Physics 121.6 2007/2008

Assignment 19 - Solutions

1. **Chapter 23, Problem 6.** Two small silver spheres, each with a mass of 10.0 g, are separated by 1.00 m. Calculate the fraction of the electrons in one sphere that must be transferred to the other in order to produce an attractive force of 1.00×10^4 N (about 1 ton) between the spheres. (The number of electrons per atom of silver is 47, and the number of atoms per gram is Avogadro's number divided by the molar mass of silver, 107.87 g/mol.)

Solution:

Some electrons are transferred from one sphere to the other. Let the charge on the electron rich sphere be $-q$. Since the two spheres where electrically neutral to begin with, the charge on the other sphere will be $+q$. The force between them will be attractive and have magnitude

$$F = k_e \frac{|-q||+q|}{r^2} = \frac{k_e q^2}{r^2}$$

$$\Rightarrow q = r\sqrt{\frac{F}{k_e}} = (1.00\text{ m})\sqrt{\frac{1.00\times10^4\text{ N}}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times10^{-3}\text{ C}$$

Thus the number of electrons transferred is

$$n_t = \frac{q}{e} = \frac{1.05\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C}} = 6.59\times10^{15}$$

The number of electrons originally in one of the spheres is

$n = Zn_{atoms}$, where Z is the number of electron per atom (the atomic number), $Z = 47$,

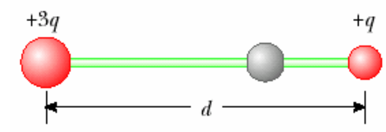
and the number of atoms $n_{atoms} = \frac{N_A m}{m_{mole}}$, so

$$n = \frac{ZN_A m}{m_{mole}} = \frac{47(6.02\times10^{23}\text{ atoms/mole})(10.0\text{ g})}{107.87\text{ g/mole}} = 2.62\times10^{24}$$

Therefore the fraction of electrons that must be transferred is

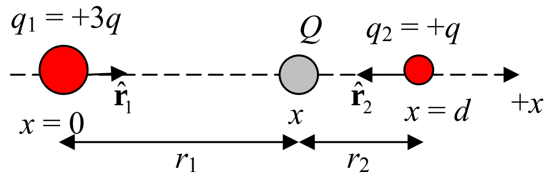
$$\frac{n_t}{n} = \frac{6.59\times10^{15}}{2.62\times10^{24}} = 2.51\times10^{-9}$$

2. **Chapter 23, Problem 8.** Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in Figure P23.8, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Explain whether it can be in stable equilibrium.



Solution

First find an expression for the net force on the charged bead, Q, as a function of its position, x.



Force on Q due to q_1:

$$\vec{\mathbf{F}}_1 = \frac{k_e q_1 Q}{r_1^2}\hat{\mathbf{r}}_1 = \frac{k_e 3qQ}{x^2}\hat{\mathbf{i}}$$

Force on Q due to q_2:

$$\vec{\mathbf{F}}_2 = \frac{k_e q_2 Q}{r_2^2}\hat{\mathbf{r}}_2 = -\frac{k_e qQ}{(d-x)^2}\hat{\mathbf{i}}$$

Therefore net force on Q is

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = k_e qQ\left(\frac{3}{x^2} - \frac{1}{(d-x)^2}\right)\hat{\mathbf{i}}$$

For this to be zero, when the bead is in equilibrium,

$$\frac{3}{x^2} = \frac{1}{(d-x)^2}$$

$$\Rightarrow \sqrt{3}(d-x) = \pm x$$

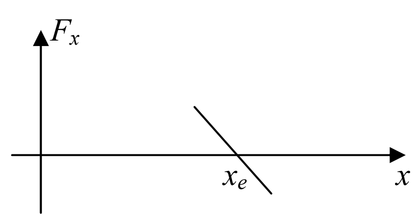
$$\Rightarrow \sqrt{3}d = \left(\sqrt{3} \pm 1\right)x$$

$$\Rightarrow x = \frac{\sqrt{3}d}{\left(\sqrt{3} \pm 1\right)} = 2.37d, \quad \text{or} \quad 0.634d$$

Only the second solution is between the two charges. Therefore the bead is in equilibrium at $x_e = 0.634d$.

