March/6/12 — Phys — Dylan Ferguson

Recap:

Faraday's law – $\mathcal{E} = -\frac{d}{dt}\overbrace{(BA\cos\theta)}^{\Phi_B}$

Lenz's law

$A\cos\theta \frac{dB}{dt}$ $\quad B\cos\theta \frac{dA}{dt}$ $\quad BA\frac{d}{dt}(\cos\theta)$, $\theta = \omega t$

• Square loop pulled out of a magnitude field with velocity

$$\mathcal{E} = \frac{-d}{dt}(BLx) = -BLv$$

velocity

Current: $i = \frac{\mathcal{E}}{R_{loop}}$

Force: $F = iLB = \frac{B^2L^2v}{R_{loop}}$

Power: $P = Fv = \frac{B^2L^2v^2}{R_{loop}}$

• Square loop rotated in a magnitude Field with freq "f"

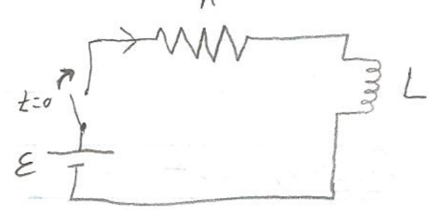
$$\mathcal{E} = -NBA\,2\pi f\,\sin[2\pi f t]$$

Inductors:

$$\mathcal{E} = -\left(\mu_0\frac{N^2A}{L}\right)\frac{di}{dt} = -L\frac{di}{dt}$$

Inductance [H]

RL circuits



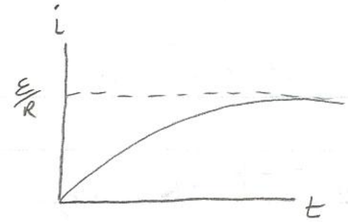
$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

$$\boxed{L\frac{di}{dt} + iR = \mathcal{E}}$$

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$$

if $t=0,\ i=0$

$t=\infty,\ i=\frac{\mathcal{E}}{R}$



Energy storage

$$\mathcal{E} = L\frac{di}{dt} + iR$$

multiply by i...

$$\mathcal{E}i = Li\frac{di}{dt} + i^2R$$

Power dissipated from battery

Power dissipated in inductor

Power dissipated in resistor

$$\frac{dU}{dt}_B = Li\frac{dv}{dt}$$

$$U_B = \int Li\frac{di}{dt} = \frac{1}{2}Li^2$$

$$\boxed{U_B = \frac{1}{2}Li^2}$$ Energy in inductor

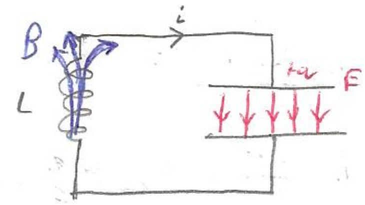
2/3

march/6/12 Phys Dylan Ferguson

LC Oscillations

E.M. Oscillations

L.C. circuits



$$U_B = \frac{1}{2} L i^2 , \quad U_E = \frac{1}{2} \frac{q^2}{C}$$

$$U_B + U_E = U = \text{constant}$$
(no resistance)

$$\frac{dU}{dt} = 0 = \frac{dU_B}{dt} + \frac{dU_E}{dt}$$

$$\frac{d}{dt}\left(\frac{1}{2}Li^2\right) + \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = 0$$

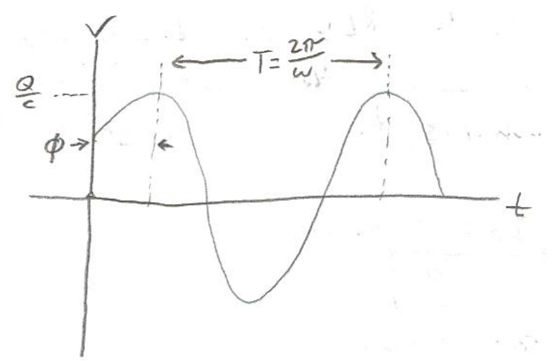
$$\frac{di}{dt} \cdot \frac{d}{di} \cdot \left(\frac{1}{2}Li^2\right) + \frac{dq}{dt}\frac{d}{dq}\left(\frac{1}{2}\frac{q^2}{C}\right) = 0$$

