

MATHEMATICS 1LS3 TEST 1

Day Class

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Duration of Examination: 60 minutes

McMaster University, 7 October 2015

First name (PLEASE PRINT): _____

Family name (PLEASE PRINT): _____

Student No.: _____

THIS TEST HAS 8 PAGES AND 7 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE.

Total number of points is 40. Marks are indicated next to the problem number. Any non-graphing calculator is allowed.

USE PEN TO WRITE YOUR TEST. IF YOU USE A PENCIL YOUR TEST WILL NOT BE ACCEPTED FOR REMARKING (IF NEEDED).

You must show work to receive full credit.

Problem	Points	Mark
1	4	
2	6	
3	8	
4	4	
5	6	
6	5	
7	7	
TOTAL	40	

Continued on next page

1. Multiple choice questions: circle ONE answer. No justification is needed.

(a)[2] Identify all correct statements about the function $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

(I) $f(x)$ is continuous at $x = 0$.

(II) $x = -1$ is a vertical asymptote of the graph of $f(x)$.

(III) $x = 1$ is a horizontal asymptote of the graph of $f(x)$.

(A) none

(B) I only

(C) II only

(D) III only

(E) I and II

(F) I and III

(G) II and III

(H) all three

(b)[2] The average half-life of acetaminophen (active ingredient in tylenol) is 2.5 hours. Assume that a patient is given a dose of 1000 mg of acetaminophen. Identify all correct statements.

(I) After 5 hours, 250 mg of acetaminophen is left unabsorbed in patient's body.

(II) After 2 hours, 450 mg of acetaminophen is left unabsorbed in patient's body.

(III) After 10 hours, less than 100 mg of acetaminophen is left unabsorbed in patient's body.

(A) none

(B) I only

(C) II only

(D) III only

(E) I and II

(F) I and III

(G) II and III

(H) all three

2. Identify each statement as true or false, or yes or no (circle your choice). You do not need to justify your answer.

(a)[2] The fact that $\lim_{x \rightarrow 2} f(x) = 3$ implies that $f(x)$ is continuous at $x = 2$, no matter what function $f(x)$ is considered.

TRUE FALSE

(b)[2] The formula $H = Mx + 4$, where M is a constant, represents a proportional relationship between H and x .

TRUE FALSE

(c)[2] The average density of a human jaw bone is 1.07 oz/in^3 , which is equivalent to approximately 1.85 g/cm^3 . [Conversion factors: $1 \text{ oz} = 28.35 \text{ g}$, $1 \text{ in} = 2.54 \text{ cm}$]

TRUE FALSE

Questions 3-7: You must show work to receive full credit.

3. Consider the function

$$f(x) = \begin{cases} \frac{x^3 - x}{x - 1} & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$$

(a)[4] Find $\lim_{x \rightarrow 1} f(x)$.(b)[2] Is $f(x)$ continuous at $x = 1$? Explain why or why not.(c)[2] Is $f(x)$ continuous at $x = -1$? Explain why or why not.

4. The following excerpt is taken from *The laminar cortex model: a new continuum cortex model incorporating laminar architecture*. J. Du, V. Vegh, and D.C. Reutens. PLoS Computational Biology. 8.10 (Oct. 2012).

the average of membrane potentials of neurons in the element, that is

$$V = \frac{N_e V_e + N_i V_i}{N_e + N_i}$$

where N_e , N_i are the numbers of excitatory and inhibitory neurons and V_e and V_i are the (average) membrane potentials of excitatory and inhibitory neuron populations respectively.

View V as a function of N_i .

By reading the text, we learn that V is the average of membrane potentials and N_i is the number of inhibitory neurons.

(a)[1] State (in one sentence) what question is answered by finding the inverse function of V .

(b)[3] Find a formula for the inverse function of V .

5. (a)[3] Consider the formula for human population growth

$$P(t) = 4.43 \left(\frac{\pi}{2} + \arctan \frac{t - 2007}{42} \right)$$

where t is a calendar year and $P(t)$ is in billions. Find the range of $P(t)$. Based on it, state the maximum world population (two decimal places suffice) predicted by this model.

(b)[3] A population of river sharks (freshwater sharks) in New Zealand changes periodically with a period of 12 months. In January, it reaches a maximum of 12,600, and in July it reaches a minimum of 5,800. By selecting an appropriate trigonometric function, find a formula which describes how the population of river sharks changes with time.

6. The survival rate (i.e., percent) $S(D)$ of clonogenic cells (cancer cells) is modelled by

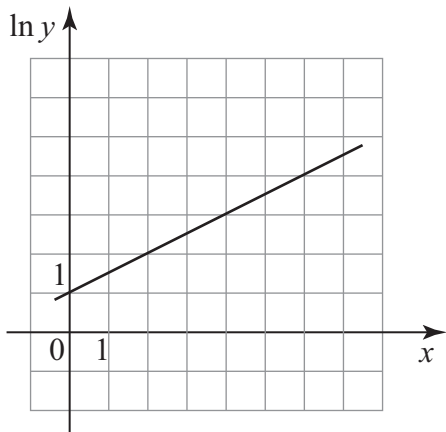
$$S(D) = e^{-0.6D}$$

where $D \geq 0$ represents the applied radiation dose (measured in grays, Gy).

(a)[1] Assume that the dose $D = 5$ Gy is applied to a cancer. What percent of cancer cells is going to survive this treatment? Round off to the nearest integer.

(b)[2] Sketch the semilog graph (use \ln) of the survival rate. Label the axes.

(c)[2] The linear graph below is a semilog graph of a function. Find an explicit formula for that function (i.e., write it in the form $y = \dots$).



7. (a)[3] Identify the domain of the function $f(x) = \ln(e^x - 1)$.

(b)[2] Find $\lim_{x \rightarrow \pi/2^+} \tan x$

(c)[2] Find $\lim_{x \rightarrow \infty} e^{-x^2}$