Econ 301: Assignment 1–Solutions
Fall 2015, Concordia University

Graphs are attached at the end.

1. a. The budget constraint is

$$10x_1 + 20x_2 \leq 800.$$

The budget line is a straight line with horizontal intercept \$800/\$10 = 80 and vertical intercept \$800/\$20 = 40. The budget set is the triangle area between the budget line and the two axes. The slope of the budget line is $-\frac{p_1}{p_2} = -\frac{10}{20} = -\frac{1}{2}$.

b. Now the consumer has to pay a price of $p_2' = 20(1 + 0.25) = 25$ for a CD. The budget constraint is

$$10x_1 + 25x_2 \leq 800.$$

The budget line has the same horizontal intercept as before, 80, since p_1 did not change, while the vertical intercept is now \$800/\$25 = 32. The budget line rotates inward around the fixed horizontal intercept.

c. This means that the consumer now only has a monthly income of \$600; the budget line shifts inward. The budget constraint is

$$10x_1 + 20x_2 \leq 600.$$

d. Pizzas are rationed: the maximum amount consumed is 20 pizzas, so the budget set is lopped off beyond $x_1 = 20$, but otherwise the budget constraint is the same as before.

2. a. Yes, it is transitive: if business A is bigger and more profitable than business B, and business B is bigger and more profitable than business C, then business A is bigger and more profitable than business C. No, it is not complete: business A may be smaller and more profitable than business B.

b. Yes, it is transitive: if person A is at least as tall as person B, and person B is at least as tall as person C, then person A is at least as tall as person C. Yes, it is complete. For any two persons A and B, we can determine whether A is at least as tall as B, or B is at least as tall as A (or perhaps both, if A and B are exactly the same height).

3. a. E.g., $(2, 7)$, $(4, 6)$, $(6, 5)$, etc. In order for Azalia to stay on the same indifference curve, when the number of popsicles consumed goes down by 2, the number of brownies must go up by 1.

b. See the attached sheet with graphs.

c. The MRS is a constant $-1/2$. Popsicles and brownies are perfect substitutes for Azalia. These indifference curves do not exhibit a diminishing marginal rate of substitution; they are constant.

d. Yes, the preferences are monotonic: more of both goods is better (i.e., both goods are *goods*, not *bads*), and thus the indifference curves have a negative slope.

4. a. The goods are improvement in philosophy grade (good 1) and improvement in mathematics grade (good 2). The prices represent the number of hours Art has to spend in order to improve his grade by one point. Thus, $p_1 = 1$ and $p_2 = 3$. The income is the number of hours Art has to spend on studying for the two subjects. Therefore, $m = 60$. Hence the budget constraint is

$$x_1 + 3x_2 \leq 60.$$

b. Art's preferences are one-to-one perfect complements, which can be described by utility functions

$$u(x_1, x_2) = \min\{x_1, x_2\} \text{ or } v(x_1, x_2) = \min\{50 + x_1, 50 + x_2\},$$

among others. Art's budget line is given by

$$x_1 + 3x_2 = 60.$$

His optimal bundle (x_1, x_2) is such that $x_1 = x_2$. Thus the budget line becomes $x_1 + 3x_1 = 60$, which gives $x_1 = 15$. Art's optimal choice of grades is $50 + 15 = 65$ points in both courses.

c. Since Art can get 50 points on each subject without any studying, he has to "consume" at least 10 units of each good in order to obtain 60 points on each subject. Thus, his new budget set is the triangle formed by the budget line, the vertical line $x_1 = 10$, and the horizontal line $x_2 = 10$. Since the original optimal bundle is still feasible (it is within the imposed constraints), the optimal bundle remains $(15, 15)$.

