Question 1- Calculate:

$$\lim_{x\to 2} \frac{x^3-8}{x-2}$$

A) 10 B) 12 C) 3 D) 14 E) 5

$$\begin{array}{r} x^2 + 2x + 4 \\ x-2 \overline{) x^3 - 8} \\ -(x^3 - 2x^2) \\ \hline 2x^2 - 8 \\ -(2x^2 - 4x) \\ \hline 4x - 8 \\ 4x-8 \\ \hline 0 \end{array}$$

So $x^3-8 = (x-2)(x^2+2x+4)$

and

$$\lim_{x\to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} x^2+2x+4 = 12$$

Question 2- For what interval or intervals is the function $f(x) = \frac{x}{x-1}$ increasing?

A) $(-\infty, 1)$ B) $(1, \infty)$ C) $(-\infty, 1) \cup (1, \infty)$ D) $(0, \infty)$ E) The function is never increasing.

$f'(x) = \frac{-1}{(x-1)^2}$, which is never positive

Question 3-Use implicit differentiation to find $\frac{dy}{dx}$ at $(1,1)$ when

$$x^3 + 4xy^2 - y^3 - 2 = 2y^4$$

A) $\frac{7}{3}$ B) $\frac{3}{2}$ C) $-\frac{1}{2}$ D) $\frac{2}{5}$ E) $-\frac{7}{2}$

$$3x^2 + 4y^2 + 8xy\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 8y^3\frac{dy}{dx}$$

Plug in (1,1)

$$3 + 4 + 8\frac{dy}{dx} - 3\frac{dy}{dx} = 8\frac{dy}{dx}$$

$$7 - 3\frac{dy}{dx} = 0 \quad \text{So } \frac{dy}{dx} = 7/3$$

Question 4- Consider the function $f(x) = (x+3)e^{x+3}$. Which of the following statements is correct?

A) There is a local max at $x = 0$. B) ~~There is a local max at $x = -3$.~~
C) There is a local min at $x = -3$. D) There is a local min at $x = -4$.
E) There is a local max at $x = -4$.

$$f'(x) = (x+3)e^{x+3} + 1e^{x+3} = (x+4)e^{x+3}$$

There is a CP at $x = -4$

		-4	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$		$\nearrow$

Question 5-Consider the following function:

$$f(x) = \begin{cases} a - x & \text{if } x < -2; \\ 3 & \text{if } x = -2; \\ bx^2 - 5 & \text{if } x > -2. \end{cases}$$

For what value of the constant a and b is $f(x)$ continuous for all real numbers?

A) $a = 1, \quad b = 3$ B) $a = -2, \quad b = 2$ C) $a = 1, \quad b = 2$ D) $a = 3, \quad b = 5$
E) $a = -3, \quad b = 6$

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} a - x = a + 2$$

~~lim f(x) = 3 so a+2=3 a=1~~

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} bx^2 - 5$$

$$= 4b - 5.$$

So we must have

$a + 2 = 3$

and

$4b - 5 = 3$

So $a = 1$ and $b = 2$

Question 6- Suppose the demand function for a product is given by $p(x) = 216 - 2x^2$. Find the elasticity of demand when $x = 10$. Is demand elastic?

A) $-\frac{3}{50}$, inelastic B) $-\frac{3}{50}$, elastic C) $-\frac{1}{25}$, inelastic D) $-\frac{1}{25}$, elastic
E) $-\frac{5}{12}$, elastic

$p(10) = 216 - 200 = 16$

$$\frac{dp}{dx} = -4x.$$

So $\frac{dp}{dx}(10) = -40$

$$\eta = \frac{p/x}{dp/dx} = \frac{16/10}{-40} = \frac{-16}{10 \cdot 40} = \frac{-1}{25}$$

$|\eta| < 1$. So inelastic

Question 7- Suppose $f'(x) = x^{\frac{2}{3}}$ and $f(1) = 1$. Find $f(8)$.