It is a stable equilibrium position if the net force always points back toward the equilibrium position when the bead is displaced a little from the equilibrium position.

This will be true when $\left.\frac{dF_x}{dx}\right|_{x=x_e} < 0$ i.e. if F_x near x_e looks like the diagram at right.



$$\frac{dF_x}{dx} = k_e qQ\left(3(-2)x^{-3} - (-2)(d-x)^{-3}(-1)\right)$$

$$= -k_e qQ\left(6x^{-3} + 2(d-x)^{-3}\right)$$

Thus $\left.\frac{dF_x}{dx}\right|_{x=x_e} < 0$ only if $Q > 0$.

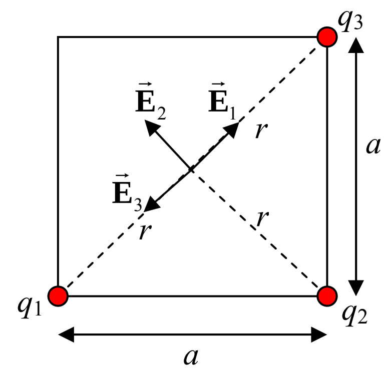
Therefore the bead is in stable equilibrium if it has a positive charge.

3. Three identical charges of $+Q$ are placed at three corners of a square with sides of length a. The magnitude of the electric field at the centre of the square is

(A) $\frac{k_e Q}{a^2}$ (B) $\frac{k_e Q^2}{2a^2}$ (C) $\frac{2k_e Q}{a^2}$ (D) $\frac{k_e Q}{2a^2}$ (E) $\frac{\sqrt{2}k_e Q}{a^2}$

Solution:

Total electric field is $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3$



$$E_1 = E_2 = E_3 = \frac{k_e Q}{r^2}$$

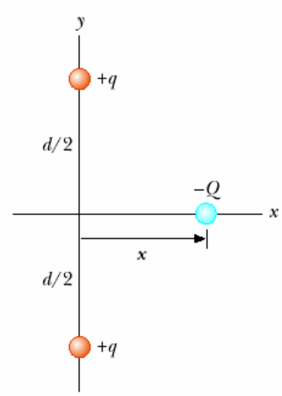
where $(2r)^2 = a^2 + a^2 \Rightarrow r^2 = \frac{a^2}{2}$

Since $\vec{\mathbf{E}}_1 = -\vec{\mathbf{E}}_3$ then $\vec{\mathbf{E}} = \vec{\mathbf{E}}_2$, so

$$E = E_2 = \frac{k_e Q}{r^2} = \frac{2k_e Q}{a^2}$$

Answer **C**.

4. **Chapter 23, Problem 10. Review problem.** Two identical particles, each having charge $+q$, are fixed in space and separated by a distance d. A third point charge $-Q$ is free to move and lies initially at rest on the perpendicular bisector of the two fixed charges a distance x from the midpoint between the two fixed charges (Fig. P23.10). (a) Show that if x is small compared with d, the motion of $-Q$ will be simple harmonic along the perpendicular bisector. Determine the period of that motion. (b) How fast will the charge $-Q$ be moving when it is at the midpoint between the two fixed charges, if initially it is released at a distance $a \ll d$ from the midpoint?



Solution:

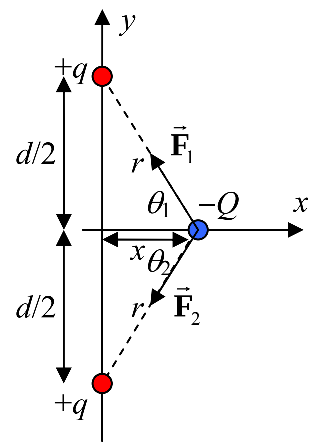
(a) The net force on the $-Q$ charge is $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$.

