Assignment 11

Due Thursday, Nov. 27, 2014

Note: there are 5 problems

1. Give a complete proof of the following statement.

The set of all irrational numbers is uncountable.

2. Prove that the set, $\mathbb{Q} \times \mathbb{Q}$, of all pairs of rational numbers, (q_1, q_2), is countable.

3. Someone tried to prove that the set of all positive integers and the set of all even positive integers have different cardinalities by the following argument:

"We argue by contradiction. Assume that the two sets have the same cardinality. Now, the function $f(2n) = 2n$, $n = 1, 2, 3, \dots$, maps the set of all even, positive integers to the set of all even positive integers. Hence it is also a map of the set of all even positive integers to the set of all positive integers, and this map is not surjective. But the existence of such a non-bijective map contradicts our assumption that the two sets have the same cardinality. Hence the two sets must have different cardinalities".

What is wrong with this argument? Explain clearly and in detail.

4. Consider the map $f : \mathbb{R} \longrightarrow \mathbb{R}$ with $f(x) = \sin x$. (a) Show that this map is neither injective nor surjective. (b) How would you restrict the domain and co-domain of this map to make it bijective?

5. Let $f : (-4, \infty) \longrightarrow \mathbb{R}^{>0}$ be the mapping

$$f(x) = \sqrt{x+4}\ .$$

Is the mapping (a) injective and (b) surjective? Give reasons for your answers. (c) Does the map have an inverse? If so, write down the inverse map and identify the domain of the inverse.