MARKS

[6] **1.** (a) Find the following limits

(i) $\lim_{x\to 2} \frac{3x^2+2x-1}{x^2+3x+2} = \frac{3(2)^2+2(2)-1}{2^2+3(2)+2} = \frac{12+4-1}{4+6+2} = \frac{15}{12} = \frac{5}{4}$

(ii) $\lim_{x\to 2} \frac{2x^2-3x-2}{x^2+x-6} = \frac{0}{0}$

(b) Where is the function $n(x) = \frac{x-2}{(x-3)(x+1)}$ continuous? $x \in \mathbb{R} \setminus \{-1, 3\}$

ii) $\lim_{x\to 2} \frac{(2x+1)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \lim_{x\to 2} \frac{2x+1}{x+3} = \frac{2(2)+1}{2+3} = \frac{5}{5} = 1$

[5] **2.** Find the derivative $f'(x)$ of the functions $f(x)$: (Do not simplify)

(a) f(x)$= 3x^4 - 4x^3 + x - 2$ $f'(x) = 12x^3 - 12x^2 + 1$

(b) f(x)$= \frac{x^{-7}}{7} + \sqrt[3]{x}$ $= \frac{1}{7}x^{-7} + x^{1/3}$ $f'(x) = -x^{-8} + \frac{1}{3}x^{-2/3}$

[9] **3.** Find $\frac{dy}{dx}$ (do not simplify):

(a) $y = \frac{e^{2x}}{x^2-4}$ $u = e^{2x}$ $v = x^2-4$ $u' = 2e^{2x}$ $v' = 2x$ $f'(x) = \frac{2e^{2x}(x^2-4) - e^{2x}(2x)}{(x^2-4)^2}$

(b) $y = \cancel{ln}(3x^2+5)$ $f'(x) = \frac{1}{3x^2+5} \cdot 6x$

(c) $y = (2x^2+1)^3(4x+6)^2$ $u = (2x^2+1)^3$ $v = (4x+6)^2$ $u' = 3(2x^2+1)^2 \cdot 4x$ $v' = 2(4x+6) \cdot 4$

$f'(x) = 3(2x^2+1)^2 4x(4x+6)^2 + (2x^2+1)^3 2(4x+6)4$

(d) $y = (4 + x^2 \ln x)^3$ $u = x^2$ $v = \ln x$ $u' = 2x$ $v' = \frac{1}{x}$

$f'(x) = 3(4 + x^2\ln x)^2 \cdot \left(2x\ln x + x^2\left(\frac{1}{x}\right)\right)$

[7] **4.** Let $f(x) = 3x^4 - 6x^2 - 7$ a) $f'(x) = 12x^3 - 12x$ $f'(2) = 12(2)^3 - 12(2) = 72$