Since $F_1 = F_2 = \dfrac{k_e qQ}{r^2}$, with $r = \sqrt{x^2 + (d/2)^2}$

and from the symmetry $\theta_1 = \theta_2 = \theta$ so the y–components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ cancel.

So the magnitude of $\vec{\mathbf{F}}$ is $F = F_1 \cos\theta + F_2 \cos\theta$

with $\cos\theta = \dfrac{x}{r}$, Therefore



$$F_x = -F = -2F_1 \cos\theta$$

$$= -2\left(\frac{k_e qQ}{r^2}\right)\frac{x}{r} = \frac{-2k_e qQx}{r^3}$$

Now, $ma_x = F_x \Rightarrow a_x = \dfrac{-2k_e qQx}{m\left(\sqrt{x^2 + (d/2)^2}\right)^3}$

If $x \ll d/2$

$$a_x \approx \frac{-2k_e qQ}{m(d/2)^3}x = -\frac{16k_e qQ}{md^3}x = -\omega^2 x$$

This is in the form for Simple Harmonic Motion with

$\omega = \sqrt{\dfrac{16k_e qQ}{md^3}}$. So the period of the motion is

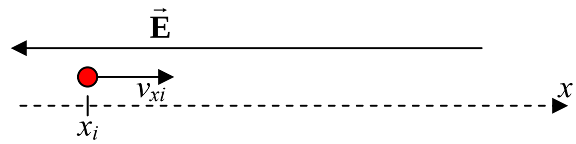
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{md^3}{16k_e qQ}} = \frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}$$

(b) If initially released from $x = a$, the amplitude of the motion will be a. The speed at $x = 0$ is its maximum speed which is

$$v_{x,\max} = a\omega = 4a\sqrt{\frac{k_e qQ}{md^3}}$$

5. **Chapter 23, Problem 36.** A proton is projected in the positive x direction into a region of a uniform electric field $\vec{\mathbf{E}} = -6.00 \times 10^5\ \hat{\mathbf{i}}$ N/C at t = 0. The proton travels 7.00 cm before coming to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time at which the proton comes to rest.

Solution:



(a) $E_x = -6.00 \times 10^5$ N/C and the charge on a proton is $+e$, so the force on the proton is in the $-x$ direction.
$F_x = eE_x = ma_x$ where m is the mass of a proton.

$$a_x = \frac{eE_x}{m} = \frac{(1.60\times10^{-19}\ \text{C})(-6.00\times10^5\ \text{N/C})}{(1.67\times10^{-27}\ \text{kg})} = -5.75\times10^{13}\ \text{m/s}^2$$

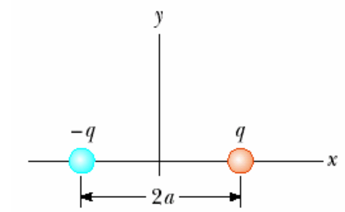
(b) Let $x_i = 0$, then $x_f = 7.00$ cm $= 7.00 \times 10^{-2}$ m, $v_{xf} = 0$, $v_{xi} = ?$, $t = ?$
Using $v_{xf}^{\ 2} = v_{xi}^{\ 2} + 2a_x(x_f - x_i)$

$$\Rightarrow v_{xi} = \sqrt{-2a_x x_f} = \sqrt{-2(-5.75\times10^{13}\ \text{m/s}^2)(7.00\times10^{-2}\ \text{m})} = 2.85\times10^6\ \text{m/s}$$

(c) Using $v_{xf} = v_{xi} + a_x t$ and since $v_{xf} = 0$,

$$t = \frac{-v_{xi}}{a_x} = \frac{-(2.85\times10^6\ \text{m/s})}{(-5.75\times10^{13}\ \text{m/s}^2)} = 4.93\times10^{-8}\ \text{s} = 49.3\ \text{ns}$$

6. **Chapter 23, Problem 22.** Consider the electric dipole shown in Figure P23.22. Show that the electric field at a *distant* point on the $+x$ axis is $E_x \approx 4k_e qa/x^3$. [*Hint:* Use the approximation $(1+x)^n \approx 1+nx$, for $x << 1$.]