5. a. Use the tangency condition: MRS = slope of the budget line, and the budget constraint to derive Jacob's optimal consumption bundle. We have

$$\text{MRS} = -\frac{\text{MU}_1}{\text{MU}_2} = -\frac{x_2}{x_1}.$$

The slope of the budget line is

$$-\frac{p_1}{p_2} = -\frac{10}{5} = -2.$$

Therefore, the optimal consumption bundle (x_1, x_2) satisfies

$$-\frac{x_2}{x_1} = -2.$$

Thus, $x_2 = 2x_1$. Substituting this into the budget equation

$$10x_1 + 5x_2 = 100,$$

we get

$$10x_1 + 5(2x_1) = 100,$$

so $x_1 = 5$. Since $x_2 = 2x_1$, $x_2 = 10$. Therefore, Jacob's optimal consumption bundle is $(x_1, x_2) = (5, 10)$.

b. The answer would not change. Let $f(u) = 7u^2 = 7(x_1x_2)^2$, where $u(x_1, x_2) = x_1x_2$. Given that $u(x_1, x_2) \geq 0$ for all $x_1, x_2 \geq 0$, this is a monotonic transformation of the utility function in part a. Alternatively, we can check that the marginal rate of substitution for $f(u)$ is

$$-\frac{14x_1x_2^2}{14x_1^2x_2} = -\frac{x_2}{x_1},$$

which is the same as the marginal rate of substitution for u. Since monotonic transformations of a utility function represent the same preferences, Jacob's optimal consumption bundle remains the same as in part a.

c. The consumption bundle in question is $(x_1, x_2) = (5, 8)$.

$$\text{For } u(5,8): \ \text{MU}_1(5,8) = \frac{\partial u}{\partial x_1} = x_2 = 8, \text{ and}$$

$$\text{for } f(u(5,8)): \ \text{MU}_1(5,8) = \frac{\partial f(u)}{\partial x_1} = 14x_1x_2^2 = 14(5)(64) = 4480$$

Observe that marginal utility depends on the particular utility representation of the preferences.

d. Now Jacob pays $p_1' = 10(1 + 0.25) = 12.5$ for movie tickets. The slope of the budget line becomes

$$-\frac{p_1'}{p_2} = -\frac{12.5}{5} = -2.5.$$

Therefore, the new optimal choice (x_1', x_2') satisfies

$$-\frac{x_2'}{x_1'} = -2.5.$$

Using this in the new budget equation

$$12.5x_1' + 5x_2' = 100,$$

we get

$$12.5x_1' + 5(2.5x_1') = 25x_1' = 100,$$

and $x_1' = 4$, $x_2' = 10$. Hence, Jacob's new optimal consumption bundle is $(4, 10)$.

e. Jacob pays \$10 in taxes when he buys 4 movie tickets ($4(\$2.5) = \$10$), the optimal number of movie tickets under the 25% value tax. Thus, the government is indifferent between this value tax and a lump-sum tax of 10\$. Jacob, however, prefers the income tax, as his optimal bundle will be adjusted to the original prices and will be different from $(4, 10)$, which he can still afford. Specifically, we can calculate his optimal bundle under the income tax as $(4.5, 9)$ which gives him the utility level of $4.5(9) = 40.5$, compared to the utility of $4(10) = 40$ from his consumption under the value tax.

Here are the two other ways to derive the optimal bundle in part a. First, set up the constrained optimization problem:

$$\max_{x_1,x_2} x_1 x_2$$

$$\text{such that } 10x_1 + 5x_2 = 100$$

The unconstrained optimization method:
From the budget constraint we have $x_2 = 20 - 2x_1$. Substituting this into the utility function yields $x_1 x_2 = 20x_1 - 2x_1^2$. Thus we can write the unconstrained maximization problem as

$$\max_{x_1} 20x_1 - 2x_1^2$$

The first-order condition is

$$20 - 4x_1 = 0.$$

This gives $x_1 = 5$, and from the budget equation $x_2 = 20 - 2(5) = 10$.

The Lagrangian method:
The Lagrange function is

$$L = x_1 x_2 - \lambda(10x_1 + 5x_2 - 100)$$

First-order conditions:

$$\frac{\partial L}{\partial x_1} = x_2 - 10\lambda = 0 \tag{1}$$

$$\frac{\partial L}{\partial x_2} = x_1 - 5\lambda = 0 \tag{2}$$

$$\frac{\partial L}{\partial \lambda} = 10x_1 + 5x_2 - 100 = 0 \tag{3}$$

From (1) we have $\lambda = x_2/10$ and from (2) we get $\lambda = x_1/5$. From these two equations, $x_2/10 = x_1/5$ or , equivalently,

