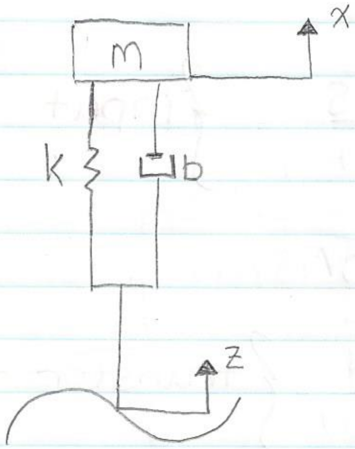
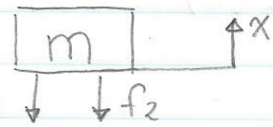
SYSC 3600 - Lecture 10

Oct 7, 2015

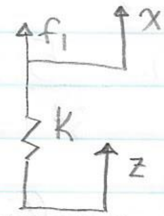
Mechanical Systems

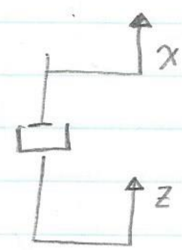


$$\frac{X(s)}{Z(s)} \;?$$



$$\boxed{m\ddot{x} = -f_1 - f_2}$$





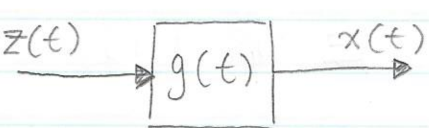
$$\boxed{f_1 = k(x-z)} \quad \boxed{f_2 = b(\dot{x} - \dot{z})}$$

Transfer Function → $x(0) = 0, \quad \dot{x}(0) = 0$
$z(0) = 0, \quad \dot{z}(0) = 0$

$$m\ddot{x} + b\dot{x} + kx = kz + b\dot{z}$$

$$(ms^2 + bs + k)X(s) = (k + bs)Z(s)$$

$$\frac{X(s)}{Z(s)} = \frac{bs + k}{ms^2 + bs + k}$$



Impulse Response

$$g(t) = \mathcal{L}^{-1}\{G(s)\}$$

Example

$$z(t) = \cos 3t \rightarrow Z(s) = \frac{s}{s^2+9}$$ } input

EXAM and MIDTERM question

$m = 1\text{ kg}$, $k = 4\text{ N/m}$ $b = 5\text{ Ns/m}$

$$G(s) = \frac{5s+4}{s^2+5s+4} = \frac{5s+4}{(s+4)(s+1)}$$ } transfer function

$$X(s) = \frac{(5s+4)s}{(s+4)(s+1)(s^2+9)} = \frac{A}{s+4} + \frac{B}{s+1} + \frac{Cs+D}{s^2+9}$$

$$A = (s+4)X(s)\Big|_{s=-4} = \frac{(5(-4)+4)(-4)}{(-4+1)(16+9)} = \frac{-64}{75}$$

$$B = (s+1)X(s)\Big|_{s=-1} = \frac{(5(-1)+4)(-1)}{(-1+4)(1+9)} = \frac{1}{30}$$

$$\boxed{A = \frac{-64}{75}} \quad \boxed{B = \frac{1}{30}}$$

$$\frac{-64}{75}(s+1)(s^2+9) + \frac{1}{30}(s+4)(s^2+9) + (s+1)(s+4)(Cs+D)$$

$$\equiv 5s^2+4s$$

Coef of s^3: $\frac{-64}{75} + \frac{1}{30} + C = 0 \rightarrow C$

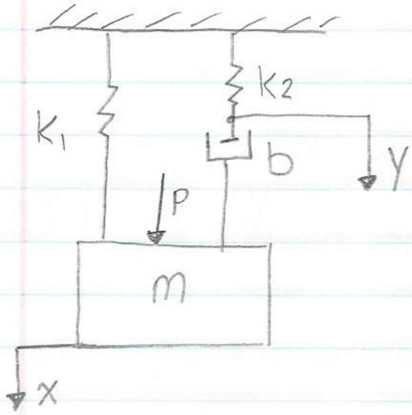
Coef of s^0: $-\frac{64}{75}9 + \frac{1}{30}(36) + 4D = 0 \rightarrow C$

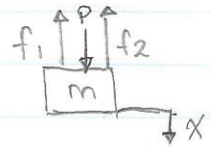
$$x(t) = Ae^{-4t} + Be^{-t} + C\cos 3t + \alpha \sin 3t$$

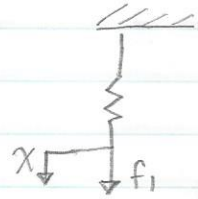
you want C and D to be small

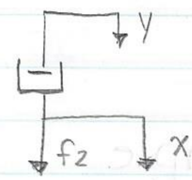
Example

Transfer Function $\dfrac{X(s)}{P(s)}$?









