Solutions

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DEPARTMENT OF MATHEMATICS
MID TERM TEST 1
MTH 140 – CALCULUS I
Date: September 26th, 2014, 4:00 pm
Duration: 1.5 hours

Last Name:_____

First Name:_____

Ryerson email: _____

Student Number(ONLY LAST 3 DIGITS): _____

Signature:_____

Section (circle one)

A. Alvarez :	1	2	3	4	
D. Ha :	6	7	8	9	10
X. Liu :	11	12	13	14	15
N. Jung :	16	17	18	19	20
J. Horowitz :	21	22	23	24	25

Instructions:

1. This is a closed-book test. **Notes, calculators and other aids are not permitted.**
2. Verify that your test has pages 1-10. The exam questions are in pages 2-8. Pages 9 and 10 are given for extra space, it does NOT contain any questions.
3. (a) Unless otherwise instructed, **make sure you include all significant steps in your solution, presented in the correct order. Unjustified answers will be given little or no credit. Cross out or erase all rough work not relevant to your solution.** Put a box around your final **answer.**
   (b) For multiple choice questions make sure to write your answers in the box at the end of each question **carefully**. There are no part marks in the multiple-choice section and **only** the answer in the box will be marked. The correct response gets full marks, an incorrect response or no response gets no marks.
   (c) Write your solutions in the space provided. **ANYTHING WRITTEN IN THE BACK OF ANY PAGE WILL NOT BE MARKED**. If you need more space, use the front part of page 9 and page 10. Indicate this fact on the original page.
4. Do not separate the sheets.
5. Have your student card available on your desk.

1. [2 marks] (Multiple choice question). The domain of the function $f(x) = \sqrt{\sqrt[3]{x} - 2}$ is the set:

A) $(8, \infty)$ B) $[8, \infty)$ C) $[2, \infty)$ D) $(-\infty, \infty)$ E) None of these

Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. [B]

2. [2 marks] (Multiple choice question). The expression $\sin(\cos^{-1}(-3/5))$ is equal to:

A) $\sqrt{24}/5$ B) $-3/5$ C) $-4/5$ D) $-5/3$ E) None of these

Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. [E]

3. [2 marks] (Multiple choice question). Given that $\log_3\left(25 + \log_2 x^2\right) = 3$. Then x is:

A) 1 B) Either 1 or -1 C) $\sqrt{2}$ D) Either 2 or -2 E) None of these

Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. [D]

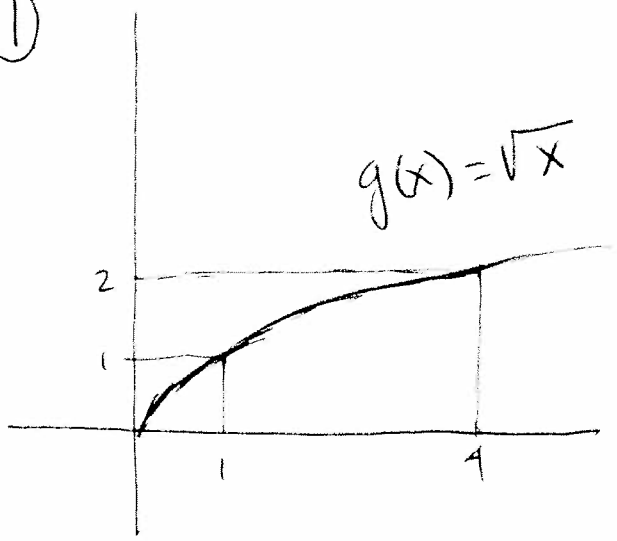
4. [2 marks] (Multiple choice question). If $f(x) = x^3 + \sqrt{x+7}$, then $f^{-1}(11)$ is equal to:

A) 2 B) -1 C) 1 D) 0 E) None of these

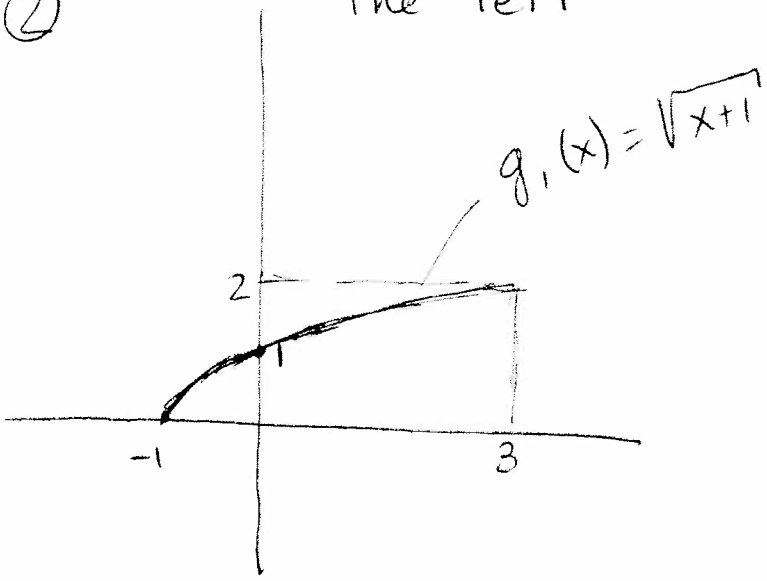
Write the (capital) letter of the answer in this box. **Only** the answer in the box will be marked. [A]

5. [5 marks] Starting with the graph of $g(x) = \sqrt{x}$, apply the appropriate transformations to sketch the graph of $f(x) = 2\left(\sqrt{x+1} - 1\right)$. Show clearly all your steps.