$$\frac{di}{dt} L \underset{\uparrow \frac{dq}{dt}}{i} + \frac{dq}{dt}\frac{q}{C} = 0$$

$$L\frac{di}{dt} + \frac{q}{C} = 0$$

$$\frac{di}{dt} = -\frac{1}{LC} q$$

$$\boxed{\frac{d^2q}{dt^2} = -\frac{1}{LC} q}$$



$$q = Q \cos(\omega t + \phi)$$

initial Charge on capacitor

angular frequency $\hookrightarrow \frac{1}{\sqrt{LC}}$

phase constant

Damped Oscillations

$\frac{dq}{dt}$

$$L\boxed{i}\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^{\boxed{2}}R \qquad \frac{dq}{dt}$$

$$L = \frac{di}{dt} + \frac{q}{c} = -iR$$

$$L = \frac{d^2q}{dt} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

Diagram 1

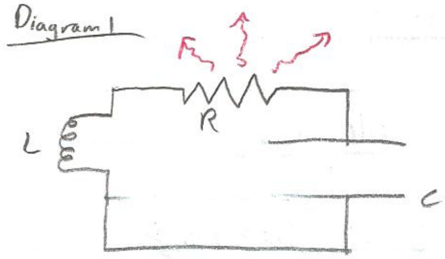
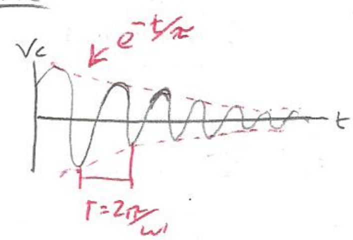


Diagram 2



Solve for q:

$$q = Qe^{-\frac{Rt}{2L}}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$$

$$\left(\frac{1}{LC}\right)$$

Q: In the RLC current displaying damped oscillations as shown (Diagram 2), when the voltage across the capacitor is a maximum, $V = \frac{Q}{C}e^{\frac{-t}{(\frac{2L}{R})}}\cos[\omega' t + \phi]$. What is the current through the resistor?

a) I_R = + maximum

$\boxed{\text{b) } I_R = 0}$

c) I_R = − minimum

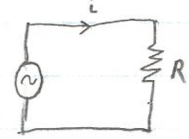
3/3

march /6/12 Phys Dylan Ferguson

AC power

- Driving currents with A.C.
- A.C. power → most common form of power because you can easily step up or down with a transformer.

Resistive load

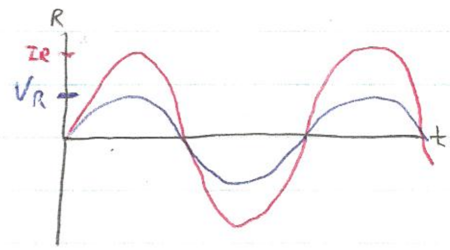


$V(t) = V_0 \sin(\omega t)$

(apply ohm's law for the current)

$V_R = I_R R$

$I_R = \frac{V_R}{R} \sin(\omega t)$



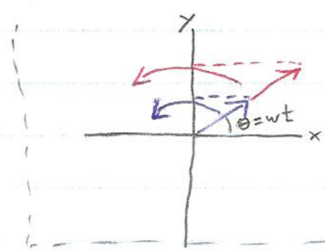
Capacitive load

$V = V_0 \sin \omega t$

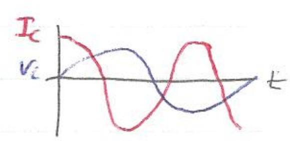
$q = C V_0 \sin \omega t$

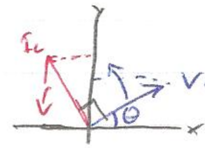
$i = \frac{dq}{dt} = \omega C V_0 \cos \omega t$

Current leads the voltage (ie ahead by 90°)



Capacitive load





$$X_c = \frac{V}{i} = \frac{V_0 \sin wt}{wCV_0 \cos wt}$$

Resistance $X_c = \left(\frac{1}{wc}\right) \frac{\sin wt}{\sin(wt+90)}$

$$|X_c| = \left|\frac{1}{wc}\right| \cdot \left|\frac{1}{2\pi f c}\right|$$