Solution:

At a distant point on the $+x$ axis the electric field due to the $-q$ charge is

$$\vec{\mathbf{E}}_1 = -\frac{k_e q}{(x+a)^2}\hat{\mathbf{i}}$$

and the electric field due to the $+q$ charge is

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{(x-a)^2}\hat{\mathbf{i}}$$

so $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

Therefore:

$$E_x = k_e q\left(\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right) = \frac{k_e q}{x^2}\left(\frac{1}{(1-a/x)^2} - \frac{1}{(1+a/x)^2}\right) = \frac{k_e q}{x^2}\left((1-a/x)^{-2} - (1+a/x)^{-2}\right)$$

From the series expansion $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$ we see that

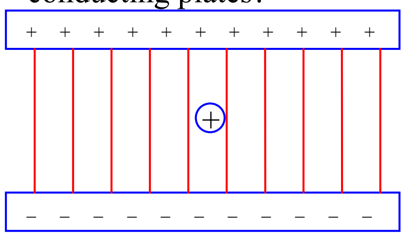
$(1+x)^n \approx 1+nx$, for $x << 1$

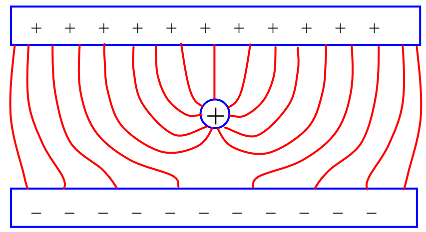
So when $a << x$, $a/x << 1$ therefore

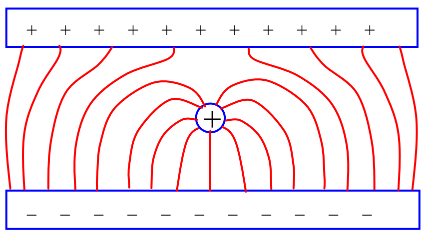
$$E_x \approx \frac{k_e q}{x^2}\left(\left(1-(-2)\frac{a}{x}\right) - \left(1+(-2)\frac{a}{x}\right)\right) = \frac{k_e q}{x^2}\left(4\frac{a}{x}\right)$$

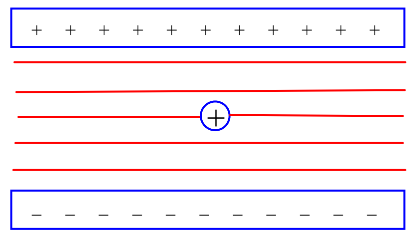
Therefore $E_x = 4\frac{k_e qa}{x^3}$, for large x.

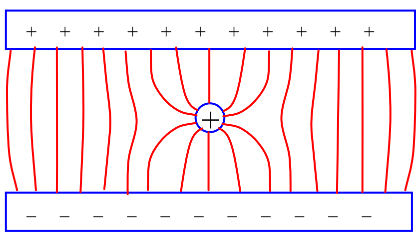
7. Which one of the following figures shows the electric field lines when a positively charged conducting sphere is placed between two parallel charged conducting plates?

(A)


(B)


(C)


(D)


(E)


(F) None of the above

Solution:

Since electric field lines come from positive charges and go to negative charges, the only possible answer is **C**.

8. Three identical charges of $+Q$ are placed at three corners of a square with sides of length a. The absolute electric potential (i.e. if $V = 0$, at $r = \infty$) at the centre of the square is

(A) $\frac{3k_eQ}{a}$ (B) $\frac{3\sqrt{2}k_eQ}{a}$ (C) $\frac{\sqrt{2}k_eQ}{a}$ (D) $\frac{\sqrt{2}k_eQ}{a^2}$ (E) $\frac{3k_eQ}{\sqrt{2}a}$

Solution:

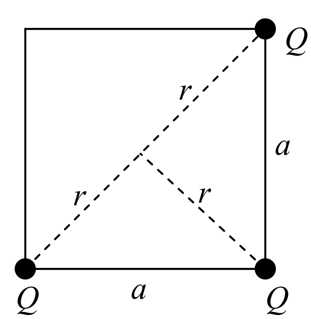
Potential at centre is:

$$V = \frac{k_eQ}{r} + \frac{k_eQ}{r} + \frac{k_eQ}{r} = 3\frac{k_eQ}{r}$$

also $a^2 = r^2 + r^2 \Rightarrow r = \frac{a}{\sqrt{2}}$

Therefore $V = \frac{3\sqrt{2}k_eQ}{a}$

Answer **B**.



