

COMP 232 Mathematics for Computer Science
Winter 2015
Midterm Exam

Name: _____

Total Points:

ID: _____

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Instructions. *This is a closed book exam. The only allowed tool is an ENCS approved calculator. Provide all answers in this booklet. Use pen, not pencil. Do not detach any pages from this exam!*

(2^{pts}_{ea.}) **1.** For each of the following propositional sentences, state whether or not it is a tautology.

10 pts

(a) $((p \vee q) \wedge (q \vee r)) \leftrightarrow (q \vee r)$

Tautology

Not tautology

Don't know!

(b) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

Not a tautology

Tautology

Don't know!

(c) $((p \rightarrow q) \wedge (q \rightarrow p)) \leftrightarrow (p \leftrightarrow q)$

Not a tautology

Tautology

Don't know!

(d) $((p \rightarrow q) \wedge (\neg r \rightarrow \neg q)) \rightarrow (p \rightarrow (q \wedge r))$

Tautology

Not a tautology

Don't know!

(e) $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$

Not a tautology

Tautology

Don't know!

10 pts

(2pts ea.) 2. Let the universe of discourse be \mathbb{Z}^+ , the set of all integers. Let

$$P(x, y) =_{\text{def}} x + 2y = xy.$$

12 pts

Determine the truth value of each of the following statements

(a) $P(1, -1)$

True

False

Don't know!

(b) $P(0, 0)$

False

True

Don't know!

(c) $\exists y P(3, y)$

True

False

Don't know!

(d) $\forall x \exists y P(x, y)$

False

True

Don't know!

(e) $\exists x \forall y P(x, y)$

True

False

Don't know!

(f) $\exists x \exists y P(x, y)$

False

True

Don't know!

12 pts

(10^{pts}) **3.** In the table below, construct a proof of the equivalence

$$\neg(p \rightarrow r) \rightarrow \neg q \equiv (p \wedge q) \rightarrow r$$

10 pts

You can only use the equivalences in the table in the handout. Make sure you any associativity or commutativity rule is used explicitly.

Step	Law applied
$\neg(p \rightarrow r) \rightarrow \neg q$	Assumption
$\equiv \neg\neg(p \rightarrow r) \vee \neg q$	Law for Implication
$\equiv (p \rightarrow r) \vee \neg q$	Double negation
$\equiv (\neg p \vee r) \vee \neg q$	Law for implication
$\equiv \neg p \vee (r \vee \neg q)$	Associativity
$\equiv \neg p \vee (\neg q \vee r)$	Commutativity
$\equiv (\neg p \vee \neg q) \vee r$	Associativity
$\equiv \neg(p \wedge q) \vee r$	DeMorgan
$\equiv (p \wedge q) \rightarrow r$	Law for implication

10 pts

(2pts
ea.)

4.

Express the negation of each of the statements (a) to (e) below, so that all negation symbols immediately precede predicates.

10 pts

$$(a) \forall x \exists y \forall z T(x, y, z)$$

$$\text{Negated: } \exists x \forall y \exists z \neg T(x, y, z)$$

$$(b) \forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$$

$$\text{Negated: } \exists x \forall y \neg P(x, y) \wedge \exists x \forall y \neg Q(x, y)$$

$$(c) \exists x \exists y (Q(x, y) \leftrightarrow Q(y, x))$$

$$\text{Negated: } \forall x \forall y ((Q(x, y) \wedge \neg Q(y, x)) \vee Q(y, x) \wedge \neg Q(x, y))$$

$$(d) \forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$$

$$\text{Negated: } \exists y \forall x \forall z (\neg T(x, y, z) \wedge \neg Q(x, y))$$

$$(e) \neg (\forall x (\exists y \forall z T(x, y, z) \wedge \exists z \forall y T(x, y, z)))$$

$$\text{Negated: } \forall x (\exists y \forall z T(x, y, z) \wedge \exists z \forall y T(x, y, z))$$

10 pts

(2pts
ea.) 5.