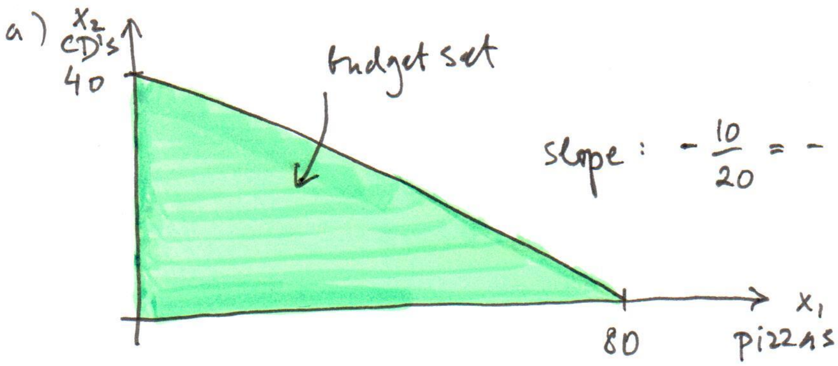
$$x_2 = 2x_1. \tag{4}$$

Finally, after substituting the above, (3) becomes

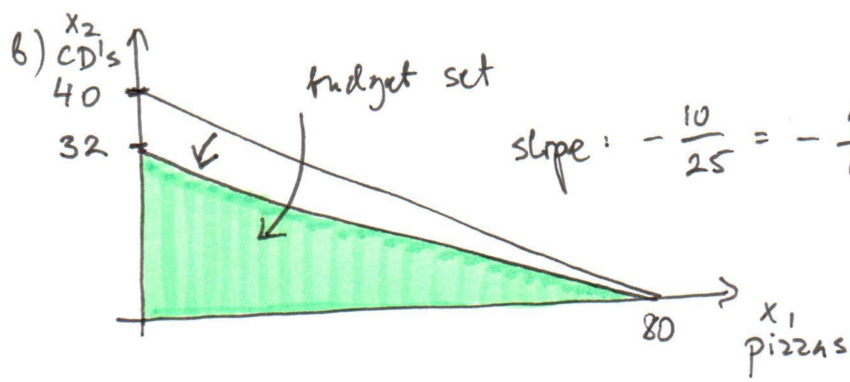
$$10x_1 + 5(2x_1) = 100,$$

which yields $x_1 = 5$. Now we can use (4) to find that $x_2 = 2x_1 = 10$.

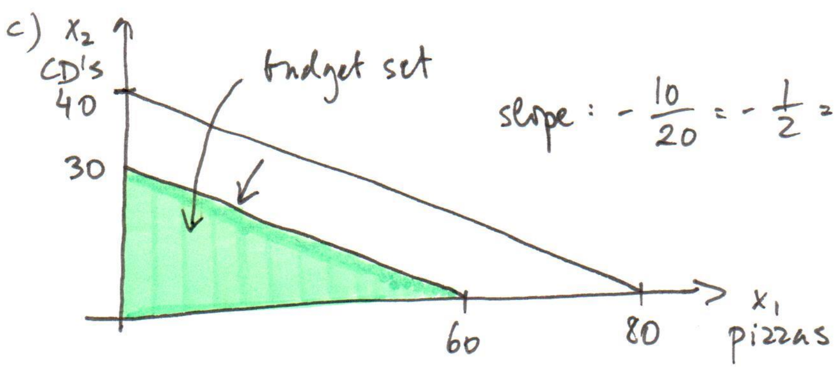
Question 1 graphs



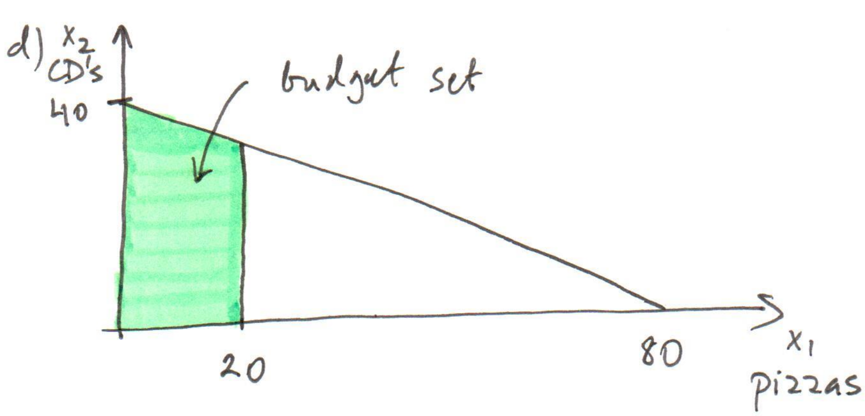
slope: $-\frac{10}{20} = -\frac{1}{2} = -\frac{40}{80}$



