Assignment 7

Due Thursday, Oct. 30, 2014

Note: there are 5 problems

1. Prove **by contradiction** that if a is an even integer and b is an odd integer, then 4 does not divide $a^2 + 2b$.

2. Disprove the statement:

If n is a positive integer and $\dfrac{n(n+1)}{2}$ *is odd, then* $\dfrac{(n+1)(n+2)}{2}$ *is odd.*

3. For which of the following statements would it not be reasonable to try an induction proof? Give reasons for your answer.

(a) For every positive integer $n, 8$ divides $5^n + 2(3^{n-1}) + 1$.

(b) For some positive integer $n, n^2 - 2n > 0$.

(c) For every integer $x > 2, x^2 - 2x > 0$.

(d) For every integer $n > 4,\ n! > n^2$.

(e) For every rational number $n > 1, n^2 > n$.

(f) For every positive integer $t, (1+x)^t \geq 1 + tx$ if $x \geq -1$.

(g) $\det(AB) = \det(A)\det(B)$ for every two $n \times n$ matrices A, B.

4. In each of the statements below identify $P(n)$ then write down the statement which corresponds to: $P(1), P(3), P(s), P(k+1), P(2k-1)$.

(i) For all integers $n \geq 1, 6$ divides $n^3 - n$.

(ii) For every integer $n \geq 1,\ 1(1!) + 2(2!) + ... + n(n!) = (n+1)! - 1$.

(iii) $2^{2^t} - 1$ is prime $\forall t \in \mathbb{Z}^{\geq 1}$.

(iv) $(1 + 2 + ... + n)^2 = 1^3 + 2^3 + \ldots + n^3,\ \forall n \in \mathbb{Z}^{\geq 1}$.

5. Prove **by induction** that for every integer $n \geq 1$ the inequality $2^n > n$ holds.