(V1)

MAT 1341E –DGD 1– Test 1- Diagnostic test 2015

21-September - 2015. Duration: 80 minutes.

Instructor: Saeid Molladavoudi

1	E
2	E
3	A
4	E
5	B
6	C
7	C
8	C
9	A
10	C
11	A
12	A
Total	

θ	$\sin\theta$	$\cos\theta$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0

Family Name:____________________

First Name:____________________

Student number:____________________

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are permitted. The use of calculators, cell phones, or similar devices is not permitted. All implanted cyber devices not necessary for life-support must be disabled at the beginning of the exam.
3. Read each question carefully – you will save yourself time and unnecessary grief later on.
4. All questions are multiple choice, are worth 1 point each and no part marks will be given. <u>Please record your answers in the spaces on this page provided next to the question numbers above.</u>
5. Where it is possible to check your work, do so.
6. Good luck! Bonne chance!

1. An equation for the plane which contains the two lines with parametric equations $x = -1 + t,\ y = -1 - t,\ z = 1 + 3t$ and $x = -3 - s,\ y = 3 + 2s,\ z = 7 + 3s$, is:

A. $7x - 11y + 2z = 6$
B. $11x - 2y + 9z = 0$
C. $6x - 2y + z = -3$
D. $3x - 6y + z = 4$
E. $9x + 6y - z = -16$
F. $9x + 6y + z = -14$

$\vec{v}_1 = (x_1, y_1, z_1)$, $x_1 = -1 + t$, $y_1 = -1 - t$, $z_1 = 1 + 3t$

$$\Rightarrow \vec{v}_1 = \underbrace{\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}_{\text{position vector}} + t \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \rightarrow \text{direction vector } \vec{d}_1$$

$V_2 = (x_2, y_2, z_2)$, $x_2 = -3 - s$, $y_2 = 3 + 2s$, $z_2 = 7 + 3s$

$$\Rightarrow V_2 = \overbrace{\begin{pmatrix} -3 \\ 3 \\ 7 \end{pmatrix}}^{\text{position vector}} + s \overbrace{\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}}^{\text{direction vector } \vec{d}_2}$$

$$\Rightarrow \vec{d}_2 \times \vec{d}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ +1 & -1 & 3 \end{vmatrix} = (+3+6)\hat{i} + (3+3)\hat{j} + (-2+1)\hat{k} = \boxed{+9\hat{i} + 6\hat{j} - \hat{k}}$$

normal vector to plane

$\Rightarrow 9x + 6y - z = d$, to find "d", plug in the position vector of $\vec{v}_1$, i.e. $(-1,-1,1)$ to the equation of plane $\Rightarrow d = -16$

2. An equation for the plane passing through the points $(0, -3, 0)$ and $(-1, 1, 2)$, and which is parallel to the x–axis is:

A. $3x + 2y + 7z = -6$
B. $2x - y = 3$
C. $x - y + z = 3$
D. $x - z = 0$
E. $y - 2z = -3$
F. $x + y + z = -3$

$B = (0, -3, 0)$, $A = (-1, 1, 2)$

$\vec{AB} = (1, -4, 2)$

the plane also contains $\hat{i} = (1, 0, 0)$,

$$\Rightarrow \vec{n} = \vec{AB} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & 2 \\ 1 & 0 & 0 \end{vmatrix} = -2\hat{j} + 4\hat{k}$$

$\Rightarrow \vec{n} = (0, -2, 4) = -2(0, +1, -2) \Rightarrow (0, +1, -2)$ is also a normal vector

$\Rightarrow 0x + y - 2z = d$, to find "d", plug in $(0, -3, 0)$

$\Rightarrow d = -3 \Rightarrow P: \boxed{y - 2z = -3}$

3. Find an equation of the plane which passes through the point $(1, 1, 1)$ and which is perpendicular to the line whose scalar parametric equations are:

$$x = -6 + 2t,\ y = 1 - 4t,\ z = -3 + 3t;\ t \in \mathbf{R}.$$

A. $2x - 4y + 3z = 1$
B. $2x + 4y + 3z = 9$
C. $6x + y - 3z = 2$
D. $2x - 4y + 3z = -25$
E. $2x - 4y + 3z = -10$
F. $2x - 4y + 3z = 10$