A) $\frac{64}{5}$ B) $\frac{82}{5}$ C) $\frac{74}{5}$ D) $\frac{98}{5}$ E) $\frac{6}{5}$

$$f(x) = \int x^{2/3} dx = \frac{3}{5} x^{5/3} + C$$

$$f(1) = \frac{3}{5} + C = 1. \text{ So } C = 2/5$$

$$f(x) = \frac{3}{5} x^{5/3} + \frac{2}{5}$$

$$f(8) = \frac{3}{5}(8)^{5/3} + 2/5 = \frac{3}{5} \cdot 32 + \frac{2}{5} = \frac{98}{5}$$

Question 8- Suppose that for a certain product, the demand function is given by $D(x) = 28 - x^2$ and the supply function is given by $S(x) = x^2 + 2x + 4$. Calculate the consumer surplus.

A) 19 B) 18 C) $\frac{26}{3}$ D) 27 E) 12

Equilibrium: $28 - x^2 = x^2 + 2x + 4$

$$0 = 2x^2 + 2x - 24$$

$$= x^2 + x - 12$$

$$= (x+4)(x-3)$$

$$x = -4, 3$$

If $x = 3$, $y = 19$

So CS =

$$\int_0^3 28 - x^2 - 19 \, dx$$

$$= \int_0^3 (9 - x^2) \, dx$$

$$= 9x - \frac{x^3}{3} \Big|_0^3$$

$$= 27 - 9 = 18$$

Question 9- Calculate:

$$\int_0^\infty e^{-3x}dx$$

A) 1 B) $\frac{1}{3}$ C) $\frac{4}{3}$ D) $\frac{1}{4}$ E) The integral is divergent.

$$= \lim_{b\to\infty} \int_0^b e^{-3x}\,dx$$

$$= \lim_{b\to\infty} \left[\frac{1}{3} - \frac{1}{3e^{-3b}}\right]$$

$$= \frac{1}{3}$$

$$\int_0^b e^{-3x}\,dx$$

$$= -\frac{1}{3}e^{-3x}\Big|_0^b$$

$$= -\frac{1}{3}e^{-3b} - \left(-\frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{1}{3e^{-3b}}$$

Question 10- Calculate:

$$\int_0^1 xe^{2x}dx$$

A) e^2 B) $\frac{e^2}{2}$ C) $\frac{e-1}{2}$ D) $\frac{e^2+1}{4}$ E) $\frac{e+3}{2}$

$$\int x e^{2x}\,dx$$

$$\boxed{\begin{array}{ll} u = x & v = \frac{1}{2}e^{2x} \\ du = dx & dv = e^{2x}\,dx \end{array}}$$

$$= \frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx$$

$$= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}$$

$$\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x}\Big|_0^1$$

$$= \left(\frac{1}{2}e^2 - \frac{1}{4}e^2\right) - \left(0 - \frac{1}{4}\right)$$

$$= \frac{1}{4}e^2 + \frac{1}{4} = \frac{e^2+1}{4}$$

Question 11- Suppose $f(x,y) = e^{xy}$. Find $f_{xy}(2,1)$.

A) e^2 B) $2e^2$ C) $3e^2$ D) $4e^2$ E) $5e^2$

$f_x = ye^{xy}$

$f_{xy} = e^{xy} + xye^{xy}$

$f_{xy}(2,1) = e^2 + 2e^2 = 3e^2$

Question 12- How many critical points does the following function of 2 variables have?

$$g(x,y) = x^2 + 2x + y^3 - 3y^2 + 7$$

A) 0 B) 1 C) 2 D) 3 E) 4

$g_x = 2x + 2 = 0 \Rightarrow x = -1$

$g_y = 3y^2 - 6y = 0 \Rightarrow 3y(y-2) = 0 \Rightarrow y = 0, 2$

So 2 CPs at

$(-1,0)$ & $(-1,2)$

Long Answer Question 1 (12 points)

Suppose you start with an initial deposit of \$4000 with an annual interest rate of 3%.