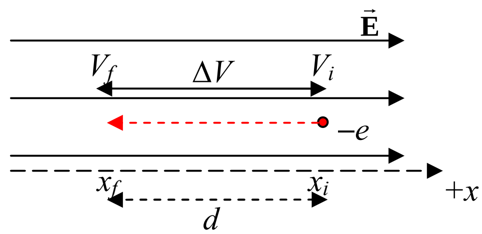
9. Suppose an electron is released from rest in a uniform electric field whose magnitude is 5.90×10^3 V/m. (a) Through what potential difference will it have passed after moving 1.00 cm? (b) How fast will the electron be moving after it has traveled 1.00 cm?

Solution:

(a) In a uniform electric field:

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{(V_f - V_i)}{(x_f - x_i)}$$

Note that the electron moves in the opposite direction to $\vec{\mathbf{E}}$, the $-x$ direction in the diagram. Therefore $\Delta x = x_f - x_i = -d$. So



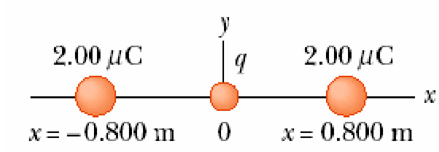
$$\Delta V = V_f - V_i = E_x d$$
$$= (5.90\times10^3 \text{ V/m})(1.00\times10^{-2} \text{ m})$$
$$= +59.0 \text{ V}$$

i.e. an increase in potential.

(b) Conservation of Energy:

$$K_f + U_f = K_i + U_i$$
$$\Rightarrow K_f = U_i - U_f = -q(V_f - V_i) = e\Delta V$$
$$\tfrac{1}{2} m v_f^2 = e\Delta V$$
$$\Rightarrow v_f = \sqrt{\frac{2e\Delta V}{m}}$$
$$v_f = \sqrt{\frac{2(1.60\times10^{-19} \text{ C})(59.0 \text{ V})}{9.11\times10^{-31} \text{ kg}}} = 4.55\times10^6 \text{ m/s}$$

10. Chapter 25, Problem 10. Given two 2.00-μC charges, as shown in Figure P25.10, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two 2.00-μC charges on the test charge q?

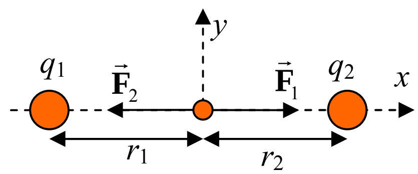


(b) What is the electric field at the origin due to the two 2.00-μC charges?
(c) What is the electrical potential at the origin due to the two 2.00-μC charges?

Solution:

(a) Forces on q:

$$\sum F_x = F_1 - F_2 = \frac{k_e q_1 q}{r_1^2} - \frac{k_e q_2 q}{r_2^2} = 0$$



Since $q_1 = q_1$ and $r_1 = r_2$.
Therefore $\sum \vec{\mathbf{F}} = 0$ on q.

(b) Electric field at origin:

$$\sum E_x = E_1 - E_2 = \frac{k_e q_1}{r_1^2} - \frac{k_e q_2}{r_2^2} = 0$$

Therefore $\vec{\mathbf{E}} = 0$ at origin.

(c) Electric potential at origin:

$$V = V_1 + V_2$$
$$= \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2}$$
$$= \frac{2k_e q_1}{r_1}$$

Since $q_1 = q_1$ and $r_1 = r_2$.

So $V = \dfrac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.00 \times 10^{-6}\ \mu\text{C})}{0.800 \text{ m}} = 4.50 \times 10^4 \text{ V} = 45.0 \text{ kV}$