

MATH 1007A — Tutorial Work 5, Dec. 1, 2014

Total Mark: \_\_\_\_\_

Note: Each of the 3-4 team members must clearly print the name and student number on the page top. You may write on both sides and indicate it. If you need to do lots of rough calculation, use your own paper.

Problem 1: (4 marks) Calculate  $\int (e^{-2x} - \frac{2}{x}) dx$ .

$$= \underbrace{-\frac{1}{2} e^{-2x}}_{[-1] \text{ if } (-\frac{1}{2}) \text{ missing}} - 2 \ln|x| + C \quad \underbrace{[-0.5] \text{ if absolute value missing}}_{\text{missing}}$$

Problem 2: (4 marks) Evaluate  $\int_0^1 \frac{1}{1+x^2} dx$ .

$$= \underbrace{\arctan x \Big|_0^1}_{[3]} = \arctan 1 - \arctan 0 = \frac{\pi}{4} \quad [1]$$

Problem 3: (4 marks) Suppose  $f'(x) = \sin(2x)$  and  $f(0) = 1$ . Find  $f$ .

$$f(x) = \int \sin 2x dx = -\frac{1}{2} \cos 2x + C \quad [2]$$

$$-\frac{1}{2} \cos 0 + C = 1 \Rightarrow C = \frac{3}{2} \quad [1] \quad f = -\frac{1}{2} \cos 2x + \frac{3}{2}$$

Problem 4: (4 marks) Evaluate  $\int_1^4 x e^{x^2} dx$  by the substitution formula ( $u = x^2$ ).

$$u = x^2 \quad [1] \quad = \int_1^4 \frac{1}{2} e^u du \quad [2]$$

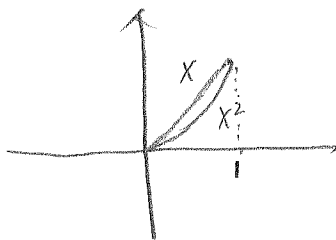
$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$= \frac{1}{2} e^u \Big|_1^4 = \frac{1}{2} (e^4 - e) \quad [1]$$

Problem 5: (4 marks) Find the area bounded by  $y = x^2$  and  $y = x$ .

$$x^2 = x \Rightarrow x = 0 \text{ or } x = 1$$



$$\text{area} = \int_0^1 (x - x^2) dx \quad [2]$$

$$= \int_0^1 x dx - \int_0^1 x^2 dx \quad [2]$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$