MAAE 2400 Final Exam

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Fall 2013

Course : MAAE 2400 Sections A, D, and E
Number of students : 301
Duration : 3 hours
Authorized memoranda : Open book, standard calculator
Students **MUST** count the number of pages in this examination question paper before beginning to write, and report any discrepancy immediately to a proctor. This question paper has 10 pages.
This examination question paper **MAY** be taken from the examination room.
In addition to this question paper, students require an examination booklet.
Students do not require a Scantron sheet.

Question 1 (20 marks)

A system employing water as its working fluid produces work at the rate of 720 MW and loses heat to the surroundings at the rate of 20 MW.

Working fluid at 5 MPa and 100°C flows into the system through an inlet with a cross-sectional area of 0.225 m^2. The velocity of the working fluid at this inlet (state 1) is 90 m/s. Working fluid also flows into the system at another inlet (state 2) at 5 MPa and 400°C, and at a velocity of 30 m/s. Working fluid exits the system at state 3 at 5 MPa, with an entropy of 4.0 kJ/kgK, and a velocity of 400 m/s. All inlets and exits are at the same elevation from the ground.

Do the following:

(a) Draw a schematic of the system that clearly illustrates the boundary between the system and the surroundings and that clearly indicates all inlets and exits through which mass flows. Indicate the direction of energy transfer by work and the direction of heat transfer between the system and its surroundings on the schematic.

(b) Write a 1^{st} law energy balance for the system that is consistent with the schematic drawn in Part (a). List all assumptions and cancel all unnecessary terms.

(c) Sketch the process on a T-v diagram. Clearly indicate states 1, 2, and 3 on the diagram. Illustrate the isobars corresponding to these states on the diagram.

(d) Indicate all the tables in the appendices of Moran and Shapiro that could be used to evaluate the required properties for state 1. Explain your choices.

(e) Indicate all the tables in the appendices of Moran and Shapiro that could be used to evaluate the required properties for state 2. Explain your choices.

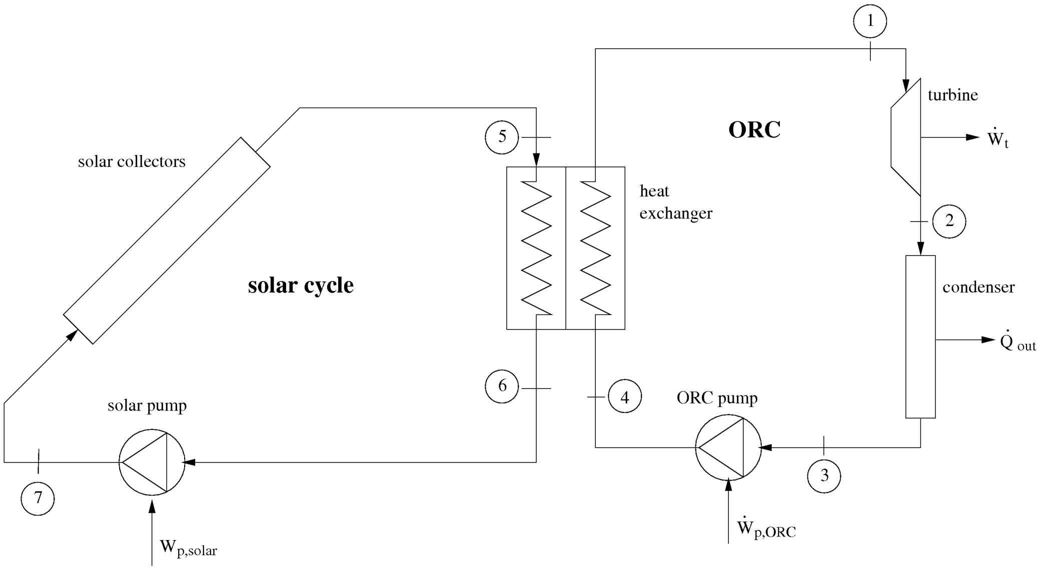
(f) Indicate all the tables in the appendices of Moran and Shapiro that could be used to evaluate the required properties for state 3. Explain your choices.