$\hat{n} = (x, y, z) = (-6+2t, 1-4t, -3+3t)$

$= (-6, 1, -3) + t(2, -4, 3)$

$\Rightarrow P: 2x - 4y + 3z = d$

$\Rightarrow$ plugin $(1,1,1)$ to the equation of P

$\Rightarrow d = 2 - 4 + 3 = 1$

$\Rightarrow P: 2x - 4y + 3z = 1$

4. Parametric equations for the line containing $(2,\ -2,\ 3)$ and $(-2,\ 4,\ 0)$ are:

A. Such a line does not exist.
B. $x = 2 - 2t,\ y = -2 + 4t,\ z = 3;\ t \in \mathbf{R}.$
C. $x = 1 - t,\ y = -1 - 6t,\ z = 4 + 3t;\ t \in \mathbf{R}.$
D. $x = 3 + 4t,\ y = -1 - 6t,\ z = 6 + t;\ t \in \mathbf{R}.$
E. $x = 2 + 4t,\ y = -2 - 6t,\ z = 3 + 3t;\ t \in \mathbf{R}.$
F. $x = -2 + 4t,\ y = 4 + 6t,\ z = 1 + 3t;\ t \in \mathbf{R}.$

$A = (2, -2, 3)$, $B = (-2, 4, 0)$

$\vec{BA}$ = direction vector

$= (4, -6, 3)$

$\Rightarrow L = \{\vec{V_0} + t\vec{d} \mid t \in \mathbb{R}\}$, $\vec{d} \equiv \vec{BA}$ = direction vector

$\Rightarrow$ choose $\vec{V_0} = A$ (non-unique)

$\Rightarrow \vec{L} = \{(2, -2, 3) + t(4, -6, 3) \mid t \in \mathbb{R}\}$

5. Find a Cartesian (scalar) equation for the plane with vector parametric equation

$$v = (0,2,-2) + s(1,-1,2) + t(2,-4,-1);\ s,t \in \mathbf{R}.$$

position vector

direction vectors

A. $4x - 9y + 6z = -30$

B. $9x + 5y - 2z = 14$

C. $9x - 5y + 2z = -14$

D. $9x + 5y + 2z = 6$

E. $9x + 2y + 5z = -6$

F. $9x - 2y + 5z = -14$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & -4 & -1 \end{vmatrix} = 9\hat{i} + 5\hat{j} - 2\hat{k}$$

$$P = \{\overbrace{(x,y,z)}^{\vec{v}} \in \mathbb{R}^3 \mid (\vec{v} - \vec{v}_0)\cdot\vec{n} = 0\}$$

$$= \{(x,y,z) \in \mathbb{R}^3 \mid ((x,y,z) - (0,2,-2))\cdot(9,5,-2) = 0\}$$

$$= \{(x,y,z) \in \mathbb{R}^3 \mid (x, y-2, z+2)\cdot(9,5,-2) = 0\}$$

$$= \{(x,y,z) \in \mathbb{R}^3 \mid 9x + 5y - 2z = 14\}$$

6. Find all vectors in $\mathbf{R}^3$ which are perpendicular to both $(-1,1,5)$ and $(2,1,2)$.

A. $\{(2,\ -8,\ 2)\}$

B. $\{(t+1,\ -8,\ t+1) \mid t \in \mathbf{R}\}$

C. $\{(t,\ -4t,\ t) \mid t \in \mathbf{R}\}$

D. $\{(-t,\ 0,\ t) \mid t \in \mathbf{R}\}$

E. $\{(0,\ 0,\ 0)\}$

F. $\{(3,\ -12,\ 3)\}$

$\vec{v}_1 = (-1,1,5)$, $\vec{v}_2 = (2,1,2)$

$$\Rightarrow \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 5 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 12\hat{j} - 3\hat{k}$$

$$\Rightarrow \vec{v}_1 \times \vec{v}_2 = -3(+1, -4, 1)$$

$t = -3 \in \mathbb{R}$

t can be any real number

7. A triangle has vertices $A = (1,\ 1,\ 1)$, $B = (2,\ 3,\ 1)$ and $C = (1,\ 2,\ 3)$. Find the cosine of the interior angle at A.

A. 0
B. 1/5
C. 2/5
D. 3/5
E. 4/5
F. 1

$\vec{AB} = +(+1, +2, 0)$

$\vec{AC} = +(0, 1, 2)$

$$\Rightarrow \cos(\theta) = \frac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\| \, \|\vec{AC}\|} = \frac{2}{\sqrt{1+4}\sqrt{1+4}} = \frac{2}{5}$$

8. The point of intersection of the line through the point $(2,\ 1,\ 0)$ parallel to the vector $u = (1,\ -1,\ 2)$ with the plane with equation $x + y + 2z = 23$ is:

A. (11, 4, 4)
B. (2, 1, 10)
C. (7, −4, 10)
D. (7, 4, 6)
E. (11, 1, 4)
F. (10, −4, 7)

$\vec{v_0}$ = position vector = $(2, 1, 0)$

$\vec{d}$ = direction vector = $(1, -1, 2)$

$$\Rightarrow L = \left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$$

$$\Rightarrow \left.\begin{array}{l} x = 2 + t \\ y = 1 - t \\ z = 2t \end{array}\right\} \Rightarrow \text{plug into } P: x + y + 2z = 23$$

