Logical connectives (operators)

Name Compound proposition	Truth value	Corresponding English expressions
Negation $\neg p$	true *if and only if* p is false	"not p" "it is not the case that p"
Conjunction $p \wedge q$	true *if and only if* both p and q are true	"p and q" "p but q"
Disjunction $p \vee q$	false *if and only if* both p and q are false	"p or q (or both)"
Exclusive or $p \oplus q$	true *if and only if* exactly one of p and q is true	"either p or q" "p or q but not both"
Biconditional $p \leftrightarrow q$	true *if and only if* p and q have the same truth value	"p if and only if q" "p is necessary and sufficient for q" "if p, then q, and conversely"
Implication $p \rightarrow q$	false *if and only if* p is true and q is false	"p implies q" "if p, then q" "p only if q" "p is sufficient for q" "q follows from p" "q if p" "q when(ever) p" "q is necessary for p" "q unless not p" "q or else not p"

Note: We will not be using *exclusive or* very frequently, since $p \oplus q \equiv \neg(p \leftrightarrow q)$.

Precedence of logical operators:

1) $\neg$; 2) $\wedge$; 3) $\vee$; 4) $\rightarrow$; 5) $\leftrightarrow$

More about the implication:

$$p \quad \rightarrow \quad q$$

(hypothesis *or* premise) (conclusion *or* consequence)

Propositions related to the implication $p \rightarrow q$:

- The **converse** of $p \rightarrow q$: $q \rightarrow p$
- The **contrapositive** of $p \rightarrow q$: $\neg q \rightarrow \neg p$
- The **inverse** of $p \rightarrow q$: $\neg p \rightarrow \neg q$

Equivalences:

$$p \rightarrow q \quad \equiv \quad \neg q \rightarrow \neg p$$
$$q \rightarrow p \quad \equiv \quad \neg p \rightarrow \neg q$$
$$p \rightarrow q \quad \not\equiv \quad q \rightarrow p$$