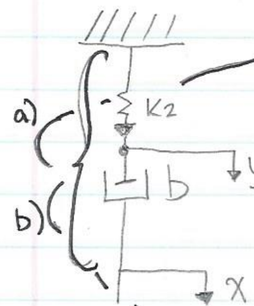
$$\boxed{f_1 = k_1 x}$$

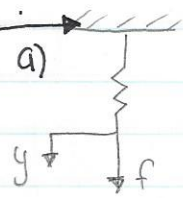
$$\boxed{f_2 = b(\dot{x} - \dot{y})}$$

$$m\ddot{x} = -f_1 - f_2 + P$$

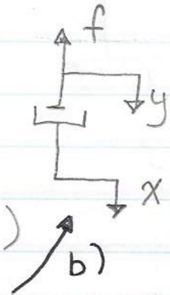
$$\boxed{m\ddot{x} = -k_1 x - b(\dot{x} - \dot{y}) + P} \rightarrow \boxed{(ms^2 + bs + k_1)X(s) = bs\,Y(s) + P(s)}$$

eqn 1





$$\begin{cases} f = ky \\ f = -b(\dot{y} - \dot{x}) \end{cases}$$



$$\boxed{k_2 y = -b(\dot{y} - \dot{x})}$$

eqn 2

$$\boxed{(k_2 + bs)Y(s) = bs\,X(s)}$$

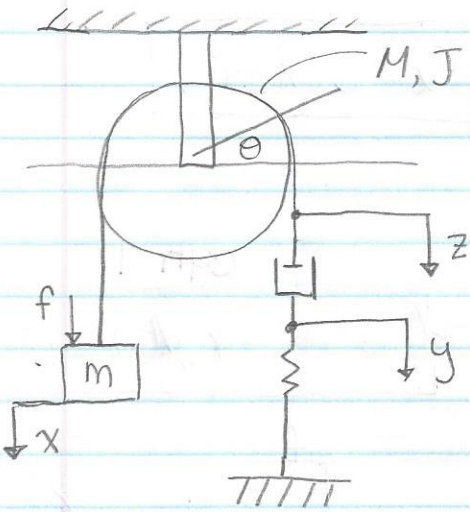
eqn 1 & eqn 2

$$(ms^2 + bs + k_1)X(s) = \frac{(bs)^2}{bs + k_2}X(s) + P(s)$$

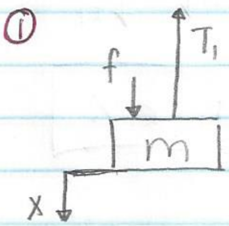
$$\left(ms^2+bs+k_1-\frac{(bs)^2}{bs+k_2}\right)X(s)=P(s)$$

$$\frac{\left((bs+k_2)(ms^2+bs+k_1)\right)-(bs)^2}{bs+k_2}X(s)=P(s)$$

Example



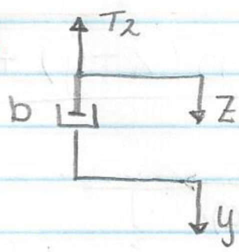
①



$$m\ddot{x}=f-T_1$$

$$Z=-Z$$

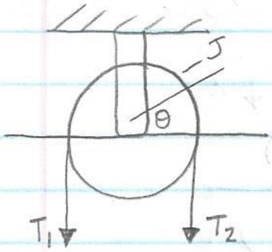
③



*

$$T_2=-b(\dot{z}-\dot{y})$$

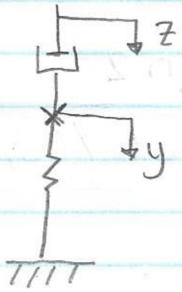
②



$$J\ddot{\theta}=(T_1-T_2)R$$

$$x=R\theta \longrightarrow \ddot{x}=R\ddot{\theta}$$

④



$$-ky=b(\dot{y}-\dot{z})$$

⑤

$$\frac{J}{R^2}\ddot{x} = T_1 - T_2 \quad **$$

$* + **$

$\downarrow$

$$\frac{J}{R^2}\ddot{x} = T_1 - b\dot{x} - b\dot{y}$$

$$m\ddot{x} = f - \frac{J}{R^2}\ddot{x} - b\dot{x} - b\dot{y} \rightarrow ((m + J/R^2)s^2 + bs)X(S) + bs\,Y(S) = F(S) \quad ②$$

$$-ky = b(\dot{y} + \dot{x}) \rightarrow (bs+k)Y(s) = bs\,X(s)$$

$$Y(S) = \frac{bs}{bs+k}X(S) \quad ①$$

eqn ① + eqn ②

$$\left((m + J/R^2)s^2 + bs + \frac{(bs)^2}{bs+k}\right)X(S) = F(S)$$

$$\frac{X(S)}{F(S)} = \frac{bs+k}{(bs+k)((m + J/R^2)S^2 + bs) + (bs)^2}$$