Assignment 1 Solution **Total: 5 points**

1. (2 points) We use 0-1 variable xj for each investment.
   - If xj = 1, then we will make investment j.
   - If it is 0, we will not make the investment j.

Let the unit be 100. Then we have

$$
\begin{aligned}
\text{Maximize} \quad & 8x1+10x2+6x3+4x4 \\
\text{Subject to} \quad & 5x1+ 7x2+4x3+3x4 <= 54 \\
& x1+ x2+ x3+ x4 >= 2 \\
& x2 - x4 <= 0 \\
& x3 <= 2-(x1+ x2) \\
& xj \in \{0,1\}, \quad j=1,\ldots,4.
\end{aligned}
$$

2. (1 point) They are not equivalent. For example, x=3, y=2, z=0 is a solution of System 2, but not a solution of System 1.

3. (2 points)

Step 1: Eliminate x: From (1)-(5) we have

$$
\begin{aligned}
x &\geq 16/3-4/3y-2/3z && (6)\\
x &\leq 14-7/4y-1/2z && (7)\\
x &\geq -5+2y+2z && (8)\\
x &\leq 15/2+7/4y && (9)\\
x &\geq 9/2+3/2y && (10)
\end{aligned}
$$

This implies that

$$
\begin{aligned}
&16/3-4/3y-2/3z \leq 14-7/4y-1/2z\\
&16/3-4/3y-2/3z \leq 15/2+7/4y\\
&-5+2y+2z \leq 14-7/4y-1/2z\\
&-5+2y+2z \leq 15/2+7/4y\\
&9/2+3/2y \leq 14-7/4y-1/2z\\
&9/2+3/2y \leq 15/2+7/4y
\end{aligned}
$$

which implies

$$
\begin{aligned}
&y \leq 104/5+2/5z\\
&y \geq -26/37-8/37z\\
&y \leq 76/15-2/3z\\
&y \leq 50-8z\\
&y \leq 38/13-2/13z\\
&y \geq -12
\end{aligned}
$$

Step 2: Eliminate y:

$-26/37-8/37z \leq 104/5+2/5z$	$\rightarrow z \geq -663/19$
$-26/37-8/37z \leq 76/15-2/3z$	$\rightarrow z <= 1601/125$
$-26/37-8/37z \leq 50-8z$	$\rightarrow z <= 469/72$
$-26/37-8/37z \leq 38/13-2/13z$	$\rightarrow z \geq -872/15$
$-12 \leq 104/5+2/5z$	$\rightarrow z \geq -82$
$-12 \leq 76/15-2/3z$	$\rightarrow z <= 128/5$
$-12 \leq 50-8z$	$\rightarrow z <= 31/4$
$-12 \leq 38/13-2/13z$	$\rightarrow z <= 97$

Therefore

$$-663/19 \leq z \leq 469/72.$$

Step 3: Final solution

$$-663/19 \leq z \leq 469/72$$

$$\max\{-26/37-8/37z,\ -12\} \leq y \leq \min\{104/5+2/5z,\ 76/15-2/3z,\ 50-8z,\ 38/13-2/13z\}$$

$$\max 16/3-4/3y-2/3z,\ -5+2y+2z,\ 9/2+3/2y\} \leq x \leq \min\{14-7/4y-1/2z,\ 15/2+7/4y\}$$