(a) Find the slope of the tangent line to the curve when $x = 2$

(b) Find the equation of the tangent line to the curve when $x = 2$

$f(2) = 3(2)^4 - 6(2)^2 - 7 = 17$ $(2, 17)$

$y = mx + b$

$17 = 72(2) + b$

$-127 = b$

$y = 72x - 127$

[13] **5.** Let $f(x) = (x-2)(x^2-4x-8) = x^3-4x^2-8x-2x^2+8x+16$

Find $= x^3-6x^2+16$

(a) the critical and inflection points of $f(x)$

(b) the intervals where $f(x)$ is increasing and where it is decreasing

(c) the intervals on which $f(x)$ is concave up and on which it is concave down

(d) use the above to sketch the graph

a) $f' = 3x^2-12x = 0$

$3x(x-4)=0$

$x=0 \quad x=4$

$f'' = 6x-12 = 0$

$6x = 12$

$x = 2$

$f(0) = (0)^3-6(0)^2+16 = 16$

$f(4) = (4)^3-6(4)^2+16 = -16$

$f(2) = 2^3-6(2)^2+16 = 0$

I. POINTS: $(2,0)$

C. POINTS: $(0,16)$ & $(4,-16)$

	-1	0	1	4	5
f'	$+$		$-$		$+$
f	↗		↘		↗

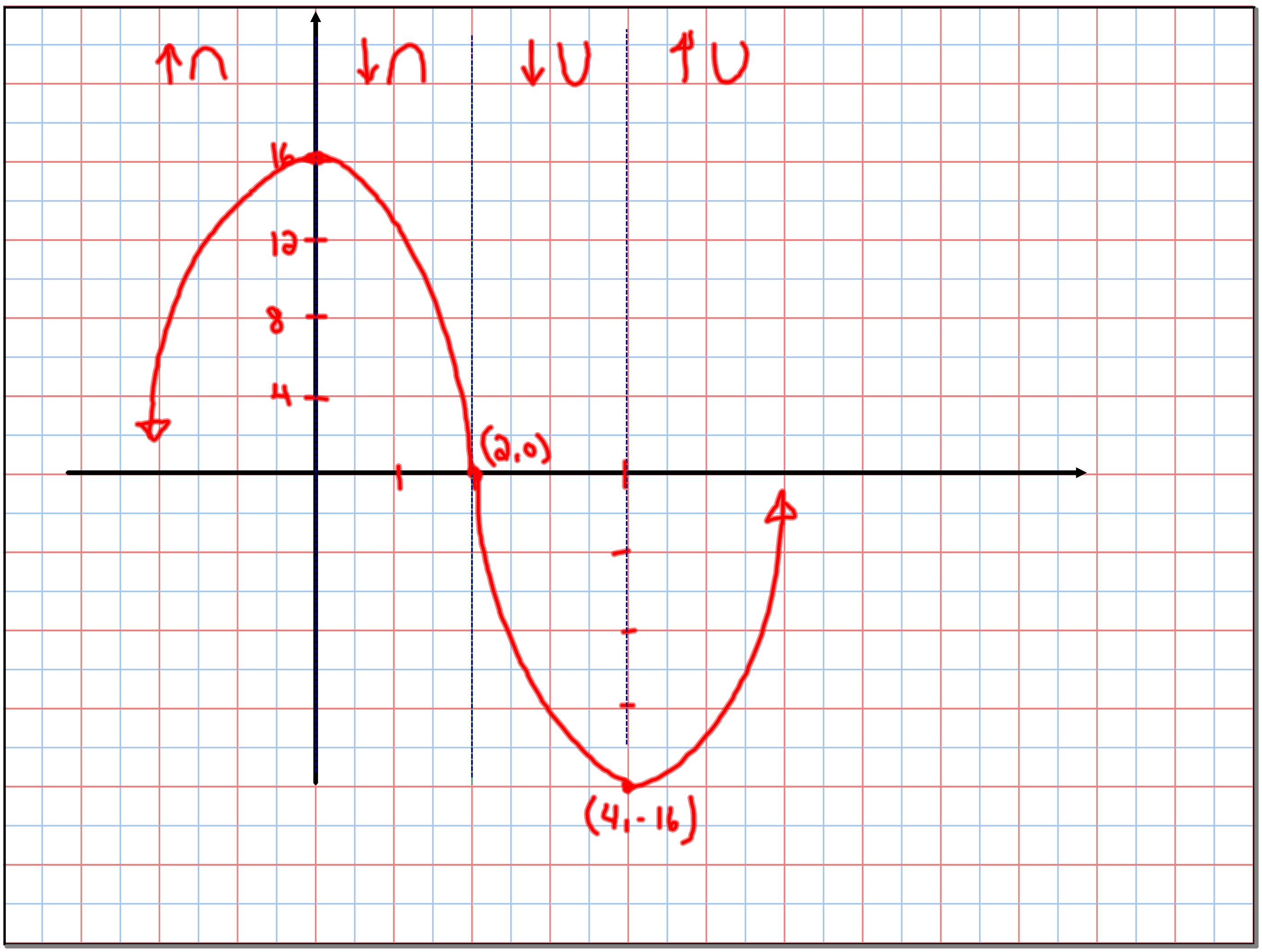
	0	2	5
f''	$-$		$+$
f	∩		∪

INT OF ↑ : $x \in (-\infty, 0) \cup (4, \infty)$

INT OF ↓ : $x \in (0,4)$

CONCAVE UP ON: $x \in (2, \infty)$

CONCAVE DOWN: $x \in (-\infty, 2)$



↑∩
↓∩
↓∪
↑∪
16
12
8
4
1
(2,0)
(4,-16)

[8] **6.** A student center sells 1600 cups of coffee per day at the price of \$2.40 per cup. A market survey shows that for every \$0.05 reduction in price per cup, 50 more cups of coffee will be sold.

How much should the student center charge for a cup of coffee in order to maximize revenue?

$R = (1600 + 50x)(2.40 - 0.05x) = 3840 + 40x - 2.5x^2$

$R' = 0: \quad 40 - 5x = 0$

$x = 8$

$P = 2.40 - 0.05x = 2.40 - 0.05(8) = \$2.00$

[7] **7.** Find the absolute extrema of the function $f(x) = x^3 - 6x^2 + 9x - 6$ on the interval $[-1, 5]$.

$f' = 0$: $3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

$f(-1) = (-1)^3 - 6(-1)^2 + 9(-1) - 6 = -22$

$f(1) = (1)^3 - 6(1)^2 + 9(1) - 6 = -2$

$f(3) = (3)^3 - 6(3)^2 + 9(3) - 6 = -6$

$f(5) = (5)^3 - 6(5)^2 + 9(5) - 6 = 14$

max of 14 at $x = 5$, min of -22 at $x = -1$

[3] 8. If interest is compounded continuously and the interest rate is 6.4%, how long will it take for money invested to double?