(g) Describe the procedure you would use to determine the mass flow rate of water entering the system at state 2. Do this in 3-4 sentences of text. It is not necessary to solve this quantity.

(h) Describe the procedure you would use to determine whether this process is possible. Use equations and/or text as necessary, but use no more than a quarter page.

Question 2 (40 marks)

A small power plant uses solar-thermal collectors to provide the heat input to an organic Rankine cycle (ORC), as illustrated in the figure below. The power plant is composed of two cycles, which are labelled *solar cycle* and *ORC* in the figure. Each cycle has its own working fluid. The two cycles are coupled through the heat exchanger that is shown in the middle of the figure. This heat exchanger does not mix the working fluids but rather transfers energy from the solar cycle's working fluid to the ORC's working fluid by heat transfer.



The collectors at the left side of the figure transfer energy from the incoming solar radiation to a working fluid of propylene glycol that is circulated by the *solar pump*. This propylene glycol working fluid can be treated as an incompressible liquid with a constant specific heat of 3 500 J/kgK. The propylene glycol exits the solar collectors at a temperature of 180^{o}C (state 5). The temperature of the propylene glycol as it exits the heat exchanger (state 6) is 50^{o}C. The isentropic efficiency of the *solar pump* is 65%.

The Rankine cycle uses n-butane as its working fluid and operates under steady conditions. The mass flow rate of n-butane through the ORC is 2 kg/s. The pressure of the n-butane at the turbine's inlet is 3.7 MPa and its quality is 65%. The condenser pressure is 400 kPa. The isentropic efficiency of the *ORC pump* is 70% and the isentropic efficiency of the turbine is 91%. The rate of heat transfer from the n-butane to the surroundings in the condenser is 740 kW.

The turbine and the two pumps can be treated as adiabatic. The pressure drops of the n-butane working fluid as it flows through the heat exchanger and through the condenser are negligible. Likewise, the pressure drops of the propylene glycol working fluid as it flows through the heat exchanger and through the solar collectors are negligible. Kinetic and potential energy effects can also be ignored. Stray heat losses from the heat exchanger can be ignored.

Property data for n-butane are tabulated on Pages 9 to 10. Use the state point numbering scheme indicated in the above schematic for your analysis. Do the following:

(a) Specify whether state 1 is in the superheated vapour, subcooled liquid, or mixed region.

(b) Specify whether state 2 is in the superheated vapour, subcooled liquid, or mixed region.

(c) Specify whether state 3 is in the superheated vapour, subcooled liquid, or mixed region.

(d) Specify whether state 4 is in the superheated vapour, subcooled liquid, or mixed region.

(e) Determine the temperature at state 1.

(f) Determine the temperature at state 2.

(g) Determine the temperature at state 3.

(h) Determine the temperature at state 4.

(i) Plot state points 1 through 4 on a T-s diagram.

(j) Calculate the power developed by the turbine, in kW.

(k) Calculate the rate of heat transfer from the propylene glycol to the n-butane in the heat exchanger, in kW.

(l) Calculate the power consumed by the ORC pump, in kW.

(m) Calculate the rate of heat transfer from the n-butane in the condenser, in kW.

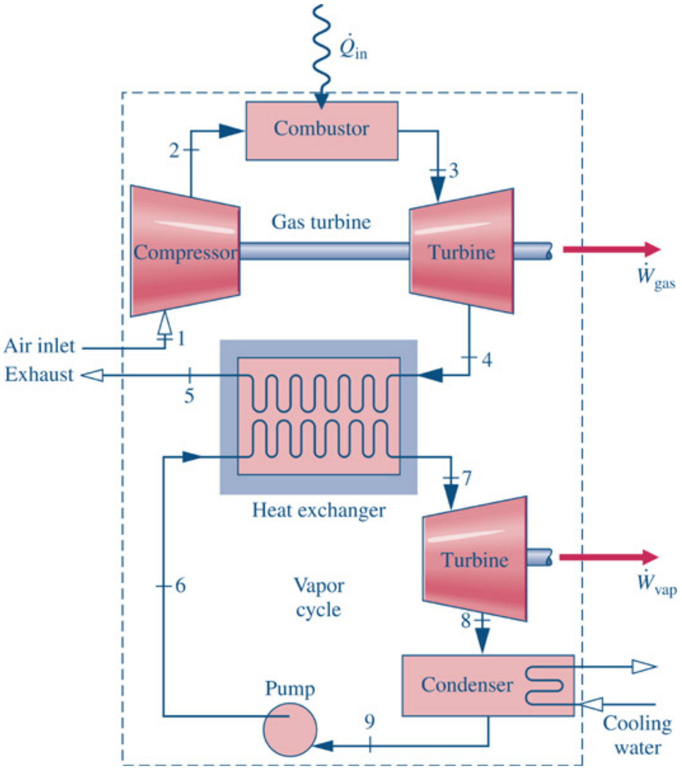
(n) Calculate the ORC cycle's back work ratio.

