MATHEMATICS 205 **Sample Midterm Test**

Total: 40 marks

Show all your work

Time allowed: 90 minutes

This is a closed book test - no notes permitted

Only approved calculators permitted

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1. (***8 marks***) **a)** Evaluate the intgral $\int_{-3}^{1} |x|\, dx$ by interpreting it in terms of area.

b) Find the derivative dF/dx of the function

$$F(x) = \int_{x^2-1}^{0} \frac{\sin(t+1)}{t+1}\, \mathrm{d}t$$

2. (***5 marks***) Find the antiderivative $F(x)$ of the function $f(x) = x\, e^{-x^2}$ such that $F(0) = 3$.

3. (***12 marks***) Calculate the following indefinite integrals

(**a**) $\displaystyle\int \frac{(\sqrt{2x}-1)^2}{x}\, \mathrm{d}x$ (**b**) $\displaystyle\int 4t^2\, \ln(t)\, \mathrm{d}t$ (**c**) $\displaystyle\int \frac{x-1}{x^2-7x+12}\, \mathrm{d}x$

4. (***8 marks***) Evaluate the following definite integrals (*do not approximate*):

(**a**) $\displaystyle\int_0^3 \frac{1+\arctan(x/3)}{9+x^2}\, \mathrm{d}x$ (**b**) $\displaystyle\int_0^1 x\, e^{-x} \mathrm{d}x$

5. (***7 marks***) Find the average value of $f(x) = 1 + \sin^2(x)$ on the interval $[0, \pi]$.

Bonus Question ***(2 marks)*** Calculate the definite integral

$$\int_0^2 [2 - \sqrt{(2-x)(2+x)}\,]\, \mathrm{d}x$$

in terms of area (HINT: sketch the function)