

**Practice problems List 1 MATH1119B**

Practicing is very important in order to do well in this course. Practice as many problems as possible until you feel comfortable with the material. Solutions to odd-value problems of the textbook can be found at the end of the textbook. Solutions to some questions that are not in the textbook will be posted on cuLearn later.

The Practice problems List 1 is about some of the basic concepts regarding matrices and elementary row operations. These subjects are important to master for the future coming weeks.

1. Determine the dimension of the following matrices.

$$\text{a) } \begin{bmatrix} 8 & 2 & 6 & 1 \\ 2 & 9 & 4 & -6 \\ 1 & -3 & -6 & -2 \\ 5 & -7 & 4 & 4 \end{bmatrix} \quad \text{b) } \begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & -3 & 4 & 1 \end{bmatrix} \quad \text{c) } \begin{bmatrix} 2 & 3 \\ 2 & -1 \\ 2 & -3 \\ 1 & -4 \end{bmatrix}$$

$$2. \text{ a) Consider the matrix } A = \begin{bmatrix} 5 & 2 & -2 & 1 \\ 2 & 3 & -1 & -3 \\ 1 & -3 & -4 & -2 \\ -1 & -1 & 3 & -5 \end{bmatrix}$$

i) What is the (3, 4) entry of  $A$     ii) What is the (2, 3) entry of  $A$ .

$$\text{b) Consider the matrix } B = \begin{bmatrix} 8 & 2 & -2 \\ 2 & -1 & -1 \\ -4 & -3 & -10 \\ -3 & -7 & 4 \end{bmatrix}$$

i) What is the (1, 4) entry of  $B$     ii) What is the (3, 3) entry of  $B$ .

3. Which of the following matrices are in echelon form ?

$$\text{a) } \begin{bmatrix} 1 & 1 & -1 & 3 \\ -2 & 1 & 1 & 4 \\ -3 & 5 & -3 & 3 \end{bmatrix} \quad \text{b) } \begin{bmatrix} 14 & 1 & -2 & -1 \\ 0 & -4 & -3 & 0 \\ 0 & 0 & 2 & 2 \end{bmatrix} \quad \text{c) } \begin{bmatrix} 1 & 1 & -2 \\ 0 & 19 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \text{d) } \begin{bmatrix} 0 & 1 & -2 \\ 1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{e) } \begin{bmatrix} 13 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{f) } \begin{bmatrix} 13 & 31 & -1 & 3 \\ 0 & 0 & 12 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{g) } \begin{bmatrix} 13 & 1 & -1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 14 & 3 \end{bmatrix}$$

4. Which of the following matrices are in reduced echelon form ?

$$\begin{array}{l} \text{a) } \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{b) } \begin{bmatrix} 1 & 1 & -2 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix} \quad \text{c) } \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{d) } \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \\ \text{e) } \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{f) } \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \text{g) } \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

5. a) Let  $A = \begin{bmatrix} 4 & 3 & 1 & 1 \\ 2 & -1 & 3 & 0 \\ -1 & 2 & -1 & 2 \end{bmatrix}$ .

- i) What matrix do we obtain if we apply the elementary row operation  $R_1 \longleftrightarrow R_3$  to  $A$ ?
- ii) What matrix do we obtain if we apply the elementary row operation  $R_2 = R_2 + 2R_3$  to  $A$ ?
- iii) What matrix do we obtain if we apply the elementary row operation  $R_1 = R_1 - 2R_2$  to  $A$ ?
- iv) What matrix do we obtain if we apply the elementary row operation  $R_1 = R_1 + 4R_3$  to  $A$ ?

b) Let  $B = \begin{bmatrix} -3 & 2 & 4 & 0 \\ 2 & 0 & 0 & 1 \\ 4 & 2 & 1 & 3 \end{bmatrix}$ .

- i) What matrix do we obtain if we apply the elementary row operation  $R_1 = 2R_1$  to  $B$ ?
- ii) What matrix do we obtain if we apply the elementary row operation  $R_2 \longleftrightarrow R_3$  to  $B$ ?
- iii) What matrix do we obtain if we apply the elementary row operation  $R_2 = 3R_2$  to  $B$ ?
- iv) What matrix do we obtain if we apply the elementary row operation  $R_3 = R_3 - 2R_1$  to  $B$ ?

c) Let  $C = \begin{bmatrix} 7 & -2 & 1 & 3 \\ 8 & -2 & 4 & 5 \\ 1 & 1 & 3 & 4 \end{bmatrix}$ .

- i) What matrix do we obtain if we apply the elementary row operation  $R_1 \longleftrightarrow R_3$  to  $C$  ?
- ii) What matrix do we obtain if we apply the elementary row operation  $R_3 = R_3 - 7R_1$  to the matrix that you obtained in i) ?
- iii) What matrix do we obtain if we apply the elementary row operation  $R_2 = R_2 - 8R_1$  to the matrix that you obtained in ii) ?
- iv) What matrix do we obtain if we apply the elementary row operation  $R_2 = -\frac{1}{10}R_2$  to the result of iii) ?
- v) What matrix do we obtain if we apply the elementary row operation  $R_3 = R_3 + 9R_2$  to iv) ?

6. Do problems 29 to 32 of Section 1.1 of the textbook.