

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)	
Mathematics	206/4	All	
Examination	Date	Time	Pages
Final	April 2011	3 Hours	2
Instructors			Course Examiner
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Special Instructions

▷ Only approved calculators are allowed.

MARKS

- [4] 1. Simplify the expressions below. Do not use a calculator.

(a) $-8\sqrt{12} + \sqrt{3} + 3\sqrt{75}$ (b) $\frac{2}{3} \log_2 8 - \log_2 (2^4 - 8)$

- [4] 2. Rationalize the denominator:

(a) $\frac{\sqrt{2}}{\sqrt{3} - 3\sqrt{2}}$ (b) $\frac{1 + \sqrt{5}}{1 - \sqrt{5}}$

- [6] 3. Simplify the expressions:

(a) $(8x^3 - 2x^2 + 6x - 2) + 4(3x^4 - 2x^3 + x^2 + x)$ (b) $\frac{2x^2 + 5x - 3}{1 - 2x}$

- [8] 4. Factor the polynomials completely:

(a) $3x^2 + 12x - 15$ (b) $3 - 27x^2$

- [4] 5. Perform the arithmetic operations and simplify:

$$\frac{3x}{x-1} - \frac{x-4}{x^2-2x+1}$$

- [9] 6. Solve the equations:

(a) $\frac{x}{x^2-1} - \frac{x+3}{x^2-x} = \frac{-3}{x^2+x}$ (b) $\log_8 x + 6 = 1 - \log_8 (x+4)$

(c) $3^{x^2-7} = 27^{2x}$

- [8] 7. Solve the inequalities, express your answer using set notation or interval notation:

(a) $\frac{2x-3}{5} + 2 \leq \frac{x}{2}$ (b) $1 - |2 - 3x| < -4$

- [4] 8. Solve the system of equations:

$$\begin{aligned}x^2 + y^2 &= 36 \\x + y &= 8\end{aligned}$$

- [8] 9. (a) Which of the points $A(3, 2)$, $B(2, 5)$ is closer to the point $C(3, 6)$?
(b) Show that the equation $x^2 + y^2 - x + 2y + 1 = 0$ represents a circle. Find coordinates of the center and radius of the circle.

- [6] 10. Find the domain and range of the functions (do not graph):

$$(a) f(x) = \frac{x}{x^2 - 9} \quad (b) g(x) = -\sqrt{x + 3} \quad (c) h(x) = |x| + 4$$

- [5] 11. Sketch the graph of the function $f(x) = 2 - \log(x + 1)$, starting from the graph of the function $g(x) = \log x$ and using appropriate transformations.

- [8] 12. Let $f(x) = \frac{2x - 1}{x - 2}$ and $g(x) = \frac{x + 4}{2x - 5}$. Find:

$$(a) fg \quad (b) \frac{f}{g} \quad (c) f \circ g \quad (d) g \circ f$$

- [8] 13. (a) Find the inverse of the function $f(x) = \frac{-3x - 4}{x - 2}$.

(b) Find the vertical and horizontal asymptotes of both f and f^{-1} above.

- [5] 14. Candy has \$70,000 to invest and requires an overall rate of return of 9%. She can invest in a safe, government-insured certificate of deposit, but it only pays 8%. To obtain 9%, she agrees to invest some of her money in non-insured corporate bonds paying 12%. How much should be placed in each investment to achieve her goal?

- [5] 15. The perimeter of a rectangle is 60 feet. Find its length and width if the length is 8 feet longer than the width.

- [8] 16. A colony of bacteria that grows according to the law of uninhibited growth is modeled by the function

$$N(t) = 100e^{0.045t}$$

where N is measured in grams and t is measured in days.

- (a) Determine the initial amount of bacteria.
(b) How long will it take for population to reach 180 grams?
(c) What is the tripling time for the population?