An Overview of Game Theory

- ***Game theory*** is a set of tools used by economists and many others to analyze players' strategic decision making.
- A **game** is an interaction between players (such as individuals or firms) in which players use strategies.
- **Cooperative** and **Non-cooperative** games
  - can you enforce contracts?

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An Overview of Game Theory

- Useful definitions:
  - The ***payoffs*** of a game are the players' valuation of the outcome of the game (e.g. profits for firms, utilities for individuals).
  - The ***rules of the game*** determine the timing of players' moves and the actions players can make at each move.
  - An ***action*** is a move that a player makes at a specified stage of a game.
  - A ***strategy*** is a battle plan that specifies the action that a player will make based on the information available at each move and for any possible contingency.
  - ***Strategic interdependence*** occurs when a player's optimal strategy depends on the actions of others.

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An Overview of Game Theory

- Assumptions:
  - Players seek to maximize their payoffs
  - Rules of the game are common (believe it or not, this is a technical term!)
  - Payoffs typically depends on actions taken by all players (is this unlike our typical approach????)
  - Complete information (payoff function is common knowledge among all players)
    - NOTE: this is NOT the same thing as perfect information (player knows full history of game up to the point he is about to move)
- **Static games**: players move once, simultaneously
- **Dynamic games**: players move sequentially, may move more than once

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Different Ways of Representing Games

There are three different ways of (visually) representing non-cooperative games:

- Normal form (used a LOT for static games)
- Strategic form (the most confusing: you want to hope that we don't write games down this way!)
- Extensive form (used a LOT for dynamic games)

- However, many games are solved without using one of these three different forms!

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Normal Form Game

- **NORMAL FORM** games:
  - Typically use this form when two (or three) players
  - use a MATRIX to show the actions available to each player: rows for one player, columns for the opponent
  - Entries in the matrix show the payoffs to each player: first entry is the row player, second entry is the column player
  - When three players, have three matrices, and the third player chooses the matrix, whereas the other two choose rows and columns

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13.1 Quantity-Setting Game

- Profits are in millions of dollars per quarter

		West Jet	
		Low	High
Air Canada	Low	(50,50)	(-5,60)
	High	(60,-5)	(0,0)

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Solving A Non-Cooperative Game

- When we try to solve a non-cooperative game, we are trying to PREDICT THE OUTCOME
- We want to know how a (rational) player would choose to play the game.
- If there is a UNIQUE sensible way to play the game, then we have a strong prediction about what the outcome should be
- If there are several different plausible strategies, then it is harder to predict the outcome

Dominant Strategy Equilibria

- Rational players will avoid strategies that are *dominated* by other strategies.
- In fact, we can precisely predict the outcome of any game in which every player has a ***dominant strategy***.
  - A strategy that produces a higher payoff than any other strategy for every possible combination of its rivals' strategies
- NOT all games can be solved using the concept of iterated elimination of dominated strategies

Quantity-Setting Game

- PRISONER'S DILEMMA

		West Jet	
		Low	High
Air Canada	Low	(50,50)	(-5,60)
	High	(60,-5)	(0,0)

Matching Pennies

- Zero sum game
- Cannot be solved using iterated elimination of dominated strategies!

		Betty	
		Heads	Tails
Ahmed	Heads	(1,-1)	(-1,1)
	Tails	(-1,1)	(1,-1)

Battle of the Sexes

- Cannot be solved using iterated elimination of dominated strategies!
- How would you predict that this game will be solved?

		Betty	
		Sailing	Yoga
Ahmed	Sailing	(10,20)	(0,5)
	Yoga	(5,0)	(20, 10)

Nash Equilibrium

- When iterative elimination fails to predict a unique outcome, we can use a related approach.
- The ***best response*** is a strategy that maximizes a player's payoff given its beliefs about its rivals' strategies.
- A set of strategies is a ***Nash equilibrium*** if, when all other players use these strategies, no player can obtain a higher playoff by choosing a different strategy.
  - No player has an incentive to deviate from a Nash equilibrium.

Dominant Strategy Equilibrium is a Nash Equilibrium

- Why is a dominant strategy equilibrium ALWAYS a Nash equilibrium?

