.78\%$

9.

a. $n=20$, $p=15\%$ $P(X=5)=\frac{20!}{5!(20-5)!}\,0.15^{5}\times 0.85^{15}=10.28\%$

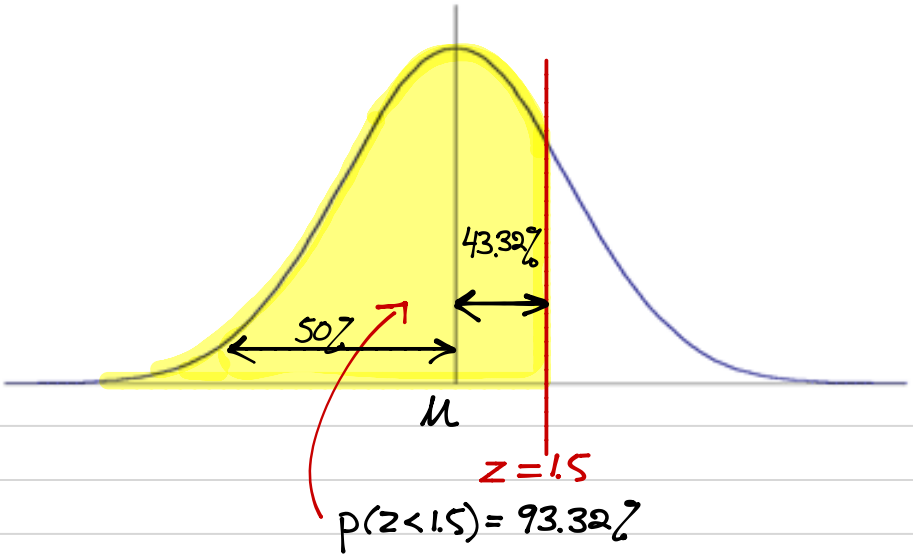
b. $E(x)=np=20(0.15)=3$

c. $P(X\geq 15)=P(15)+P(16)+P(17)+P(18)+P(19)+P(20)=93.27\%$

But now, the question concerns "good" items so $p=0.85$

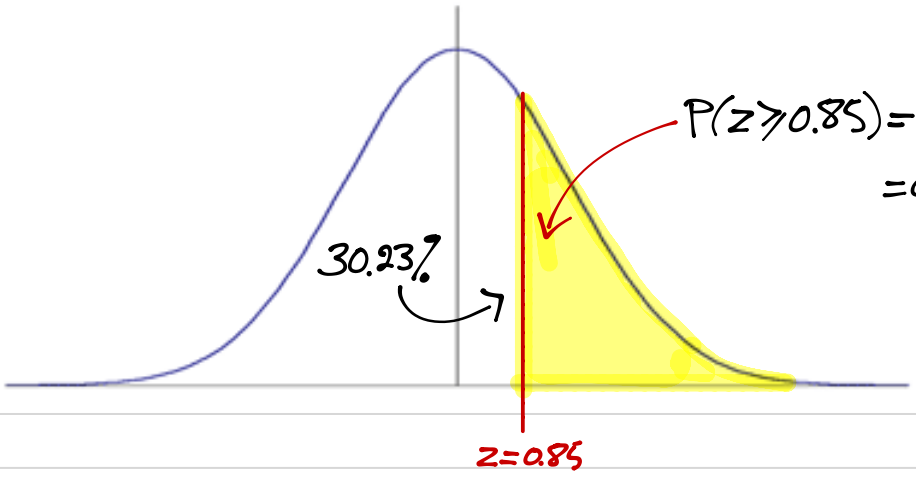
10.

a.



$P(z<1.5)=93.32\%$

b.

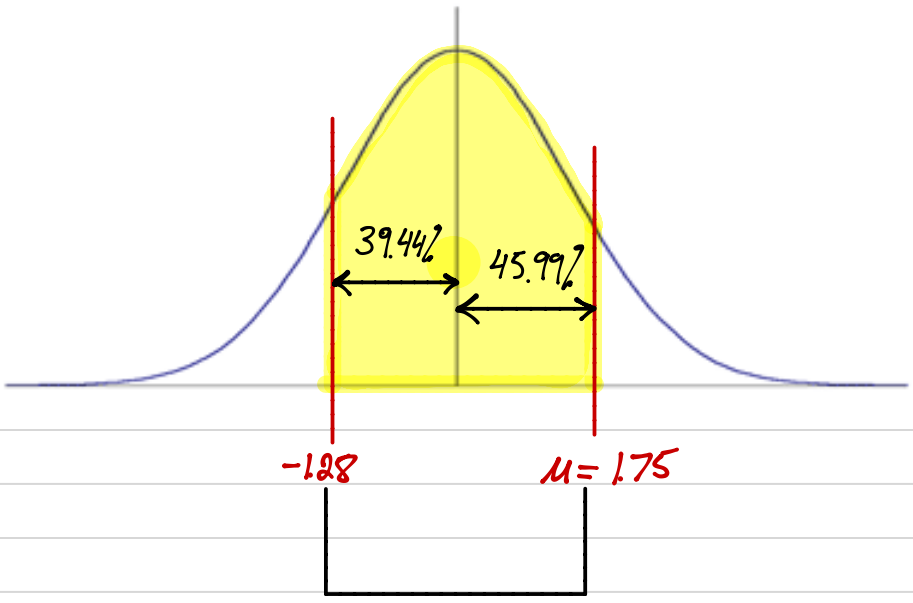


$P(z\geq 0.85)=0.5-P(0\leq z\leq 0.85)$

$=0.5-0.3023=19.77\%$

cont...