(o) Calculate the ORC cycle's thermal efficiency.

(p) Calculate the mass flow rate of propylene glycol through the solar cycle.

Question 3 (30 marks)

A combined Brayton and Ranking cycle power plant is illustrated in the figure below. The Brayton cycle portion of the power plant can be treated with an air-standard analysis. Air enters the compressor (state 1) at 310 K and 101 kPa and is compressed to 1.2 MPa by the compressor which has an isentropic efficiency of 88%.



The specific heat addition at the combustor is 800 kJ/kg. The air is then expanded to 101 kPa (state 4) by the turbine, which has an isentropic efficiency of 91%. Finally the air is cooled to 150°C (state 5) by the heat exchanger that couples the Brayton and Rankine portions of the cycle before it is exhausted to the environment. The Brayton cycle transfers energy at the rate of 32 MW to the Rankine cycle through this heat exchanger.

The specific heat of air cannot be treated as a constant. List all assumptions used in your analysis and indicate all sources of data (e.g. Table A-1). Marks will be deducted if you neglect to adequately justify the form of the 1st law energy balances used to represent each process.

Do the following:

(a) Indicate state points 1 through 5 on a T-s diagram. You must follow the state point numbering scheme utilized in the above figure.

(b) Calculate the mass flow rate of air through the Brayton cycle.

(c) Calculate the power input to the compressor in MW.

(d) Determine the temperature of the air exiting the compressor (state 2), in $^{\mathrm{o}}$C.

(e) Determine the temperature of the air entering the turbine (state 3), in $^{\mathrm{o}}$C.

(f) Calculate power output from the Brayton cycles turbine ($\dot{W}_{\mathrm{gas}}$) in MW.

(g) Determine the temperature of the air entering the heat exchanger that couples the Brayton and Rankine cycles (state 4), in $^{\mathrm{o}}$C.

Question 4 (10 marks)

A 35 m length of pipe is used to transport steam from the boiler to the turbine in a Rankine cycle power plant. The rate of heat addition at the Rankine cycle's boiler is 1.3 GW. The pipe is 2.2 m in diameter and its surface temperature is 440$^{\mathrm{o}}$C.

Energy is lost from the pipe to the surroundings due to heat transfer by convection and by radiation. The convection coefficient between the pipe and the surrounding air is 9 W/m^{2}K and the air temperature is 19$^{\mathrm{o}}$C.

The radiation portion of the heat transfer can be treated as emission from a grey body to an enclosure. The enclosure temperature (i.e. the surfaces formed by the power plant containing the pipe and turbine) can be taken as 16$^{\mathrm{o}}$C. The emissivity of the surface of the pipe is 0.2.

Do the following:

(a) Calculate the rate of convective heat transfer per surface area of the pipe, in W/m^2.

(b) Calculate the rate of radiation heat transfer per surface area of the pipe, in W/m^2.

(c) Calculate the total rate of heat transfer from pipe to the surroundings, in MW.

(d) Comment on whether it would be a reasonable assumption to neglect heat losses from the pipe when analyzing the Rankine cycle to determine the cycle's thermal efficiency.