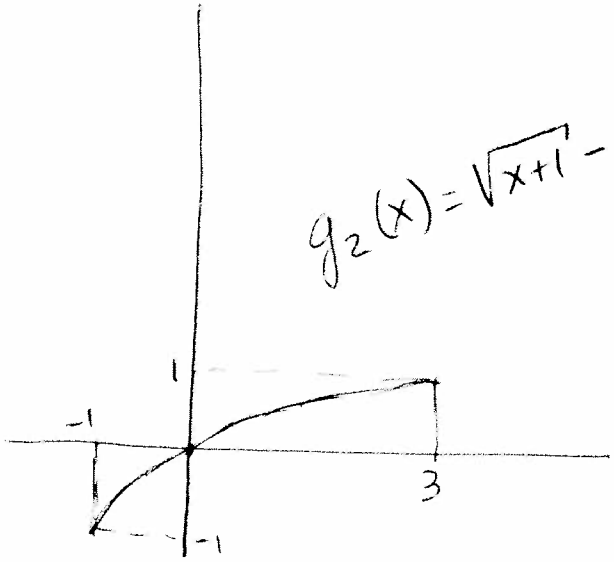
(1)



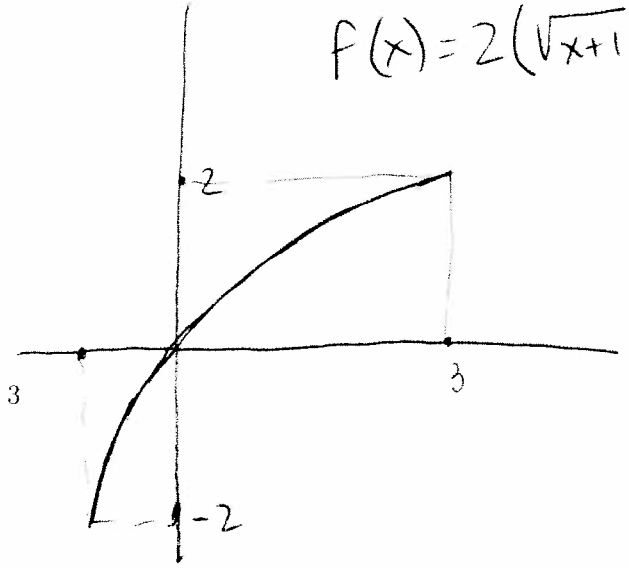
(2) move one unit to the left



(3) move one unit down



(4) stretch by a factor of two (vertically)



6. [7 marks] Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation

$$\csc^2 x - 1 = 4\cos^2 x$$

$$\frac{1}{\sin^2 x} - 1 = 4\cos^2 x$$

$$\frac{1-\sin^2 x}{\sin^2 x} - 4\cos^2 x = 0$$

$$\frac{\cos^2 x}{\sin^2 x} - 4\cos^2 x = 0$$

$$\cos^2 x \left(\frac{1}{\sin^2 x} - 4\right) = 0 \qquad \text{so}$$

$$\cos^2 x = 0 \quad \text{or} \quad \frac{1}{\sin^2 x} = 4$$

$\cos x = 0$ at

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin^2 x = \frac{1}{4} \Rightarrow \sin x = \pm\frac{1}{2}$ so

$x = \frac{\pi}{6}, x = \frac{5\pi}{6}, x = \frac{7\pi}{6}, x = \frac{11\pi}{6}$

S: $\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

7. [7 marks] Consider the function

$$f(x) = 1 + \sqrt{1 - e^x}$$

a) Find a formula for the inverse function f^{-1}
b) What is the range of f^{-1}? Explain.

a) $y = 1 + \sqrt{1 - e^x}$

$y - 1 = \sqrt{1 - e^x}$

$(y-1)^2 = 1 - e^x$

$e^x = 1 - (y-1)^2$

$x = \ln\left(1 - (y-1)^2\right)$

$y = \ln\left(1 - (x-1)^2\right)$

$\boxed{f^{-1}(x) = \ln\left(1 - (x-1)^2\right) \text{ or } = \ln\left(-x^2 + 2x\right)}$

b) Range of f^{-1} is domain of f.

