**Assignment 3**

*Due Thursday, Sep 25, 2014*

**Note: there are 5 problems**

1. Let $n(x) = 5x^7 - 3x^5 + x^3 + x - 1$ and $m(x) = 2x^2 + 2$ be two polynomials. Find polynomials $q(x)$ and $r(x)$ such that $n(x) = m(x)q(x) + r(x)$.

2. Factor the polynomial $n(x) = 2x^4 + 2x^3 - 6x^2 - 8x - 8$ into prime polynomials.

3. Write a convincing argument that if $a$ and $b$ are positive real numbers, $a < b$, then

$$a^2 < \frac{(a+b)^2}{4} < b^2 \ .$$

4. Give a direct proof of the statement: if $n$ is any positive integer or zero then $(n^3 + 5)(n^2 + 6) + 1$ is an odd number.

5. Consider the statement: $q_1 \in \mathbb{Q}$ and $q_2 \in \mathbb{Q} \Longrightarrow (q_1)^m + (q_2)^n \in \mathbb{Q}$, where $m$ and $n$ are positive, non-zero integers.

$(i)$ Identify the context, premise and conclusion.

$(ii)$ Give a direct proof.

$(iii)$ Would the statement be true if either $m$ or $n$ or both are zero? Explain.