Carleton University School of Mathematics and Statistics

MATH1119B FALL 2015 TEST 1

Name: Student ID: /25

Non-programmable calculators are allowed. This test has 7 questions.

1. [5 Marks] Find the solution(s) of the following linear systems. Write out the name (**Gaussian Elimination or Gauss-Jordan Elimination**) of the method that you are using. **Show all your work**. If the linear system has infinitely many solutions then find the formula that expresses all the solutions (parametric vector form).

$$\left.\begin{aligned} 2x_1 + 4x_2 + 6x_3 &= 18 \\ 4x_1 + 5x_2 + 6x_3 &= 24 \\ 3x_1 + x_2 - 2x_3 &= 4 \end{aligned}\right\}$$

Gauss-Jordan Eliminatn

Augmented matrix of the system

2

$$\begin{bmatrix} 2 & 4 & 6 & 18 \\ 4 & 5 & 6 & 24 \\ 3 & 1 & -2 & 4 \end{bmatrix} \xrightarrow[R_1 \frac{1}{2}]{R_1 \text{ by}} \begin{bmatrix} 1 & 2 & 3 & 9 \\ 4 & 5 & 6 & 24 \\ 3 & 1 & -2 & 4 \end{bmatrix} \xrightarrow[R_3 \text{ by } R_3 - 3R_1]{R_2 \text{ by } R_2 - 4R_1}$$

$$\begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & -3 & -6 & -12 \\ 0 & -5 & -11 & -23 \end{bmatrix} \xrightarrow[R_2(-\frac{1}{3})]{R_2 \text{ by}} \begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 1 & 2 & 4 \\ 0 & 5 & 11 & 23 \end{bmatrix} \xrightarrow[R_3 - 5R_2]{R_3 \text{ by}}$$

1

Gaussian Elimination stops here

$$\begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow[R_2 - 2R_3]{R_2 \text{ by}} \begin{bmatrix} 1 & 2 & 3 & 9 \\ 0 & 1 & \cancel{2}\ 0 & \cancel{4}\ -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_1 \text{ by } R_1 - 3R_3}$$

1

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow[R_1 - 2R_2]{R_1 \text{ by}} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

or

1

Solution $(4, -2, 3)$.

OR

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 9 \\ x_2 + 2x_4 &= 4 \\ x_3 &= 3 \end{aligned}$$

1

$x_2 = 4 - 2x_2$
$= 4 - 6 = -2$

$x_1 = 9 - 2x_2 - 3x_3$
$= 9 - 2(-2) - 9 = 4$

$(4, -2, 3)$

1

2. [5 Marks] Determine whether $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \ \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}. \ \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix},$$

Is $\underline{b}$ is linear combination of $\underline{a}_1, \underline{a}_2$ & $\underline{a}_3$.
is equivclent to the question.
Does ~~this~~ the vecter equation $x_1\underline{a}_1 + x_2\underline{a}_2 + x_3\underline{a}_3 = \underline{b}$
have a solution.
The equation

$$x_1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3\begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix} \quad (1)$$

(2)

has the same. solution set as the linear system whose augmented matrix is

$$M = \begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix} \xrightarrow[R_2+2R_1]{R_2 \text{ by}} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix} \xrightarrow[R_3 \frac{1}{2}]{R_3 \text{ by}}$$

(1)

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 1 & 4 & 3 \end{bmatrix} \xrightarrow[R_3 - R_2]{R_3 \text{ by}} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(1)

$\therefore$ The linear system corresponding to M has a solution. so the vecter equation (1) has a soln.

(1)

$\therefore$ $\underline{b}$ is a linear combination of $\underline{a}_1$, $\underline{a}_2$, & $\underline{a}_3$

3. [5 Marks] If the following matrix A is the reduced echelon form of an augmented matrix of a linear system of equations with variables x_1, x_2, x_3, x_4 and x_5, then write out the solution in parametric vector form (i.e. $\mathbf{x} = \mathbf{p} + s\mathbf{u} + t\mathbf{v}$)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -3 & 5 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(2)

Basic variables: x_1, x_2, x_4

Free variables: x_3, x_5 (1)

The corresponding system

$$\left.\begin{aligned} x_1 \quad - 3x_5 &= 5 \\ x_2 \quad - 4x_5 &= 1 \\ x_4 + 9x_5 &= 4 \end{aligned}\right\} \Rightarrow$$

(1)

$$\left.\begin{aligned} x_1 &= 5 + 3x_5 \\ x_2 &= 1 + 4x_5 \\ x_3 &\text{ free.} \\ x_4 &= 4 - 9x_5 \\ x_5 &\text{ free.} \end{aligned}\right\} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 + 3x_5 \\ 1 + 4x_5 \\ x_3 \\ 4 - 9x_5 \\ x_5 \end{bmatrix}$$

(1)

$$= \begin{bmatrix} 5 \\ 1 \\ 0 \\ 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + x_5 \begin{bmatrix} 3 \\ 4 \\ 0 \\ -9 \\ 1 \end{bmatrix} = \underline{p} + s\underline{u} + t\underline{v}$$

Where $\underline{p} = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 4 \\ 0 \end{bmatrix}$, $\underline{u} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ & $\underline{v} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -9 \\ 1 \end{bmatrix}$

4. [1.5 Marks] Suppose that $Ax = \mathbf{0}$ has only a trivial solution. What can you say about the solution to $Ax = b$?

(a) Unique solutions **(b)** infinitely many solutions

(c) no solutions **(d)** Not enough information

5. [1.5 Marks] If the columns of an $m \times n$ matrix A span $\mathcal{R}^m$, then what can you say about the equation $Ax = b$ for each **b** in $\mathcal{R}^m$?

(a) Empty solutions **(b)** Inconsistent

(c) Consistent **(d)** Not enough information

6. [2.5 Marks] Circle the matrices that are in reduced echelon form. (equivalently called row-reduced echelon form)

a) $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & -2 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

Questions 7 to 9 are true or false questions. Write True after the question if you think that the statement is true and False if you think that the statement is false. **IMPORTANT: YOU <u>DO NOT NEED</u> TO JUSTIFY YOUR ANSWERS. JUST TO WRITE TRUE OR FALSE.** (4 points)

5) [1.5 Marks] Elementary row operations on an augmented matrix never change the solution set of the associated linear system. True

6) [1.5 Marks] Reducing matrix to echelon from is called the forward phase of the row reduction process. True

7) [1.5 Marks] If the augmented matrix $[Ab]$ has a pivot position in every row, then the equation $Ax = b$ is inconsistent False.