		West Jet	
		Low	High
Air Canada	Low	(50,50)	(-5,60)
	High	(60,-5)	(0,0)

Finding the Best Responses and Nash Equilibrium in Normal Form

- In a game without dominant strategies, calculate best responses to determine Nash equilibrium

		Betty	
		Sailing	Yoga
Ahmed	Sailing	(10,20)	(0,5)
	Yoga	(5,0)	(20, 10)

Inefficiency of Nash Equilibrium

- Nash equilibria are not necessarily *efficient,* i.e., they do not necessarily maximize total surplus. (Actually, most of the time they are indeed inefficient. Cooperatives games always have efficient outcomes)

		Betty	
		Sailing	Yoga
Ahmed	Sailing	(9,20)	(-5,0)
	Yoga	(0,-10)	(11, 5)

Mixed Strategies

- **Pure strategies:** each player chooses a single action given the actions chosen by the other player(s).
- A **mixed strategy** is when a player chooses to play more than one action with positive probability, given the actions chosen by the other players
  - MUST have same expected return to each pure strategy in the mixed strategy
  - Choose probability weights optimally, i.e., so that the mixed strategy is a best response
- When a game has multiple pure-strategy Nash equilibria, or no equilibria in pure strategies, a mixed-strategy Nash equilibrium can help to predict the outcome of the game.

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Calculating Mixed Strategy Equilibrium

- Let's go back to Matching Pennies….
- Suppose Betty plays 'Heads' with probability α, 'Tails with probability $1-\alpha$
- Payoff to Ahmed from Heads: $\alpha(1) + (1-\alpha)(-1) = 2\alpha - 1$
- Payoff to Ahmed from Tails: $\alpha(-1) + (1 - \alpha)(1) = 1 - 2\alpha$
- When is Ahmed indifferent?
  If $(2\alpha - 1) = (1 - 2\alpha)$ i.e., $\alpha = 1/2$
- Similarly for Betty: indifferent if $\beta = 0.5$

Exists mixed strategy Nash equilibrium: $\alpha = 0.5, \beta = 0.5$

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Challenge

- Consider the Battle of the Sexes.
- Is there a mixed strategy Nash equilibrium?

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Dynamic Games

- In ***dynamic games***:
  - players move either sequentially or repeatedly
  - players have complete information about payoff functions
  - at each move, players have perfect information about previous moves of all players
- Dynamic games are analyzed in their ***extensive form***, which specifies
  - the *n* players
  - the sequence of their moves
  - the actions they can take at each move
  - the information each player has about players' previous moves
  - the payoff function over all possible strategies.

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Actions and Strategies

- In games where players move sequentially, we distinguish between an action and a strategy.
  - An **action** is a move that a player makes at a specified point.
  - A **strategy** is a battle plan that specifies the action a player will make based on information available at each move.

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Repeated Game

A repeated game is the SAME (STAGE) game repeated a known number of times

- Finitely versus infinitely repeated games

- A game can have many stages *without* being a repeated game: in this case, it is a dynamic game, but not a repeated game.
- In a repeated game, a firm can try to influence its rival's behavior by **signaling** and **threatening to punish**
  - **Will the firm choose to do so? Will the signal or threat be believed, i.e., is it credible?**

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Repeated Airline Game

In the airline game, both firms would like to choose 'Low', but 'High' is the unique NE of the stage game.

Suppose that Air Canada announces the following strategy:

I am going to play 'Low'. If you play 'Low', I will continue to play 'Low' until such time as you play 'High'. If you play 'High', I will then play 'High' forever. (aka Grim Trigger Strategy or Tit for Tat)

Will West Jet play 'High' or 'Low'?

Answer depends on T!

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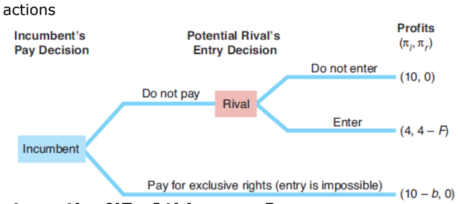
Sequential Games

- This ***game tree*** shows
  - *decision nodes*: indicates which player's turn it is
  - *branches:* indicates all possible actions available
  - *subgames*: subsequent decisions available given previous actions



What are the NE of this game?

