

**University of British Columbia
Faculty of Applied Science
Department of Electrical and Computer Engineering**

**EECE 261/MATH 264 — Engineering Electromagnetics — Quiz #1
Monday 20 January 2014**

Duration: 60 minutes

Materials admitted: Writing supplies, approved calculator. Nothing else. (A sheet of formulas is provided with the test.)

This exam has **6 pages**, including this cover. Check that you have them all. Show all your work and solutions on these pages. You may write on both sides of every page. The last page is blank, to provide extra space.

Write your UBC ID on every page. Do this before the exam ends. Writing anything after the end of the exam period is not allowed.

Turn off and put away all cell phones, pagers, alarms, etc. before the exam begins. Any such device that disrupts the exam will be confiscated.

Rules Governing Formal Examinations

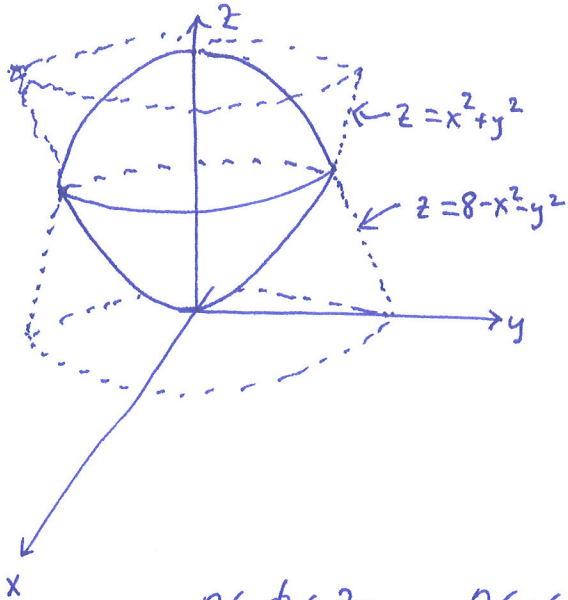
1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Question	Marks	Score
Prob 1	10	
Prob 2	10	
Prob 3	10	
Prob 4	10	
TOTAL	40	

Question 1 [10 marks]: Evaluate

$$\iiint_V \sqrt{x^2 + y^2} \, dx \, dy \, dz,$$

where V is the region bounded by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$. *Hint:* Use cylindrical coordinates.



Given paraboloids intersect in a ring:

$$x^2 + y^2 = z = 8 - x^2 - y^2$$

$$\Leftrightarrow 2(x^2 + y^2) = 8$$

$$\Leftrightarrow x^2 + y^2 = 4 = 2^2$$

Cyl coord region:

$$0 \leq \phi \leq 2\pi, \quad 0 \leq \rho \leq 2, \quad \rho^2 \leq z \leq 8 - \rho^2$$

EVAL:
$$\iiint_V \sqrt{x^2 + y^2} \, dx \, dy \, dz = \int_{\rho=0}^2 \int_{\phi=0}^{2\pi} \int_{z=\rho^2}^{8-\rho^2} [\rho] \, dz \, d\phi \, \rho \, d\rho$$

$$= \int_{\rho=0}^2 \int_{\phi=0}^{2\pi} [z]_{z=\rho^2}^{8-\rho^2} \rho^2 \, d\phi \, d\rho$$

$$= 2\pi \int_{\rho=0}^2 (8 - 2\rho^2) \rho^2 \, d\rho$$

$$= 2\pi \left[\frac{8}{3} \rho^3 - \frac{2}{5} \rho^5 \right] = 2\pi \left[\frac{64}{3} - \frac{64}{5} \right]$$

$$= \boxed{\frac{256\pi}{15}} \text{ ANS}$$

$$\left(\frac{256\pi}{15} \approx 53.6165 \right)$$

Question 1 [10 marks]: Evaluate

$$\iiint_V \sqrt{x^2 + y^2} \, dx \, dy \, dz,$$

where V is the region bounded by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$. *Hint:* Use cylindrical coordinates.

ALTERNATIVE APPROACH

Given paraboloids intersect in a ring:

$$x^2 + y^2 = z = 8 - x^2 - y^2$$

$$\Leftrightarrow 2(x^2 + y^2) = 8$$

$$\Leftrightarrow x^2 + y^2 = 4 \quad \text{so } z = 4.$$

(Same sketch)

Solid region V has reflection symmetry across horizontal plane $z = 4$.
So does integrand, $\sqrt{x^2 + y^2}$.
Therefore

$$\begin{aligned} \iiint_V \sqrt{x^2 + y^2} \, dV &= 2 \iiint_{V_{\text{BOT}}} \sqrt{x^2 + y^2} \, dV \\ &= 2 \int_{z=0}^4 \int_{\phi=0}^{2\pi} \int_{\rho=0}^{\sqrt{z}} \rho \, (\rho \, d\rho \, d\phi \, dz) \\ &= 2 \int_{z=0}^4 \int_{\phi=0}^{2\pi} \frac{1}{3} z^{3/2} \, d\phi \, dz \\ &= 2 \cdot \frac{1}{3} \cdot 2\pi \int_{z=0}^4 z^{3/2} \, dz \\ &= \frac{4\pi}{3} \cdot \frac{2}{5} z^{5/2} \Big|_{z=0}^4 = \frac{256\pi}{15}. \end{aligned}$$

Question 2 [10 marks]: Empty space (where $\epsilon = \epsilon_0$) contains two point charges. Charge Q_1 is located at $P_1(-5, -4, 8)$, and charge $Q_2 = 4Q_1$ is located at $P_2(-8, 10, -2)$.

