Concordia University
Department of Mathematics & Statistics

Course	**Number**	**Section**
MATH	205	AA
Examination	**Date**	**Pages**
Final	June 2015	2
Instructor		
Dr. D. Dryanov		

Instructions. *Evaluation out of 100 marks. Answer the numbered questions. The value of each question is indicated in square brackets. Show all your steps. Only approved calculators may be used. This is a closed book exam. Any outside materials, books, notes, or recorded materials may not be used.*

[7 marks] 1. (a) Evaluate the definite integral

$$\int_{-2}^{2}\left(1+\sqrt{4-x^2}\right)dx$$

by interpreting it in terms of signed area.
(b) Use Fundamental Theorem of Calculus, Part 1 to evaluate the derivative $F'(x)$ of the function

$$F(x) = \int_{\sqrt{x}}^{1} \frac{t\cos(\pi t^2)}{t^2+1}\,dt.$$

and use it to determine whether $F(x)$ is increasing or decreasing at $x=1$.

[10 marks] 2. Find the antiderivative $F(x)$ of the function $f(x)$ that satisfies the given condition

$$\textbf{(a)}\quad f(x) = xe^{-x^2},\ F(0)=\frac{1}{2}; \qquad \textbf{(b)}\quad f(x)=x^2\ln(x),\ F(1)=\frac{8}{9}.$$

[15 marks] 3. Find the following indefinite integrals

$$\textbf{(a)}\ \int (1-\cos(x))^2\,dx \qquad \textbf{(b)}\ \int \frac{[\ln(x)]^3}{x}dx \qquad \textbf{(c)}\ \int \frac{x}{x^2+2x-3}dx\,.$$

[15 marks] 4. Evaluate the following definite integrals (*do not approximate, give the exact value*)

$$\textbf{(a)}\ \int_1^2 \frac{e^{1/x}}{x^2}dx \qquad \textbf{(b)}\ \int_0^{\pi/2} x^2\cos(2x)dx \qquad \textbf{(c)}\ \int_0^1 \frac{dx}{(1+\sqrt{x})^2}\,.$$

[10 marks] 5. Evaluate the improper integral or show that it diverges

$$\textbf{(a)}\ \int_e^{\infty} \frac{1}{x\ln^2(x)}dx \qquad \textbf{(b)}\ \int_1^3 \frac{1}{\sqrt[3]{x-1}}dx.$$

[14 marks] 6. (a) Sketch the curves $y=5x-x^2$ and $y=x$, find their points of intersection and then, find the area of the region enclosed by the curves.
(b) Find the average value of the function $f(x)=\sin^2(x)$ on the interval $[0,\pi/2]$.

(c) Sketch the region bounded by the curves $y = x^2$ and $x = y^2$. Then, find the volume of the solid obtained by rotating this region about the x-axis.

[8 marks] 7. Find the limit of the given sequence $\{a_n\}, n = 1, 2, \ldots$ or show that it does not exist

$$\textbf{(a)} \quad a_n = \sqrt{n+10} - \sqrt{n} \qquad\qquad \textbf{(b)} \quad a_n = \frac{\sqrt{1+4n^2+n^4}}{(1+2n\sqrt{n})(5+\sqrt{n})}.$$

[12 marks] 8. Determine whether the given series is convergent or divergent and if convergent, then is it convergent absolutely or conditionally

$$\textbf{(a)} \sum_{n=1}^{\infty} \frac{2-\cos(n)}{n} \qquad \textbf{(b)} \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n\ln(n)} \qquad \textbf{(c)} \sum_{n=1}^{\infty} \left(e^{-3n} - e^{-3(n+1)}\right).$$

[9 marks] 9. (a) Find the radius and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}\,2^n}.$$

(b) Derive the MacLaurin series of the function

$$F(x) = \int_0^x e^{-t^2}\,dt.$$

Then, use the MacLaurin polynomial of degree 7 for $F(x)$ in order to approximate the exact value of the definite integral

$$\int_0^{0.5} e^{-t^2}\,dt.$$

[5 marks] Bonus question 1. Show that the area of the region below the graph of $f(x) = e^{\sqrt{x}}$ and above the x-axis from $x = 0$ to $x = 1$ is equal to the area of the region below the graph of $g(x) = e^{\sin(x)}\sin(2x)$ and above the x-axis from $x = 0$ to $x = \pi/2$.

[5 marks] Bonus question 2. Verify that $f(x) = \sin(\sqrt[3]{x})$ is an odd function and use this fact to show that

$$0 \leq \int_{-1}^{2} \sin(\sqrt[3]{x})\,dx \leq 1.$$

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