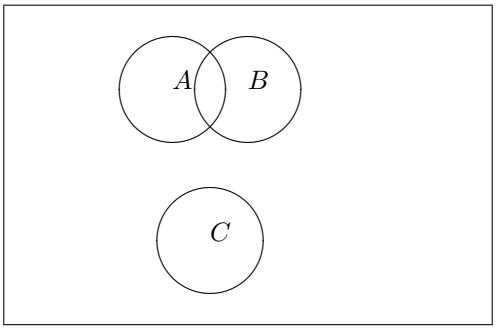
MAT2377C - Assignment 1

Solutions

Total number of points: 19

Q1. (5 points) The three events are shown on the Venn diagram:



Reproduce the figure and shade the region corresponding to the following events:

(a) A^c (b) $(A \cap B) \cup (A \cap B^c)$ (c) $(A \cap B) \cup C$

(d) $(B \cup C)^c$ (e) $(A \cap B)^c \cup C$

Marking scheme for Q1:

1 point for each correct answer. Total - 5 points.

Q2. In a group of 16 candidates for for laboratory research positions, 7 are chemists and 9 are physicians. In how many ways one can choose 2 chemists and 3 physicians?

Solution to Q2:

$$\binom{7}{2} * \binom{9}{3} = 21 * 84 = 1764$$

Marking scheme for Q2:

(This question will not be marked)

Q3. (9 points) Pieces of aluminium are classified according to the finishing of the surface and according to the finishing of edge. The results from 85 samples are summarized as follows:

	Edge	
Surface	excellent	good
excellent	60	5
good	16	4

Let A denote the event that a selected piece has "excellent" surface, and let B denote the event that a selected piece has "excellent" edge. If samples are selected randomly, determine the following probabilities:

(a) $P(A)$ (b) $P(B)$ (c) $P(A^c)$

(d) $P(A \cap B)$ (e) $P(A \cup B)$ (f) $P(A^c \cup B)$

(g) If the selected piece has excellent edge finishing, what is the probability that it has excellent surface finishing?

(h) If the selected piece has good surface finishing, what is the probability that it has excellent edge finishing?

(i) Are A and B independent?

Solution to Q3:

(a) $P(A) = 65/85$

(b) $P(B) = 76/85$

(c) $P(A^c) = 1 - 65/85 = 20/85$

(d) $P(A \cap B) = 60/85$

(e) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 65/85 + 76/85 - 60/85 = 81/85$

(f) $P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B) = 20/85 + 76/85 - 16/85 = 80/85$

(g) $P(A|B) = P(A \cap B)/P(B) = 60/76$

(h) $P(B|A^c) = P(B \cap A^c)/P(A^c) = 16/20$

(i) $P(A|B) \neq P(A)$ - events are not independent.

Marking scheme for Q3:

1 point for each correct answer. Total - 9 points.

Q4. If $P(A) = 0.1$, $P(B) = 0.3$, $P(C) = 0.3$, and events A, B, C are mutually exclusive (disjoint), determine the following probabilities:

(a) $P(A \cup B \cup C)$ (b) $P(A \cap B \cap C)$ (c) $P(A \cap B)$

(d) $P((A \cup B) \cap C)$ (e) $P(A^c \cap B^c \cap C^c)$ (f) $P[(A \cup B \cup C)^c]$

Solution to Q4:

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.7$, since the events are mutually exclusive.

(b) $P(A \cap B \cap C) = 0$

(c) $P(A \cap B) = 0$

(d) $P((A \cup B) \cap C) = 0$

(e) $P(A^c \cap B^c \cap C^c) = P[(A \cup B \cup C)^c] = 1 - P(A \cup B \cup C) = 0.3$ (draw Venn diagram)

(f) $P[(A \cup B \cup C)^c] = 1 - P(A \cup B \cup C) = 0.3$

Marking scheme for Q4:

(This question will not be marked)

Q5. (2 points) Probability that an electrical switch, which is kept in dryness, fails during the guarantee period, is 1%. If the switch is humid, the failure probability is 8%. Assume that 90% of switches are kept in dry conditions, whereas remaining 10% are kept in humid conditions.

