Assignment 8

Due Thursday, Nov. 6, 2014

Note: there are 5 problems

1. Prove by induction: $1(1!) + 2(2!) + \ldots + n(n!) = (n+1)! - 1$, $\forall n \in \mathbb{Z}^{\geq 1}$.

2. Prove by induction: For every positive integer t, $(1+x)^t \geq 1 + tx$ if $x \geq -1$.

3. Prove by induction: For each positive integer t, $11^t - 1$ is divisible by 10.

4. Prove, by induction, that for all $n > 1$, if $x_1, x_2, \ldots, x_n$ are real numbers strictly between 0 and 1, then $(1 - x_1)(1 - x_2) \ldots (1 - x_n) > 1 - x_1 - x_2 - \ldots - x_n$.

5.(i) Prove that for all positive integers n, $x^n - 1 = (x-1)(x^{n-1} + \ldots + x + 1)$, indicating clearly the method of proof you are using.

(ii) Use the above result to show that if m is not a prime, then $2^m - 1$ is not a prime.