Concordia University
Faculty of Engineering and Computer Science
Department of Mechanical and Industrial Engineering

ENGR 311- Final Examination

Total Marks – 60 December 12, 2005

Professors: Dr. Wen-Fang Xie & Dr. Ashok Kaushal

1. (5 marks) Find the Laplace Transform for each of the following functions:

a) $e^{-t} t \cos(2t)$ b) $F(t) = (t-3)\sin(2t)$

(5 marks) Find the Inverse Laplace Transforms of

c) $F(s) = e^{\frac{-\pi}{2}s}\left[\frac{s}{s^2+10s+29}\right]$ d) $F(s) = \frac{8}{s^3(s^2-s-2)}$

2. (a). (5 marks) Solve the following Integral Equation

$$\int_0^t y(x)y(t-x)dx = 6t^3$$

(b). (5 marks) Use the Laplace transform to solve the given system of differential equations.

$$z'' + y' = \cos t$$
$$y'' - z = \sin t$$

where z(0) = z '(0) =-1; y(0)=1 and y '(0)=0

3) (7 marks) Find the Fourier Series of the function defined by:

$$F(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi & 0 \le x \le \pi \end{cases}$$

(3 marks) If the above function changes to $F(x) = \begin{cases} -\frac{\pi}{2} & -\pi < x < 0 \\ \frac{\pi}{2} & 0 \le x \le \pi \end{cases}$

What happens to the Fourier Series?

4) (10 marks) Solve, using the method of separation of variables, the following wave equation.

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} - 4u$$

BCs: $u(0,t) = 0, \quad u(\pi,t) = 0, \quad t > 0$

ICs: $u(x,0) = 0$, and $\frac{\partial u(x,0)}{\partial t} = x(\pi - x)$, $0 < x < \pi$

5) (10 marks) Consider a thin rod of length L with an initial temperature $f(x) = x$ throughout and whose ends are held at temperature zero for all time. Solve the following heat equation which describes the variations of temperature in the rod.

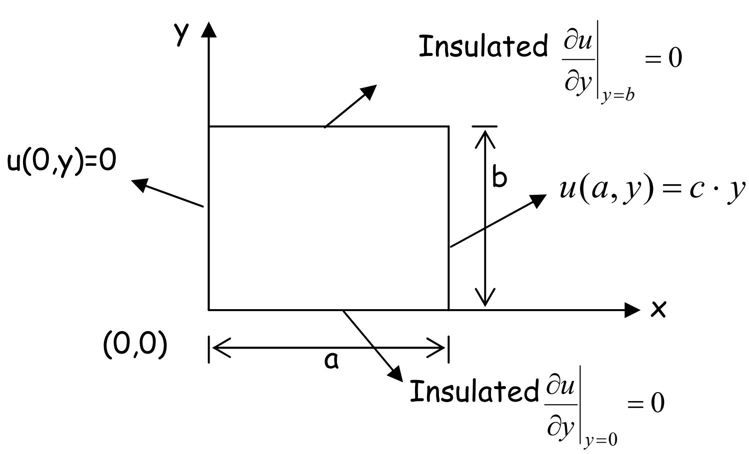
$$K\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}, \quad 0 < x < L, \quad t > 0$$

BCs: $u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0$

IC: $u(x,0) = x, \quad 0 < x < L$

6) (10 marks) Consider a rectangular plate of size $a \times b$. It is positioned along x-y axis as shown in the following. The edges of the plate parallel to the x-axis are insulated. The edge of the plate along y-axis at x=0 is maintained at zero temperature while the temperature at the other parallel edge is given by the function $u(a, y) = c \cdot y$. Solve for the steady-state temperature $u(x, y)$ on the plate using the Laplace equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, 0 < y < b$$



Happy Holidays!!!