$1 - e^x \geq 0$ or $e^x \leq 1$

$\boxed{x \leq 0}$ so $\boxed{\text{Range of } f^{-1}: (-\infty, 0]}$

8. [8 marks] Let $f(x) = \arctan(x)$ and $g(x) = 4\ln(x+1) - \sqrt{3}$.

a) Evaluate $(f \circ g)(0)$

b) Evaluate $(g \circ f)(0)$

c) Find the domain of the composite function $(f \circ g)(x)$

a) $(f \circ g)(0) = f(g(0))$

$= \arctan(-\sqrt{3})$

$= \boxed{-\frac{\pi}{3}}$

b) $(g \circ f)(0) = g(f(0)) = g(0) = \boxed{-\sqrt{3}}$

c) As domain of f is $\mathbb{R}$, the expression is defined if g is defined.

$\boxed{\text{Domain } \{x > -1\}}$

9. [8 marks] Consider the function

$$f(x) = \frac{1 - \cos^2 x}{|\sin x|}$$

a) Evaluate the limit, if it exists $\lim_{x \to 0} f(x)$

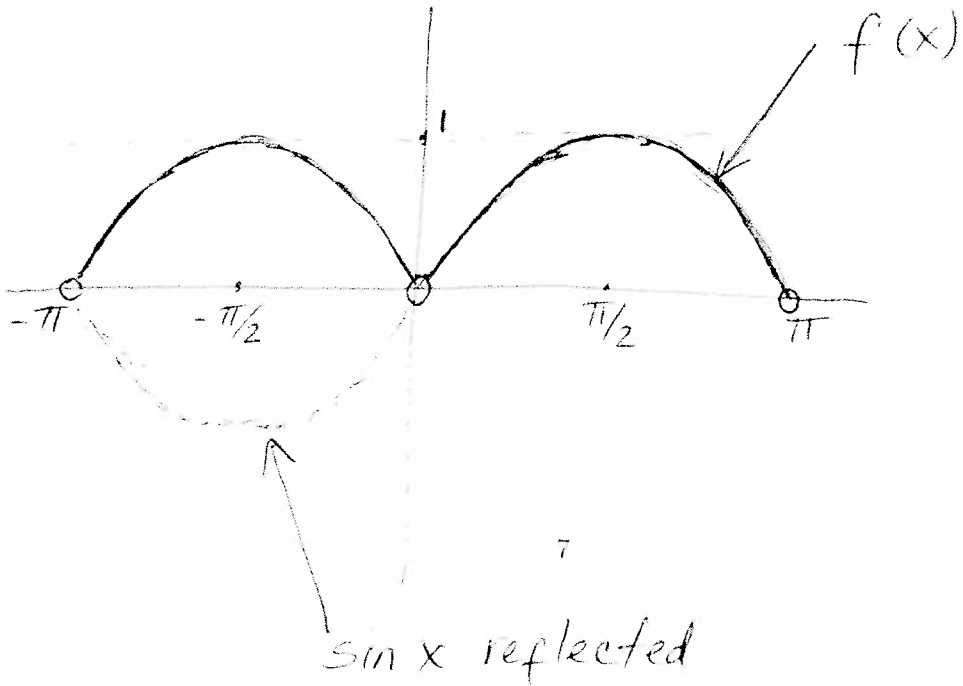
b) Sketch the graph of $f(x)$ in the interval $[-\pi, \pi]$

a) $f(x) = \dfrac{1 - \cos^2 x}{|\sin x|} = \dfrac{\sin^2 x}{|\sin x|} = \dfrac{|\sin x|^2}{|\sin x|}$

$= |\sin x|$ if $\sin x \neq 0$

So $\lim_{x \to 0} f(x) = \lim_{x \to 0} |\sin x| = |\sin 0| = \boxed{0}$

b)



10. [7 marks] Let the function

$$f(x) = \frac{\sqrt{x^2+9}-5}{x^2+5x+4} \cdot \cot x$$

a) Is $x = 0$ a vertical asymptote of f? Show your work.

b) Is $x = -4$ a vertical asymptote of f? Show your work.

a) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x^2+9}-5}{x^2+5x+4} \cdot \lim_{x\to 0^+} \frac{\cos x}{\sin x}$

$= \frac{-2}{4} \lim_{x\to 0^+} \frac{1}{\sin x} = -\infty$

So $\boxed{x=0 \text{ is a V.A}}$

b) $\lim_{x\to -4} f(x) = \lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{(x+4)(x+1)} \cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5} \cot x$

$= \lim_{x\to -4} \frac{x^2-16}{(x+4)(x+1)} \cdot \frac{1}{\sqrt{x^2+9}+5} \cot x$

$= \lim_{x\to -4} \frac{(x+4)(x-4)}{(x+4)(x+1)} \cdot \frac{1}{\sqrt{x^2+9}+5} \cot x$

$= \lim_{x\to -4} \frac{x-4}{x+1} \cdot \frac{1}{\sqrt{x^2+9}+5} \cdot \cot x$

$= \frac{-8}{-3} \cdot \frac{1}{10} \cdot \cot(-4)$

$= \frac{4}{15} \cot(-4)$

So $\boxed{x=-4 \text{ is NOT a V.A.}}$