Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	208/2	All
Examination	**Date**	**Pages**
Final	December 2006	3

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **No programmable calculators allowed!**
  Non-programmable calculators are permitted.

MARKS

[10] **1.** Assume that supply S and demand D are linear functions of price p (in dollars). At \$0.60 per bushel, the daily supply for wheat is 450 bushels, and the daily demand is 570 bushels. When the price p is raised by \$0.75 per bushel, the daily supply increases to 600 bushels, and the daily demand decreases to 495 bushels.

(A) Find the supply $S = S(p)$ as a linear function of price p.

(B) Find the demand $D = D(p)$ as a linear function of price p.

(C) Find the equilibrium price p – i.e., $S(p) = D(p)$ and the equilibrium demand (supply).

[10] **2.** Solve for x in the following equations:

(A) $\pi^{12x-795+2x} = \pi^{-16x+105}$.

(B) $19x^2 - 24x - 95 = 7x^2 - 48x + 85$.

(C) $\log_7 x^2 + 4\log_7 3 - \log_7 9 = \log_7 243 - \log_7 \frac{1}{4} + \frac{1}{3}\log_7 1$.

(D) $e^{3x^2+3x} = e^{6x+60}$.

(E) $\ln(x+3) - \ln(x-4) = \ln 5$.

[10] **3.** Your garage shall need \$250000 five years from now for a new automobile diagnostic system, and the annual rate of interest at your favourite bank is 7.5%, which is compounded **weekly** (52 weeks in a year).

(A) What are your weekly payments such that you have \$250000 five years from now?

(B) What interest has your garage made?

(C) If you add \$1000 to the weekly payment, then how soon shall you have the \$250000?

[10] **4.** You are buying a \$300000 house, for which you make a downpayment of \$60000 and mortgage the rest for 25 years at 7.2% compounded **monthly**.

(A) What are your monthly payments?

(B) How much interest are you paying?

(C) What is the remaining balance after 15 years?

(D) How much are you saving by paying the remaining balance after 15 years?

[10] **5.** Given the functions $f(x) = 4x - 15$ and $g(x) = 5(1.2)^x$ find:

(A) $\displaystyle\sum_{i=1}^{20} f(i) = f(1) + f(2) + \cdots + f(20).$

(B) $\displaystyle\sum_{i=0}^{10} g(i) = g(0) + g(1) + \cdots + g(10).$

[10] **6.** Find the inverse A^{-1} of the matrix $A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

by means of Gauss-Jordan Row Reduction.
No other method of finding A^{-1} shall be accepted!

[10] **7.** An economy is based on agriculture, construction and energy. Production of a dollar's worth of agriculture requires an input of \$0.30, \$0.10 and \$0.20 from these respective sectors taken in the order: agriculture, construction and energy. Production of a dollar's worth of construction requires an input of \$0.20, \$0.10 and \$0.10 from these sectors taken in the above order. Whereas a dollar's worth of energy requires an input of \$0.20, \$0.10 and \$0.10 from each of these sectors taken in the same order.

(A) Write the technological matrix M for the input-output analysis.

(B) Given the demand vector $D = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$,
write the equation for the output vector $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

(C) Given that $D = \begin{pmatrix} 20 \\ 15 \\ 10 \end{pmatrix}$, find the corresponding output $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

[10] **8.** Extremize (find the extrema of) $z = -30x + 40y$ subject to

$$3x + 2y \leq 30,\ 5x + 4y \leq 40,\ x \leq 12,\ x \geq 0 \text{ and } y \geq 0\,.$$

[10] **9.** Given a standard 52-card deck, find the probability of getting:

(A) 5 black cards in a 7-card hand.

(B) 7 number cards in a 7-card hand (number cards are 2, 3, 4, 5, 6, 7, 8, 9, 10).

(C) 4 face cards in a 7-card hand (face cards are jacks, queens and kings).

(D) 7 diamonds in a 7-card hand.

[10] **10.** A shipment of 50 snow-blowers include 5 defective ones. The receiving department selects 7 at random, and rejects the entire shipment if one or more are defective.

(A) What is the probability that the shipment of snow-blowers shall be accepted?

(B) What is the probability that the shipment of snow-blowers shall be rejected?