$\Rightarrow (2 + \cancel{t}) + (1 - \cancel{t}) + 2(2t) = 23 \Rightarrow 4t = 20 \Rightarrow t = 5$

$\Rightarrow x = 7, y = -4, z = 10$

9. If $u = (3,\ 3,\ 6)$ and $v = (2,\ -1,\ 1)$ then the length of the projection of u along v is:

A. $\frac{3\sqrt{6}}{2}$

B. $\frac{3\sqrt{2}}{2}$

C. 0

D. $\frac{\sqrt{6}}{2}$

E. $\frac{2\sqrt{6}}{3}$

F. $\frac{2\sqrt{2}}{3}$

$$\mathrm{Proj}_{\vec{v}}(\vec{u}) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{(3,3,6)\cdot(2,-1,1)}{(4+1+1)}\vec{v}$$

$$= \frac{9}{6}(2,-1,1)$$

$$\Rightarrow \|\mathrm{Proj}_{\vec{v}}(\vec{u})\| = \frac{9}{6}\|\vec{v}\| = \frac{9}{6}\sqrt{6} = \frac{3}{2}\sqrt{6}$$

10. Find the area of the triangle whose vertices are the points $P = (3,\ -1,\ 2)$, $Q = (1,\ 1,\ 0)$ and $R = (1,\ 2,\ -1)$.

A. 4

B. $2\sqrt{2}$

C. $\sqrt{2}$

D. 0

E. $4\sqrt{2}$

F. 2

$$\vec{PQ} = (-2, +2, -2)$$

$$\vec{PR} = (-2, +3, -3)$$

$$\Rightarrow \text{area} = \frac{1}{2}\|\vec{PQ}\times\vec{PR}\| = \frac{1}{2}\left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & +2 & -2 \\ -2 & +3 & -3 \end{vmatrix} \right\|$$

$$\Rightarrow \text{area} = \frac{1}{2}\|0\hat{i} - 2\hat{j} - 2\hat{k}\| = \frac{1}{2}\sqrt{4+4} = \sqrt{2}$$

11. Write the complex number

$$\frac{(1+3i)(5+10i)}{4+3i} = z = \frac{5+10i+15i-30}{4+3i}$$

in Cartesian form: $a+bi$, with $a, b \in \mathbf{R}$.

$$= \frac{-25+25i}{4+3i}$$

A. $-1+7i$
~~B.~~ -1
C. $1+7i$
D. $7i$
E. $-5+35i$
F. $-\frac{1}{5}+\frac{7}{5}i$

$$z \cdot \frac{(4-3i)}{4-3i} = \frac{-25+25i}{4+3i} \cdot \frac{(4-3i)}{(4-3i)}$$

$$= \frac{\cancel{25}(-1+i)(4-3i)}{\cancel{25}} = \boxed{-1+7i}$$

12. What is the polar form of $\frac{-\sqrt{2}+\sqrt{2}i}{3+3\sqrt{3}i}$?

$z_1 = -\sqrt{2}+\sqrt{2}i \Rightarrow |z_1| = \sqrt{2+2} = 2$

A. $\frac{1}{3}(\cos(5\pi/12) + i\sin(5\pi/12))$
B. $\frac{1}{3}(\cos(5\pi/12) - i\sin(5\pi/12))$
C. $3(\cos(5\pi/12) - i\sin(5\pi/12))$
D. $3(\cos(5\pi/12) + i\sin(5\pi/12))$
E. $\cos(11\pi/12) + i\sin(11\pi/12)$
F. $\cos(5\pi/12) + i\sin(5\pi/12)$

$\cos\theta_1 = -\frac{\sqrt{2}}{2}$, $\sin\theta_1 = \frac{\sqrt{2}}{2}$

$\Rightarrow \theta_1 = \frac{3\pi}{4}$

$z_2 = 3+3\sqrt{3}i \Rightarrow |z_2| = \sqrt{9+27} = \sqrt{36} = 6$

$\cos\theta_2 = \frac{3}{6} = \frac{1}{2}$, $\sin\theta_2 = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta_2 = \pi/3$

$$\Rightarrow \frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} e^{i(\theta_1-\theta_2)} = \frac{2}{6} e^{i\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)} = \frac{1}{3} e^{i(5\pi/12)}$$

$$= \frac{1}{3} e^{i(+5\pi/12)} = \frac{1}{3}\left(\cos\left(+\frac{5\pi}{12}\right) + i\sin\left(+\frac{5\pi}{12}\right)\right)$$