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Subgame Perfect Nash Equilibrium

- To predict the outcome of the Stackelberg game, we use a strong version of Nash equilibrium.
- A set of strategies forms a ***subgame perfect Nash equilibrium*** if the players' strategies are a Nash equilibrium in every subgame.
  - This game has four subgames; three subgames at the second stage where United makes a decision and an additional subgame at the time of the first-stage decision.
  - We can solve for the subgame perfect Nash equilibrium using ***backward induction (remember block pricing!)***.

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Backward Induction to Solve Extensive Form Game

- ***Backward induction*** is where we determine:
  - the best response by the last player to move
  - the best response for the player who made the next-to-last move
  - repeat the process until we reach the beginning of the game

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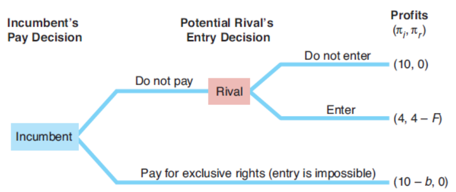
Dynamic Entry Games

- Two NE but only one subgame-perfect NE: Entry occurs unless the incumbent acts to deter entry by paying for exclusive rights to be the only firm in the market.



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Auctions

- What if the players in a game don't have complete information about payoffs?
  - Players have to devise bidding strategies without this knowledge.
- An ***auction*** is a sale in which a good or service is sold to the highest bidder.
- Examples of things that are exchanged via auction:
  - Airwaves for radio stations, mobile phones, and wireless internet access
  - Houses, cars, horses, antiques, art

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Elements of Auctions

- Rules of the Game:
1. Number of units
   - Focus on auctions of a single, indivisible item
2. Format
   - English auction: ascending-bid auction; last bid wins
   - Dutch auction: descending-bid auction; first bid wins
   - Sealed-bid auction: private, simultaneous bids submitted
3. Value
   - Private value: each potential bidder values item differently
   - Common value: good has same fundamental value to all

Bidding Strategies in Private-Value Auctions

- In a ***second-price sealed-bid auction***, the winner pays the amount bid by the second-highest bidder.
- In a second-price auction, should you bid the maximum amount you are willing to spend?
  - If you bid more, you may receive negative consumer surplus.
  - If you bid less, you only lower the odds of winning without affecting the price that you pay if you do win.
  - So, yes, you should bid your true maximum amount.
- In an ***English auction***, you should raise the current highest bid as long as your bid is less than your value.
- In a ***Dutch*** or ***first-price sealed bid auction***, you should shave your bid below your value.

Bidding Strategies in Private-Value Auctions

- English Auction Strategy
  - Strategy is to raise your bid by smallest permitted amount until you reach the value you place on the good being auctioned.
  - The winner pays slightly more than the value of the second-highest bidder.
- Dutch Auction Strategy
  - Strategy is to bid an amount that is equal to or slightly greater than what you expect will be the second-highest bid.
  - Reducing your bid reduces probability of winning but increases consumer surplus if you win.

Revenue Equivalence

- In auctions with private values and other plausible conditions:
  - The expected revenue is the same across all auction formats in which the winner is the person with the highest value.

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Common Values and the Winner's Curse

- The ***winner's curse*** is that the auction winner's bid exceeds the common-value item's value.
  - Overbidding occurs when there is uncertainty about the true value of the good
  - Occurs in common-value but not private-value auctions
- Example:
  - Government auctions of timber on a plot of land
  - Bidders may differ on their estimates of how many board feet of lumber are on the plot
  - If average bid is accurate, then high bid is probably excessive
  - Winner's curse is paying too much

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Behavioral Game Theory

- Behavioral economics seeks to augment the rational economic model so as to better understand and predict economic decision-making.
- Example: ***The Ultimatum Game***
  - Proposer makes a take-it-or-leave-it offer to a responder.
  - In the subgame perfect equilibrium, proposer makes the lowest possible offer and responder accepts.
  - But, such rational behavior is not a good predictor of actual outcomes.
  - Experimentally, the lowest-possible offer is rarely made, and low offers frequently are rejected.
  - Responders reject low offers due to notions of fairness and reciprocity.

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