Concordia University March 23, 2007

Engineering Differential Equations
Section J
Exam II (B)

Directions: You have **60 minutes** to solve the following **4** problems. You may use a calculator. No cell phones are allowed during the exam.

(1) (10 points) Solve the equation

$$xy' + y = \frac{1}{y^2}, \quad x > 0.$$

(2) (10 points) A tank, with total capacity 300 liters, contains 200 liters of fluid in which 30 grams of salt is dissolved. Pure water is then pumped into the tank at a rate of 4 L/min; the well-mixed solution is pumped out at a rate of 3 L/min. Answer the following questions, giving the exact value calculated with this model, even if you also approximate your answer.
   (a) Find the number of grams of salt in the tank after 5 minutes.
   (b) When will the tank overflow?
   (c) What will be the number of grams of salt in the tank at the instant it overflows?

(3) (10 points)
   (a) State, without explaining your choice, whether the following statement is TRUE or FALSE: "The functions $y_1(x) = x$ and $y_2(x) = x^2$ cannot form a fundamental set of any second-order homogeneous equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$."
   (b) State, without explaining your choice, whether the following statement is TRUE or FALSE: "If y_p is a particular solution of the nonhomogeneous second-order differential equation with constant coefficients $y'' - 4y' + 3y = g(x)$, then $y(x) = e^x + y_p(x)$ is also a solution to this differential equation."
   (c) Determine whether the set of functions $f_1(x) = e$, $f_2(x) = e^x$, $f_3(x) = e^{x^2}$ is linearly dependent or linearly independent on the interval $(-\infty, \infty)$.
   (d) Solve the initial value problem $y'' + y = 0$, $y(0) = 1$, $y'(0) = -1$.

(4) (10 points) Find the general solution of the differential equation

$$y'' + 2y' = 2x.$$