



FINAL  
EXAMINATION  
APRIL 2010

DURATION: **3 HOURS**

Number of Students: **90**

Department Name: Mathematics & Statistics

Course Number: MATH 1401 A

Course Instructors: Luc Bourbonnais

Non-Programmable, Non-Graphing Calculators Allowed

- Students **MUST** count the number of pages in this examination question paper before beginning to write, and report any discrepancy to a proctor.
- This question paper has six (6) double-sided sheets, beginning with this header page and ending with a blank page on page 12.
- In addition, 2 blank sheets of paper are supplied for rough notes and calculations (you may **gently** detach these last 2 sheets from the rest of the document).

This examination question paper **may not** be taken from the examination room.

This examination question paper **may not** be released to the library.

In addition to this question paper, students require: A positive attitude!

Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

- There are seven (7) multiple choice questions worth a total of **20** marks.
- There are nine (9) written questions worth a total of **80** marks.
- The total for this examination is therefore **100**

You have **3 hours** to complete the exam. **GOOD LUCK!**

---

**DO NOT OPEN THE EXAM  
UNTIL YOU HAVE BEEN  
TOLD TO DO SO**

## Part A – Multiple Choice Questions

**INSTRUCTIONS:** Correct answers are worth 2 marks. Incorrect answers are worth **0 marks**. Please read each question carefully and **clearly enter your answers in the table provided below**. Only the answers written in this table will be marked.

Question	1	2	3	4	5	6	7
Answer							

1. [3 marks] Let:  $q^s = 4p - 12$  be the rule of a supply function. The rule of the inverse function is:

- a.  $p^s = 4q + 12$
- b.  $p^s = 0.25q + 12$
- c.  $p^s = 0.25q + 3$
- d.  $p^s = 12q - 4$

2. [3] The 2 curves  $y = 3x + 8$  and  $y = x^2 + 4x - 4$  intersect at the following points:

- a.  $(-4, -4)$  and  $(3, 17)$
- b.  $(4, 20)$  and  $(-3, -1)$
- c.  $(-4, -36)$  and  $(3, 17)$
- d. The 2 curves do not meet; there are no intersection points.

3. [2] The expressions  $\lim_{x \rightarrow 0} \left(1 + \frac{1}{x}\right)^x$  and  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$  are **respectively** equal to:

- a. 0 and 1
- b. 0 and  $\infty$
- c.  $e$  and 1
- d. 1 and  $e$

4. [3] The linearization of  $y = f(x) = 4x^2 - 6x - 2$  at  $x=3$  is :

- a.  $y = f(3) = 16$
- b.  $y = L(x) = f'(x) = 8x - 6$
- c.  $y = L(x) = 18x - 38$
- d. Because  $f$  represents a parabola, linearization is not possible.

5. [3] The function  $f$  such that  $y = f(x) = 2x^3 - 15x^2 - 144x + 10$  has:

- a. A local minimum at  $(-3, 253)$  and a local maximum at  $(8, -1078)$
- b. A local maximum at  $(-3, 253)$  and a local minimum at  $(8, -1078)$
- c. A local minimum at  $(3, -503)$  and a local maximum at  $(-8, -822)$
- d. A local maximum at  $(3, -503)$  and a local minimum at  $(-8, -822)$

6. [3] The solution to the equation  $16^{x-5} = 2^{x+16}$  is:

- a. 12
- b.  $\frac{21}{2}$
- c. -12
- d. There is no solution

7. [3] The sum of  $2 + \frac{5}{3} + \frac{25}{18} + \frac{125}{108} + \dots$  is:

- a. 12
- b.  $\infty$
- c. 3
- d.  $e$

## Part B – Written questions

**INSTRUCTIONS:** Each question outlines its own marking scheme. Please read each question carefully. **For full marks, be sure to show all of your steps and properly justify any procedures that you use.**

1. [12 marks] Evaluate the derivative of the following functions whose rule is given below.

a. [3]  $y = f(x) = \frac{3x}{x^2 + 16}$  ... Simplify the numerator as much as possible.

b. [3]  $y = f(x) = \sqrt{\frac{3x^3}{x^2 - 4}}$  ... Simplify the numerator as much as possible.

c. [3]  $y = f(x) = (2x^2 - 3x)^3(8 - 2x)^5$  ... Simplify as much as possible.

d. [3]  $y = f(x) = e^{x^2} \cdot \ln(\sqrt{x})$  ... No need to simplify.