- (a) Find \mathbf{E} , the electric field, at the point $P_3(3, 0, 0)$.
- (b) Find all points on the y -axis, if any, at which $E_z = 0$.

Strategy: To earn full marks, express your answer as a multiple of the symbolic factor $Q_1/(4\pi\epsilon_0)$. Alternatively, accept a 1-mark penalty for working with the specific value $Q_1 = 25 \text{ nC}$.

At general point (x, y, z) , respective point charges #1 and 2 give fields

$$\vec{E}_1(x, y, z) = \frac{Q_1}{4\pi\epsilon_0} \frac{\langle x-(-5), y-(-4), z-8 \rangle}{|\langle x-(-5), y-(-4), z-8 \rangle|^3}$$

$$\vec{E}_2(x, y, z) = \frac{Q_2}{4\pi\epsilon_0} \frac{\langle x-(-8), y-10, z-(-2) \rangle}{|\langle x-(-8), y-10, z-(-2) \rangle|^3}$$

NOTE: $Q_2 = 4Q_1$

(a) Total field at given pt P_3 is

$$\begin{aligned} \vec{E}_{\text{TOT}}(3, 0, 0) &= \frac{Q_1}{4\pi\epsilon_0} \left\{ \frac{\langle 8, 4, -8 \rangle}{|\langle 8, 4, -8 \rangle|^3} + 4 \frac{\langle 11, -10, 2 \rangle}{|\langle 11, -10, 2 \rangle|^3} \right\} \\ &= \frac{Q_1}{4\pi\epsilon_0} \left\{ \frac{\langle 8, 4, -8 \rangle}{12^3} + 4 \frac{\langle 11, -10, 2 \rangle}{15^3} \right\} = \frac{Q_1}{4\pi\epsilon_0} \langle 0.01767, -0.00954, -0.00226 \rangle \end{aligned}$$

$|\langle 8, 4, -8 \rangle|^2 = 64 + 16 + 64 = 144$
 $|\langle 11, -10, 2 \rangle|^2 = 121 + 100 + 4 = 225$

(b) On y -axis, both $x=0$ and $z=0$. Plug these in above, and then solve

$$0 = \vec{E}_{\text{TOT}}(0, y, 0) \cdot \hat{a}_z = \frac{Q_1}{4\pi\epsilon_0} \left\{ \frac{-8}{|\langle -5, y+4, -8 \rangle|^3} + 4 \frac{2}{|\langle -8, y-10, 2 \rangle|^3} \right\}$$

$$\Leftrightarrow |\langle -5, y+4, -8 \rangle|^3 = |\langle -8, y-10, 2 \rangle|^3$$

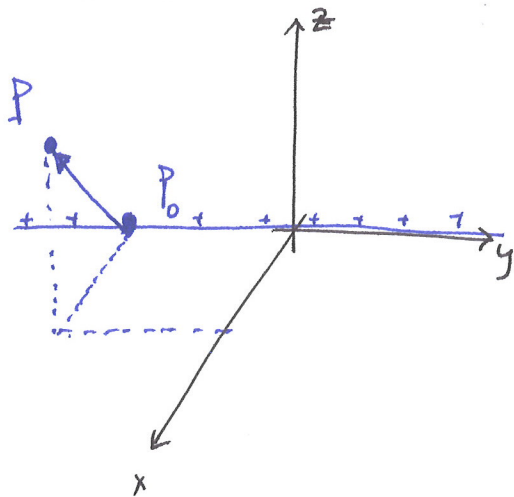
$$\Leftrightarrow (-5)^2 + (y+4)^2 + (-8)^2 = (-8)^2 + (y-10)^2 + (2)^2$$

$$\Leftrightarrow 25 + (y^2 + 8y + 16) = (y^2 - 20y + 100) + 4$$

$$\Leftrightarrow 28y = -63. \quad \text{ONE POINT WORKS: } (0, -\frac{63}{28}, 0).$$

Raise both sides to power $2/3$

Question 3 [10 marks]: Charge is distributed along the y -axis with a uniform linear density of 5 C/m . At the point $P(3, -6, 4)$ is a sensitive piece of equipment that must be protected from the electric field. An engineer is hired to arrange $\mathbf{E} = \mathbf{0}$ at point P by specifying the location for a given point charge $Q = 32 \text{ C}$. Find the correct location for the charge Q .



At $P(3, -6, 4)$, direction of \vec{E} is radially away from y -axis.