(a) How much do you have after 8 years if you compound every 6 months? (You do not need to simplify your answer)

(b) How long will it take for the balance to double if interest is compounded continuously?

a) $P(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$

$= 4000\left(1+\frac{.03}{2}\right)^{2\cdot 8}$

b) $P(t) = P_0 e^{rt} = 4000\, e^{.03t}$

We must solve

$$8000 = 4000 e^{.03t}$$

$$2 = e^{.03t}$$

$$\ln(2) = \ln(e^{.03t})$$

$$\ln(2) = .03t$$

$$t = \frac{\ln(2)}{.03}$$

Long Answer Question 2 (12 points)
Evaluate the following indefinite integrals:

$$\int x^7 \ln(x)\, dx$$

$u = \ln(x)$ $\quad v = x^8/8$

$du = \frac{1}{x} dx$ $\quad dv = x^7 dx$

$$\int x^7 \ln(x) dx = \frac{x^8}{8}\ln(x) - \int \frac{x^8}{8} \cdot \frac{1}{x} dx$$

$$= \frac{x^8}{8}\ln(x) - \int \frac{x^7}{8} dx = \frac{x^8}{8}\ln(x) - \frac{x^8}{64} + C$$

$$\int \frac{-3x-3}{x^2+2x+2}\, dx$$

$$= -3\int \frac{x+1}{x^2+2x+2} dx$$

U-substitution

$u = x^2+2x+2$

$du = 2x+2\ dx$

$= 2(x+1) dx$

$$= \frac{-3}{2}\int \frac{2(x+1)\ dx}{x^2+2x+2}$$

$$= \frac{-3}{2}\int \frac{du}{u} = \frac{-3}{2}\ln(u)$$

$$= \frac{-3}{2}\ln(x^2+2x+2) + C$$

Long Answer Question 3 (14 points)
Consider the two functions:
$f(x) = x^2 - 2x \quad$ and $\quad g(x) = x$

(a) **(2 points)** Find the intersection points of the graphs of the two functions.

(b) **(6 points)** On the next page, graph these functions, and shade the region between the graphs of f and g for x such that $0 \leq x \leq 4$.

(c) **(6 points)** Find the area of the shaded region.