In each of (a) to (e) below, fill in the blank, so that you obtain a valid argument.

- (a) If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers a and b .
There are no integers a and b , such that $\sqrt{2} = a/b$

$\therefore \sqrt{2}$ is not rational.

- (b) If this is a while loop, then the body of the loop may never be executed,
This is a while loop.

\therefore The body of the loop may never be executed.

- (c) If logic is easy, then I'm a monkey's uncle.
I'm not a monkey's uncle.

\therefore Logic is not easy.

- (d) If this polygon is a triangle, then the sum of its interior angles is 180° .
The sum of the interior angles of this polygon is not 180° .

\therefore This polygon is not a triangle.

- (e) If they were unsure of the address, then they would have telephoned.
They did not telephoned.

\therefore They were sure of the address.

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10 pts

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10 pts

(6_{ea.}^{pts})

6. (a) Consider the assertion

“If x is an odd integer and y is an even integer, then $x + y$ is odd.”

In the box below, give a direct proof of the assertion.

$$\begin{aligned} x \text{ odd} &\Rightarrow x = 2k + 1 \text{ for some } k \in \mathbb{Z} \\ y \text{ even} &\Rightarrow y = 2k' \text{ for some } k' \in \mathbb{Z} \\ &\Rightarrow x + y = 2k + 1 + 2k' = 2(k + k') + 1 \\ &\Rightarrow x + y \text{ odd.} \end{aligned}$$

24 pts

(b) Consider the assertion “If $n^3 + 2n + 1$ is odd, then n is even.”

In the box below, give an indirect proof of the assertion.

$$\begin{aligned} &\text{We need to show: } n \text{ odd} \rightarrow n^3 + 2n + 1 \text{ even.} \\ n \text{ odd} &\Rightarrow n = 2k + 1 \text{ for some } k \in \mathbb{Z} \\ &\Rightarrow n^3 + 2n + 1 = (2k + 1)^3 + 2(2k + 1) + 1 = 8k^3 + 12k^2 + 6k + 1 + 4k + 2 + 1 \\ &= 8k^3 + 12k^2 + 10k + 4 = 2(4k^3 + 6k^2 + 5k + 2) \\ &\Rightarrow n^3 + 2n + 1 \text{ even.} \end{aligned}$$

(c) Consider the assertion

“Let x and y be real numbers. If $5x + 25y = 1723$, then x or y is not an integer.”

In the box below, prove the assertion by contradiction.

$$\begin{aligned} &\text{We need to show } 5x + 25y = 1723 \text{ and } x \text{ and } y \text{ are integers} \Rightarrow \text{contradiction.} \\ x \text{ integer} &\Rightarrow 5x \text{ is an integer.} \\ y \text{ integer} &\Rightarrow 25y \text{ is an integer.} \\ &\Rightarrow 5x + 25y = 5(x + 5y) \text{ is an integer.} \\ &\Rightarrow \frac{1723}{5} \text{ is an integer; contradiction.} \end{aligned}$$

(d) Consider the assertion

“For all integers n , if 3 does not divide n , then n^2 leaves remainder 1 when it is divided by 3.”

In the box below, give a proof by cases of the assertion.

$$\begin{aligned} &\bullet \text{ Case 1: } n \bmod 3 = 1 \\ &\Rightarrow n = 3k + 1 \text{ for some } k \in \mathbb{Z} \\ &\Rightarrow n^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1 \\ &\Rightarrow n^2 \bmod 3 = 1 \\ &\bullet \text{ Case 2: } n \bmod 3 = 2 \\ &\Rightarrow n = 3k + 2 \text{ for some } k \in \mathbb{Z} \\ &\Rightarrow n^2 = 9k^2 + 12k + 4 = 9k^2 + 12k + 3 + 1 = 3(3k^2 + 2k) + 1 \\ &\Rightarrow n^2 \bmod 3 = 1 \end{aligned}$$

— End of Exam —

24 pts