2. [10] The following describes the short-run Total Cost curve of a firm:  $TC = f(q) = 2q^2 + 10q + 1800$  (in dollars, with  $q > 0$ ).
- [1] What is the fixed cost?
  - [2] Determine a rule for the Marginal Cost ( $MC$ ) and the Average Cost ( $AC$ ) functions.
  - [3] Determine the value of  $q$  for which  $AC$  is minimized. Confirm that it is indeed a minimum.
  - [2] Determine the value of the minimum  $AC$  and show that it is equal to  $MC$  at that point.
  - [2] Show that when  $MC > AC$ ,  $AC$  is an increasing function. Do not use specific numbers; show algebraically.

3. [10] A monopoly operates under the following conditions:

- The firm's short-run Total Cost function is:  $TC = q^2 + 8q + 60$
- The market inverse demand function is:  $p^D = p = 48 - 3q$  (in dollars, with  $q > 0$ )
  - a. [2] Determine the Total Revenue ( $TR$ ) and the Marginal Revenue ( $MR$ ) functions.
  - b. [3] Find the level of production that maximizes  $TR$ . Also, determine the maximum  $TR$ .
  - c. [2] Find a rule for  $E^D$ , the elasticity of demand. Express it as a function of  $q$  only.
  - d. [2] Find the optimum level of production  $q$  that maximizes  $\Pi$ .
  - e. [1] Determine the maximum profit.

NB – in d. and e. you do not need to confirm that  $\Pi$  is a maximum.

4. [8] Let:  $y = f(x) = x^3 + 10x + 100 \dots (x > 0)$ .
- [3] Develop a general expression for  $E^y$  (in terms of “ $x$ ” only).
  - [2] Determine  $E^y$  when  $x=2$ . Is  $y$  elastic then?
  - [3] Determine the value of  $x$  for which  $E^y=1$ .

5. [8] You are required to make 2 future payments: \$7,500 in 5 years from now and \$22,500 in 12 years from now. Determine the amount of money you need to invest now (rounded off to the nearest dollar) for the following cases:
- [4] You can invest in a fund that pays 6% annually, compounded every year.
  - [4] You are offered a term deposit that pays 4% per annum, compounded every month.

6. [7] You borrow \$20,000 to buy a car. The terms of the deal are: a 5–year loan paid in monthly instalments, at a 6% annual rate of interest.
- [4] Determine, to the nearest dollar, the amount of your monthly instalments.
  - [3] How much total interest will you pay?
7. [8] Compute the future compounded value (*FCV*) of the following investments after 5 years:
- [3] You invest \$2,000 in a fund which pays a 1% monthly dividend. The interest is not re-invested (silly you!).
  - [3] You place \$1,000 in a term account which pays 4% interest per annum, compounded 4 times a year.
  - [2] What is your Average Rate of Return (EAR) after one year, for case “b.”?

8. [5] On your fifth birthday, you are given a dollar as an investment gift. You leave the dollar in the capable hands of Eric Sprott, one of Canada's leading investors. On your 65<sup>th</sup> birthday, you go to the bank to cash in your investment. The bank hands you \$20,000. What was the average annual rate of return from Sprott's mutual fund if the fund's growth is compounded continuously?