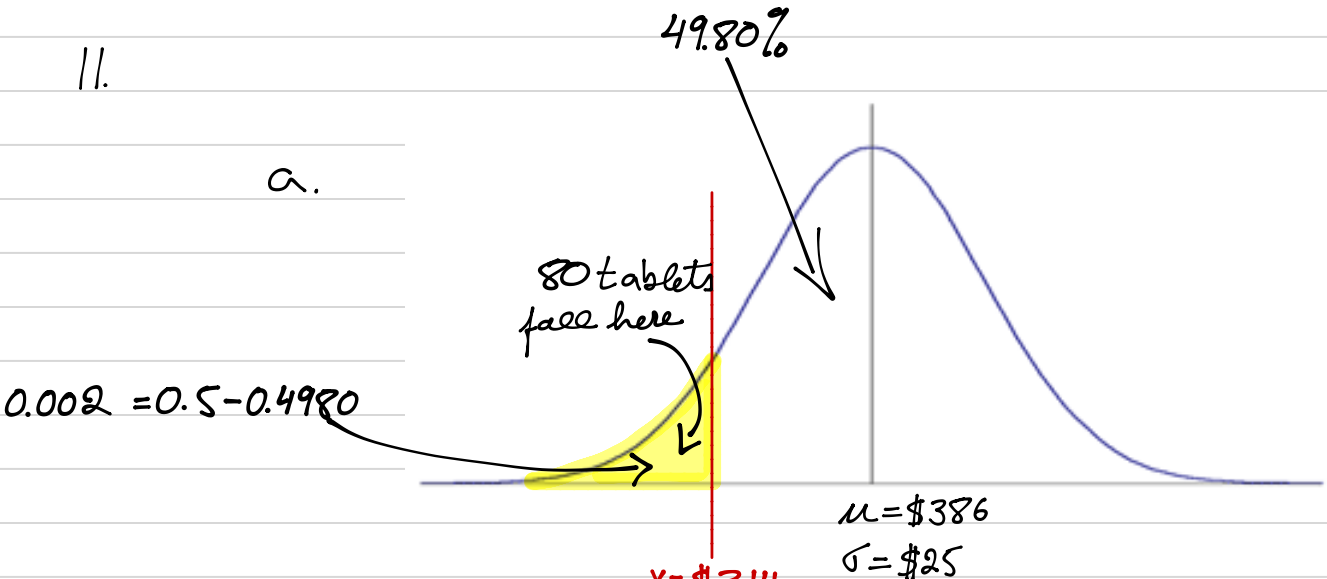
C.



$p(-1.28 < z < 1.75) = 85.43\%$

11.

a.



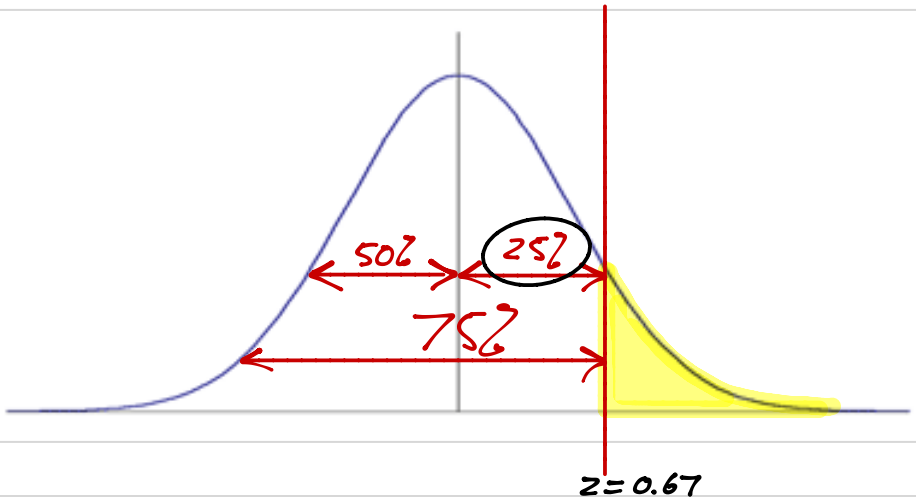
$$Z = \frac{\$314 - \$386}{\$25}$$

$Z = -2.88$

Since 80 tablets = 0.002 of all tablets

then $\frac{80}{X} = 0.002$, $X =$ 40,000 tablets

b. $Q_3 = 75\%$



look up 0.25 in the Z table

Z score = 0.67 (approx)

$$0.67 = \frac{X - \$386}{\$25}$$

$X = \underline{\underline{\$402.75}}$ is the cut-off price for the 75th percentile

C. $P(X \geq \$386(1.15)) = P(X \geq \$443.9)$

$$Z = \frac{443.9 - 386}{25} = 2.316 = 2.32$$

$P(Z \geq 2.32) = 0.5 - 0.4898 = 0.0102 = \underline{\underline{1.02\%}}$