$f(x) = g(x)$

$x^2 - 2x = x$

$x^2 - 3x = 0$

$x = 0 \quad x = 3$

b) $f(x) = x^2 - 2x = 0 \Rightarrow x = 0, 2$ x-intercept

$f'(x) = 2x - 2 = 0$

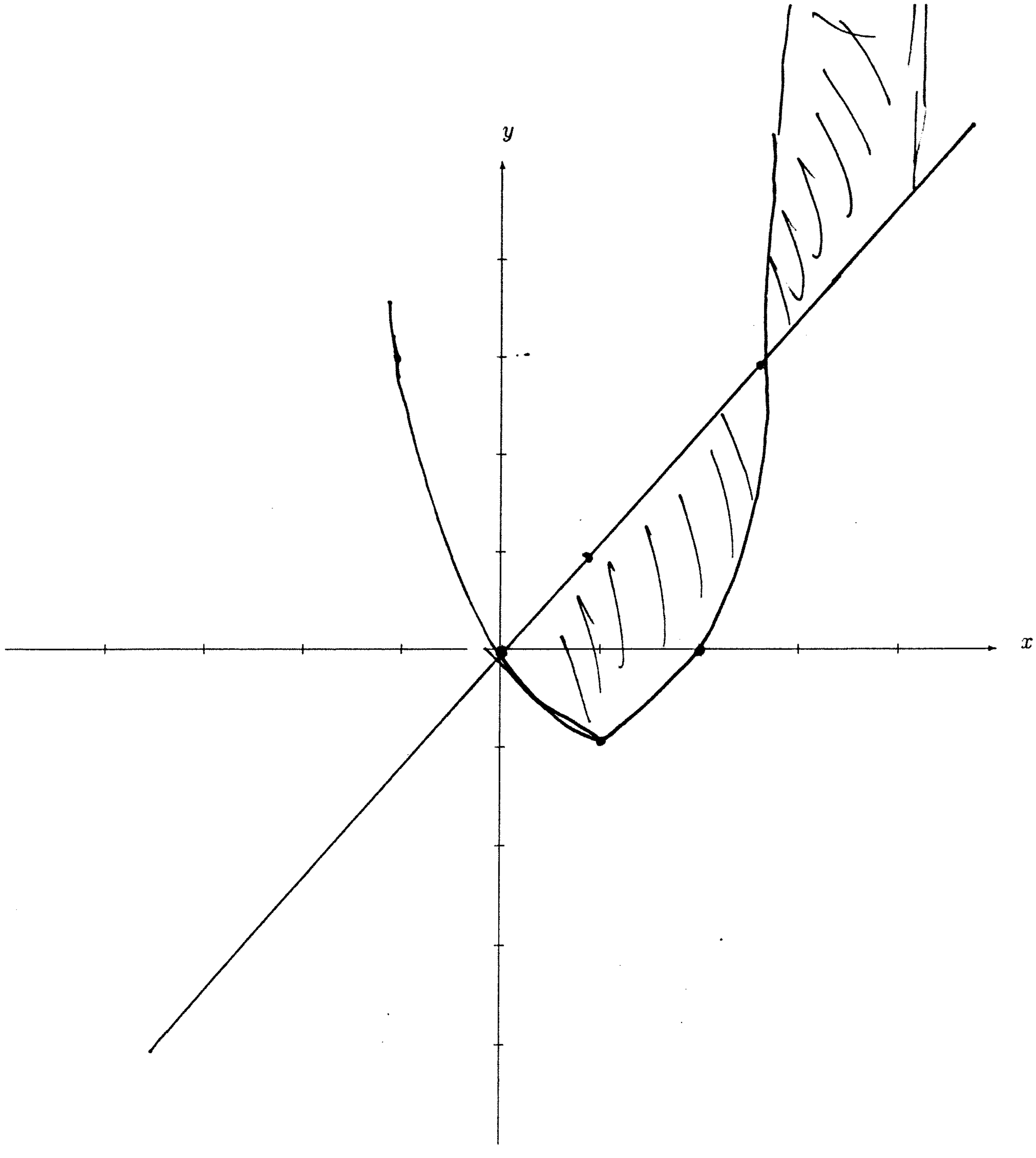
$x = 1 \quad (1, -1)$ local min

$f''(x) = 2 > 0$

c) $A_1 = \int_0^3 (x - (x^2 - 2x))\,dx = \frac{9}{2}$

$A_2 = \int_3^4 (x^2 - 2x) - x \, dx = 11/6$

$\frac{9}{2} + \frac{11}{6} = \frac{27+11}{6} = \frac{38}{6} = \frac{19}{3}$



y
x

Long Answer Question 4 (14 points)

Consider the function of two variables

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy + 20$$

(a) **(4 points)** Calculate the first-order partial derivatives.

(b) **(4 points)** Find all critical points.

(c) **(6 points)** Identify what type of critical points they are (local max, local min or saddle point).

a) $\begin{cases} f_x = 12x - 6x^2 + 6y \\ f_y = 6y + 6x \end{cases}$

b) $y = -x \implies 12x - 6x^2 - 6x = 0$

$6x - 6x^2 = 0 \qquad x = 0, \; x = 1$

$(0,0) \quad (1,-1)$

c) $f_{xx} = 12 - 12x \quad f_{yy} = 6$

$f_{xy} = 6 \quad f_{yx} = 6$

2

$D(x,y) = 6(12 - 12x) - 36 = 36(1 - 2x)$

$D(0,0) = 36 > 0 \qquad f_{xx}(0,0) = 12 > 0 \qquad (0,0)$ local min

$D(1,-1) = -36 < 0 \rightarrow$ saddle point