Let $P_0 = (0, -6, 0)$, so at P ,

$$\frac{\vec{E}}{|\vec{E}|} = \frac{\vec{P_0P}}{|\vec{P_0P}|} = \frac{\langle 3, 0, 4 \rangle}{|\langle 3, 0, 4 \rangle|} = \left\langle \frac{3}{5}, 0, \frac{4}{5} \right\rangle.$$

Distance from P_0 to charged line is $|\vec{P_0P}| = 5$, so field strength is

$$|\vec{E}| = \frac{\rho_L}{2\pi\epsilon_0} \cdot \left(\frac{1}{5\text{m}}\right) = \frac{5 \text{ C/m}}{2\pi\epsilon_0} \left(\frac{1}{5\text{m}}\right) = \frac{1}{2\pi\epsilon_0} \frac{\text{C}}{\text{m}^2}$$

This gives the field at P :

$$\vec{E}(P) = \left(\frac{\vec{E}}{|\vec{E}|}\right) |\vec{E}| = \frac{1}{2\pi\epsilon_0} \left\langle \frac{3}{5}, 0, \frac{4}{5} \right\rangle.$$

Charge Q must cancel this. So distance R from Q to P

must satisfy $\frac{1}{2\pi\epsilon_0} = \frac{Q}{4\pi\epsilon_0} \frac{1}{R^2} \Leftrightarrow R^2 = \frac{Q}{2} = \frac{32}{2} = 16$, giving $R=4$.

Position for Q must be 4m beyond P along the line that extends segment P_0P . That exact location is

$$\begin{aligned} (x, y, z) &= (3, -6, 4) + 4 \left\langle \frac{3}{5}, 0, \frac{4}{5} \right\rangle = \left\langle 3 + \frac{12}{5}, -6, 4 + \frac{16}{5} \right\rangle \\ &= \langle 5.4, -6.0, 7.2 \rangle. \end{aligned}$$

Question 4 [10 marks]: Four charges are arranged in the plane $z = 0$ as shown in the sketch. The electrostatic force \vec{F} exerted on the charge at point P makes an angle of 30° with \hat{a}_x . Use the following data to find Q , the charge at the origin.

- The charge at point $A(2, 0, 0)$ is $Q_A = -50$ C.
- The charge at point $B(0, 2, 0)$ is $Q_B = 100$ C.
- The charge at point $P(2, 2, 0)$ is $Q_P = 707$ nC.

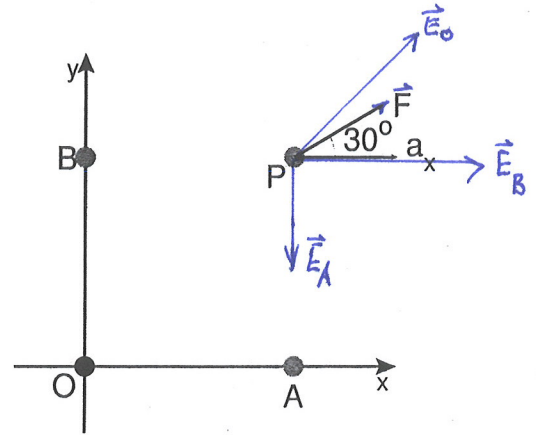
NOTE: Knowing $Q_P > 0$ helps set directions, but size of Q_P has no effect on result!

Electric field at P due to O, A, B :

$$\vec{E}_A = \frac{Q_A}{4\pi\epsilon_0} \frac{1}{2^2} \hat{a}_y \quad (Q_A = -50 \text{ C})$$

$$\vec{E}_B = \frac{Q_B}{4\pi\epsilon_0} \frac{1}{2^2} \hat{a}_x \quad (Q_B = +100 \text{ C})$$

$$\vec{E}_O = \frac{Q}{4\pi\epsilon_0} \frac{1}{(2\sqrt{2})^2} \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \right)$$



ADD:

$$\vec{E}_{TOT} = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{2^2} \left\{ Q_A \hat{a}_y + Q_B \hat{a}_x + \frac{Q}{2} \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}} \right) \right\}$$

At point P , force on Q_P is $\vec{F} = Q_P \vec{E}_{TOT}$.

Also, $\vec{F} = F_x \hat{a}_x + F_y \hat{a}_y$ with angle $\frac{F_y}{F_x} = \tan(30^\circ) = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$.

That is, $\frac{1}{\sqrt{3}} = \frac{F_y}{F_x} = \frac{Q_P E_y}{Q_P E_x} = \frac{E_y}{E_x} = \frac{Q_A + Q/2\sqrt{2}}{Q_B + Q/2\sqrt{2}}$.

Solve this for Q :

$$Q_B + \frac{Q}{2\sqrt{2}} = \sqrt{3} \left(Q_A + \frac{Q}{2\sqrt{2}} \right) \Leftrightarrow Q_B - \sqrt{3} Q_A = (\sqrt{3} - 1) \frac{Q}{2\sqrt{2}}$$

$$\Leftrightarrow Q = (2\sqrt{2}) \frac{Q_B - \sqrt{3} Q_A}{\sqrt{3} - 1} = (2\sqrt{2}) \frac{100 \text{ C} - \sqrt{3}(-50 \text{ C})}{\sqrt{3} - 1} \approx 721.000 \text{ C}$$