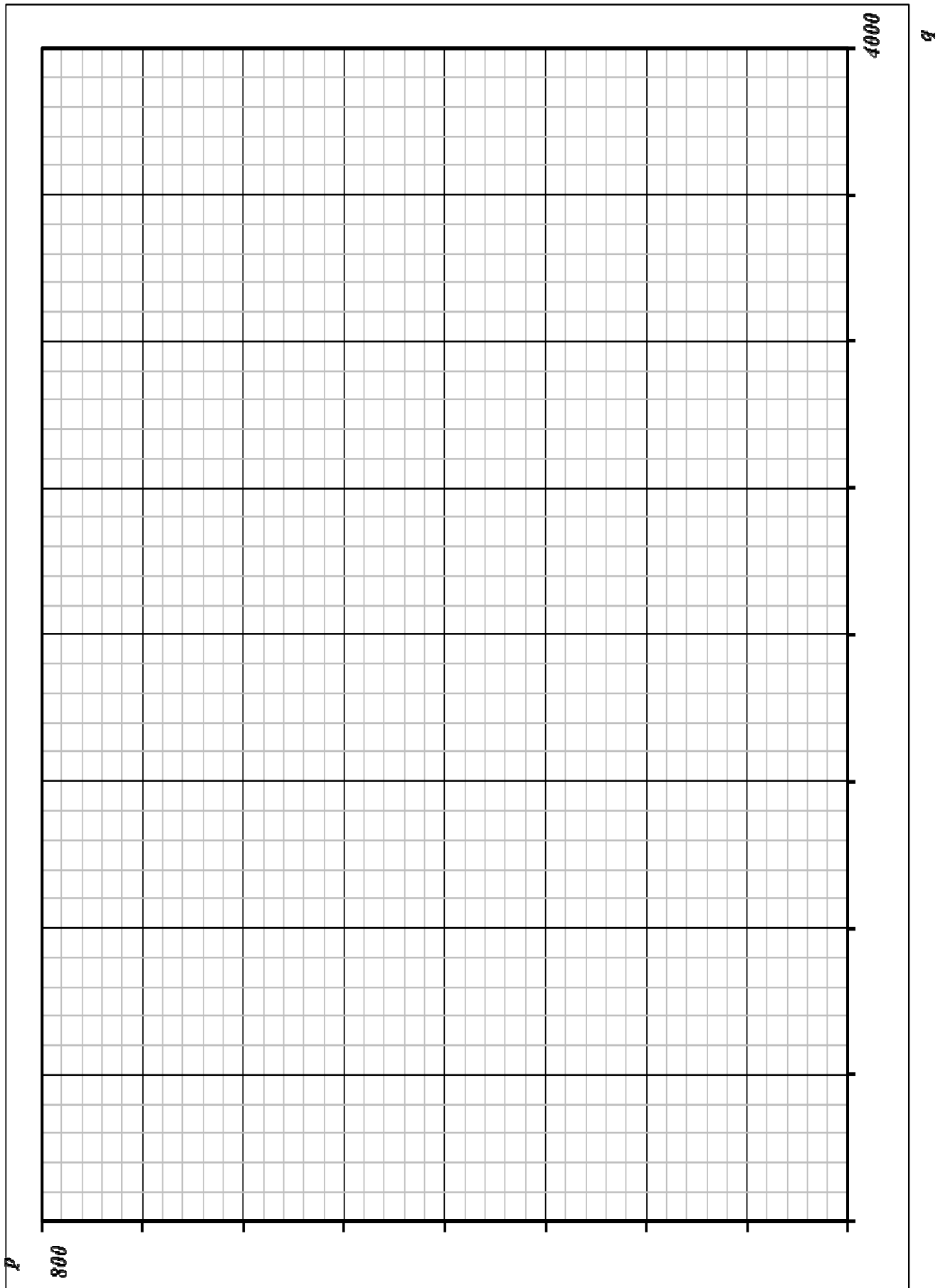
9. [12] Use the following information to answer the questions below:

A product is sold by the litre and supply or demand is measured in litres/month. The *inverse demand* function for that product is given by:  $p^D = -0.125q + 750$ . On the *supply* side, vendors require a minimum price of \$150/litre to make this product and when it is sold for \$650/litre, the supply is 2500 litres/month.

- a. [3] Plot the inverse supply and demand curves on the same graph (next page). Label appropriately.
- b. [3] Find the equilibrium point.
- c. [3] Determine the new equilibrium point if a 15% ad valorem tax is imposed on sellers.
- d. [3] Determine the tax revenue.

NB – **You do not need to plot the new curve(s).**

Use the suggested upper limits on each axis. Note that the graph is on its side.



THIS PAGE IS LEFT INTENTIONALLY BLANK