Table 1: Properties of saturated n-butane

P_{sat} (kPa)	T_{sat} (^oC)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kgK)	s_g (kJ/kgK)	v_f (m^3/kg)	v_g (m^3/kg)
100	-0.8664	198.2	583.3	0.9935	2.408	0.001663	0.3734
200	18.79	244.4	611.3	1.156	2.413	0.001725	0.1949
300	31.85	276.2	629.9	1.262	2.422	0.001771	0.1326
400	41.93	301.4	644.2	1.343	2.431	0.001811	0.1006
500	50.26	322.8	655.9	1.41	2.439	0.001846	0.08097
600	57.43	341.6	665.9	1.467	2.447	0.001878	0.06766
700	63.76	358.5	674.6	1.517	2.455	0.001909	0.05802
800	69.45	374	682.3	1.562	2.462	0.001939	0.05069
900	74.64	388.4	689.3	1.603	2.468	0.001967	0.04492
1000	79.42	401.8	695.5	1.641	2.474	0.001995	0.04027
1100	83.86	414.5	701.3	1.676	2.479	0.002023	0.03642
1200	88.01	426.5	706.5	1.709	2.484	0.002051	0.03318
1300	91.92	438.1	711.3	1.74	2.488	0.002079	0.03041
1400	95.61	449.1	715.8	1.769	2.493	0.002106	0.02802
1500	99.11	459.7	719.9	1.798	2.496	0.002134	0.02593
1600	102.4	470	723.7	1.824	2.5	0.002163	0.02409
1700	105.6	479.9	727.1	1.85	2.503	0.002191	0.02245
1800	108.7	489.6	730.3	1.875	2.506	0.002221	0.02097
1900	111.6	499	733.3	1.899	2.508	0.002251	0.01964
2000	114.4	508.2	735.9	1.922	2.51	0.002281	0.01844
2100	117.1	517.2	738.4	1.945	2.512	0.002313	0.01733
2200	119.7	526	740.5	1.967	2.513	0.002346	0.01632
2300	122.2	534.7	742.5	1.988	2.514	0.00238	0.01538
2400	124.6	543.3	744.2	2.009	2.514	0.002416	0.01451
2500	127	551.7	745.6	2.03	2.514	0.002453	0.0137
2600	129.3	560.1	746.8	2.05	2.514	0.002493	0.01294
2700	131.5	568.3	747.7	2.07	2.513	0.002535	0.01222
2800	133.6	576.6	748.3	2.089	2.512	0.002579	0.01154
2900	135.7	584.8	748.6	2.109	2.51	0.002627	0.01089
3000	137.8	593	748.5	2.128	2.507	0.002679	0.01027
3100	139.7	601.4	748.1	2.148	2.503	0.002737	0.009674
3200	141.6	609.8	747.1	2.168	2.499	0.002801	0.009091
3300	143.5	618.4	745.6	2.188	2.493	0.002873	0.008516
3400	145.3	627.3	743.3	2.208	2.485	0.002958	0.007939
3500	147.1	636.8	739.9	2.23	2.475	0.003062	0.007345
3600	148.8	647.2	734.5	2.254	2.461	0.003197	0.006698
3700	150.4	659.8	724.8	2.283	2.437	0.003402	0.005892

Table 2: Properties of subcooled liquid n-butane at P = 400 kPa

T (^{o}C)	v (m^3/kg)	h (kJ/kg)	s (kJ/kgK)
10	0.001694	223.5	1.083
15	0.00171	235.3	1.124
20	0.001727	247.2	1.165
25	0.001745	259.3	1.206
30	0.001763	271.5	1.247
35	0.001782	283.9	1.287
40	0.001802	296.5	1.328

Table 3: Properties of subcooled liquid n-butane at P = 3.7 MPa

T (^{o}C)	v (m^3/kg)	h (kJ/kg)	s (kJ/kgK)
10	0.001681	226.2	1.072
15	0.001696	237.9	1.113
20	0.001712	249.7	1.154
25	0.001728	261.7	1.195
30	0.001745	273.8	1.235
35	0.001763	286	1.275
40	0.001781	298.4	1.315
45	0.0018	310.9	1.354
50	0.00182	323.6	1.394
55	0.00184	336.4	1.433