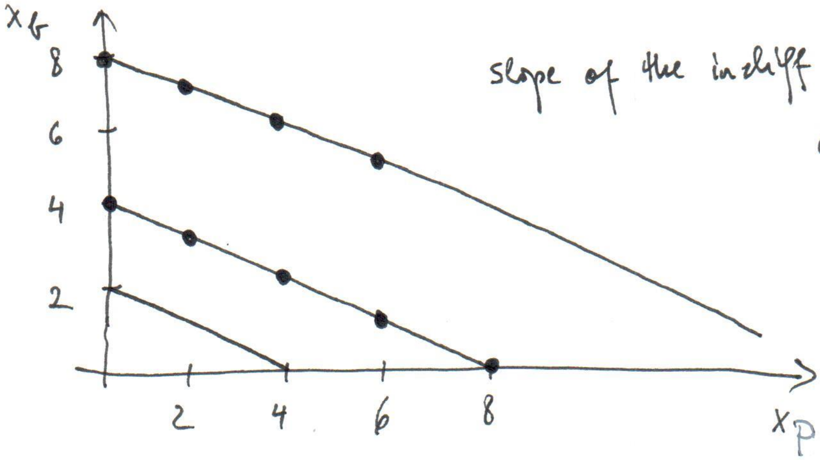
slope: $-\frac{10}{25} = -\frac{4}{10} = -\frac{32}{80}$