$A = Pe^{rt}$

$2\cancel{P} = \cancel{P}e^{0.064t}$

$\ln 2 = 0.064t \rightarrow t = 10.83$ yrs

[10] 9. Find the equation(s) of the tangent line(s) to the graph of $y^2 - xy - 6 = 0$ at the point(s) with $x = 1$.

find point: plug-in $x=1$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$ OR $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = 3 \quad y = -2$

2 points! $(1,-2)$ & $(1,3)$

find Slope: $y^2 - \underline{xy} - 6 = 0$ $\quad u = -x \quad v = y$; $\quad u' = -1 \quad v = y'$

$2yy' - y - xy' = 0$

plug-in $(1,-2) \rightarrow 2(-2)y' - (-2) - (1)y' = 0 \quad -4y' + 2 - y' = 0 \quad y' = \frac{2}{5}$

$(1,3) \rightarrow 2(3)y' - 3 - (1)y' = 0 \quad 6y' - 3 - y' = 0 \quad y' = \frac{3}{5}$

2 lines: ① $(1,-2)$ $m = 2/5$

② $(1,3)$ $m = 3/5$

$y = mx + b$

$-2 = \frac{2}{5}(1) + b$

$-\frac{12}{5} = b$

$y = \frac{2}{5}x - \frac{12}{5}$

$y = mx + b$

$3 = \frac{3}{5}(1) + b$

$\frac{12}{5} = b$

$y = \frac{3}{5}x + \frac{12}{5}$

[10] **10.** Compute these antiderivatives:

(a) $\int (3x^5 - 2x^3 - 7)\,dx = \frac{3x^6}{6} - \frac{2x^4}{4} - 7x + C = \frac{x^6}{2} - \frac{x^4}{2} - 7x + C$

(b) $\int \frac{e^{-7x}}{2+e^{-7x}}\,dx$

$u = 2+e^{-7x}$

$dx = \frac{du}{-7e^{-7x}}$

$\int \frac{\cancel{e^{-7x}}}{u}\cdot\frac{du}{-7\cancel{e^{-7x}}} = \frac{-1}{7}\int \frac{1}{u}\,du = -\frac{1}{7}\ln|u| + C = -\frac{1}{7}\ln|2+e^{-7x}| + C$

(c) $\int \frac{x^2}{\sqrt{x-3}}\,dx$

[10] **11.** Evaluate the integrals:

(a) $\int_0^1 (x^3 - 4)\,dx = \frac{x^4}{4} - 4x\Big|_0^1 = \left(\frac{1}{4} - 4\right) - (0 - 0) = -3.75$

(b) $\int_2^7 \frac{1}{x-5}\,dx$

$u = x-5$ $x=2$ $u=-3$

$dx = du$ $x=7$ $u=2$

$\int_{-3}^{2} \frac{1}{u}\,du = \ln|u|\Big|_{-3}^{2} = \ln 2 - \ln 3 = \ln\frac{2}{3} = -0.40547$

(c) $\int_4^6 \sqrt{x-3}\,dx$

$u = x-3$ $x=4$ $u=1$

$dx = du$ $x=6$ $u=3$

$= \int_1^3 u^{1/2}\,du = \frac{2u^{3/2}}{3}\Big|_1^3 = \frac{2(3)^{3/2}}{3} - \frac{2}{3} = 2.7974$

$\int \frac{x^2}{\sqrt{x-3}}\,dx$

$u = x-3 \rightarrow x = u+3$

$dx = du$

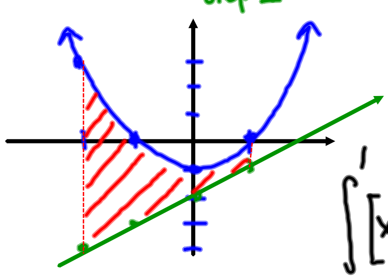
$\int \frac{x^2}{u^{1/2}}\,du = \int \frac{(u+3)^2}{u^{1/2}}\,du = \int \frac{(u+3)(u+3)}{u^{1/2}}\,du$

$= \int\left(\frac{u^2}{u^{1/2}} + \frac{6u}{u^{1/2}} + \frac{9}{u^{1/2}}\right)du = \int\left(u^{3/2} + 6u^{1/2} + 9u^{-1/2}\right)du = \frac{2u^{5/2}}{5} + \frac{12u^{3/2}}{3} + 18u^{1/2} + C$

$= \frac{2}{5}(x-3)^{5/2} + 4(x-3)^{3/2} + 18(x-3)^{1/2} + C$

[10] **12.** Find the area bounded by the graphs of $f(x) = x^2 - 1$ and $y = x - 2$ over the interval $-2 \le x \le 1$.

Step 1



x	y
-2	3
-1	0
0	-1
1	0

x	y
-2	-4
-1	-3
0	-2
1	-1

$\int_{-2}^{1}\left[x^2 - 1 - (x-2)\right]dx = \int_{-2}^{1}(x^2 - 1 - x + 2)\,dx = \int_{-2}^{1}(x^2 - x + 1)\,dx$

$= \frac{x^3}{3} - \frac{x^2}{2} + x\Big|_{-2}^{1} = \left(\frac{1}{3} - \frac{1}{2} + 1\right) - \left(\frac{-8}{3} - 2 - 2\right)$

$= 7.5$