(a) What is the probability that the switch fails during the guarantee period?

(b) If the switch failed during the guarantee period, what is the probability that it was kept in humid conditions?

Solution to Q5:

Let F-"failure", H-"humid", D-"dry". Given: $P(F|D) = 0.01$, $P(F|H) = 0.08$, $P(D) = 0.9$, $P(H) = 0.1$

(a)

$$P(F) = P(F|D)P(D) + P(F|H)P(H) = 0.01 * 0.9 + 0.08 * 0.1 = 0.009 + 0.008 = 0.017$$

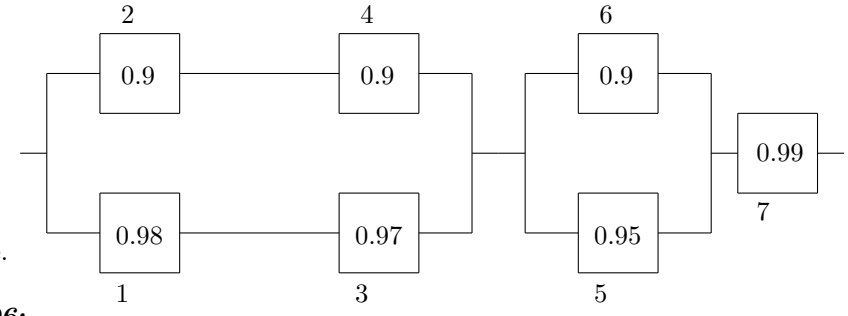
(b)

$$P(H|F) = \frac{P(H \cap F)}{P(F)} = \frac{P(F|H)P(H)}{P(F)} = 0.4706$$

Marking scheme for Q5:

1 point for each part. Total - 2 points.

Q6. (3 points) The following system operates only if there is a path of functional device from left to the right. The probability that each device functions is as shown. What is the probability that the circuit operates? Assume independence.



Solution to Q6:

Let Box A: components 1,2,3,4; Box B: components 5,6; Box C: component 7.

$$P(\text{system works}) = P(\text{A works})P(\text{B works})P(\text{C works}).$$

Now, B is just parallel system, so that

$$P(\text{B works}) = 0.9 + 0.95 - 0.9 * 0.95 = 1.85 - 0.855 = 0.995.$$

Furthermore, $P(2 \text{ and } 4 \text{ work}) = 0.9 * 0.9 = 0.81$, $P(1 \text{ and } 3 \text{ work}) = 0.9506$. Now, A is the parallel system of $2, 4$ and $1, 3$, thus

$$P(\text{A works}) = P(2 \text{ and } 4 \text{ work}) + P(1 \text{ and } 3 \text{ work}) - P(2 \text{ and } 4 \text{ work})P(1 \text{ and } 3 \text{ work}) = 0.9906.$$

Final answer: 0.9578.

Marking scheme for Q6:

Total - 3 points.

Q7. An inspector working for a manufacturing company has a 95% chance of correctly identifying defective items and 2% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 1% of nonconforming items.

(a) What is the probability that an item selected for inspection is classified as defective?

(b) If an item selected at random is classified as nondefective, what is the probability that it is indeed good?

Solution to Q7:

Let A - the event that an item is classified as defective, D - the event that an item is defective; so that D^c is the event that an item is 'good'. What is known is: $P(D) = 0.01$; $P(A|D) = 0.95$, $P(A|D^c) = 0.02$.

(a)

$$P(A) = P(A \cap D) + P(A \cap D^c) = P(A|D)P(D) + P(A|D^c)P(D^c) \approx 0.0293.$$

(b) To compute $P(D^c|A^c)$. From Bayes' formula:

$$P(D^c|A^c) = \frac{P(A^c|D^c)P(D^c)}{P(A^c)} = \frac{(1 - P(A|D^c))P(D^c)}{1 - P(A)} \approx 0.999$$

Marking scheme for Q7:

(This question will not be marked)