slope: $-\frac{10}{20} = -\frac{1}{2} = -\frac{30}{60}$



Question 3 graphs

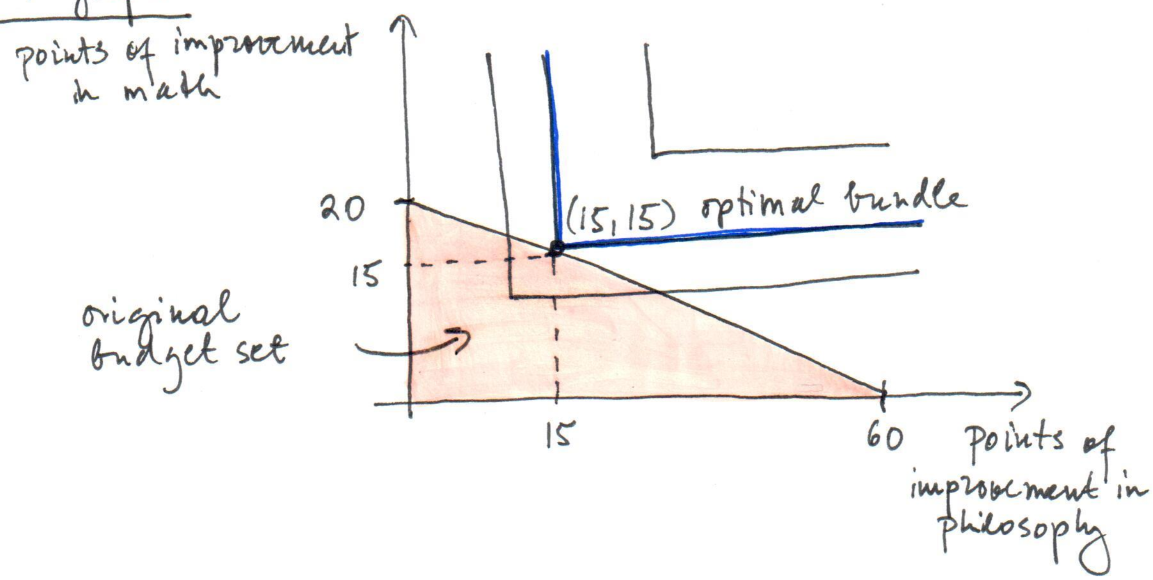


slope of the indiff curves: $-\frac{1}{2}$

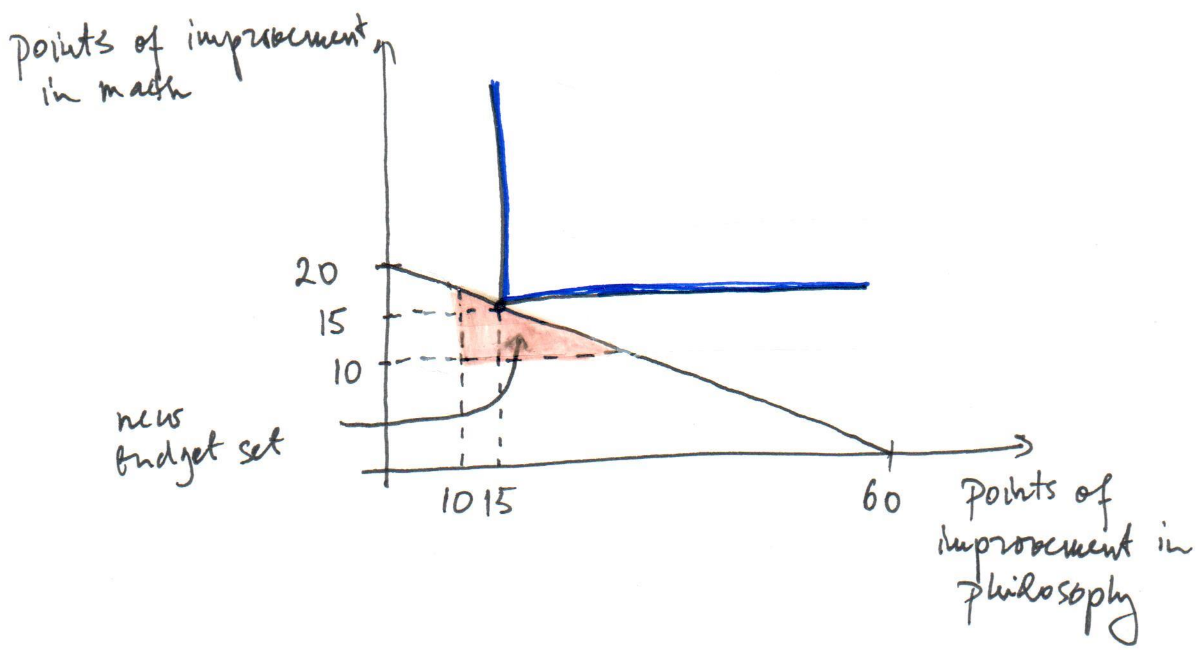
constant MRS

Question 4 graphs

a-b)

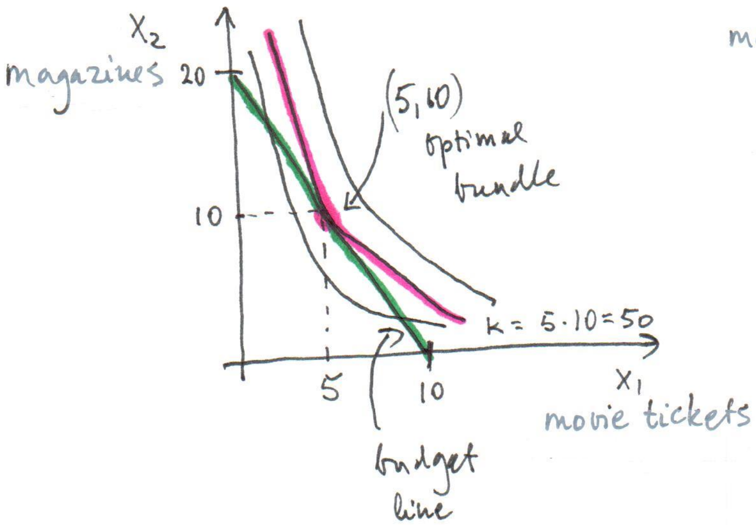


c)



Question 5 graphs

a.



d.