

Concordia University

Department of Mathematics and Statistics

Course Mathematics	Number 208/1	Section All except EC	
Examination Final	Date April 2014	Time 3 Hours	Pages 3
Instructors Dube, Hughes, Kalvin, Kokotov, Romanelli, White		Course Coordinator Cummins	
Materials Allowed:		Approved Calculators allowed.	
Special Instructions:		Answer all questions and justify your answers.	
Formulas: $A = P(1+i)^n$, $A = Pe^{rt}$, $FV = PMT \frac{(1+i)^n - 1}{i}$, $PV = PMT \frac{1 - (1+i)^{-n}}{i}$			

☺ ok

Q1 (10 Marks) A manufacturer of tablet computers, after extensive research, established the following price-demand, and cost functions:

Price demand $p(x) = 360 - 20x$

Cost $C(x) = 300 + 95x$

where $p(x)$ is the wholesale price in dollars at which x million tablet computers can be sold. The cost $C(x)$ is in millions of dollars. The domain of each function is $1 \leq x \leq 15$.

- Find the revenue function $R(x)$ and sketch the graph of $R(x)$.
- Find the output which will produce the maximum revenue. What is the maximum revenue? What is the wholesale price, per tablet computer, that produces the maximum revenue?
- For what outputs will a loss occur? For what outputs will a profit occur?

Q2 (10 Marks) Solve the following equations for x .

a) $x^2 - x = \log_2(4)$

b) $36^{2x} - 6^{x^2-12} = 0$

c) $2\log_3(x+1) - 2\log_3(9) = 2$

d) $\log_8(x-1) + \log_8(x+1) = 1$

e) $\log_{10}(x+7) - \log_{10}(x-2) = 1$

Q3 (10 Marks)

- a) An arithmetic sequence has 100 terms. If the 100th term is 1000 and the sum of all 100 terms is 50000, what is the first term?
- b) If $h(n) = 10(0.9)^n$, compute the sum

$$\sum_{n=2}^{20} h(n) = h(2) + h(3) + \dots + h(20)$$

Q4 (10 Marks)

- a) Solve the following linear system using Gauss-Jordan elimination:

$$3x + 6y = -9z + 3$$

$$2x + 3y = -4z + 3$$

$$3x + 6y = -3z + 9$$

- b) Check your solution to part a by substituting it into the linear system and verifying that it solves the linear system.

Q5 (10 Marks) A company takes out a loan for \$150000 at an interest rate of 4% compounded monthly.

- a) If the loan is due to be repaid in a single payment after 2 years, what is the total amount, including interest, which the company will have to repay?
- b) The company has insufficient funds to repay this all of this loan. So it takes a second loan of \$8000 at an interest rate of 3% compounded monthly which is paid in equal monthly installments over a period of 5 years. What is the amount of the monthly payments? What is the total amount of interest paid on the second loan?

Q6 (10 Marks)

- a) You invest \$5000 at the end of each of 30 years into an account that earns 8% compounded annually. How much money is in the account at the end of this time?
- b) You then use this money to purchase an annuity which pays equal monthly amounts for 10 years at a rate of 6% compounded monthly. What is the amount of the monthly payments?

Q7 (10 Marks) Extremize the function $P(x, y) = 10x + 12y$ subject to the constraints:

$$2x + 3y \leq 12$$

$$4x - 5y \leq 2$$

$$x \geq 1$$

$$y \geq 1$$

Q8 (10 Marks) A family has a \$130000, 25 year mortgage at 4.2% compounding monthly.

- a) Find the monthly payment.
- b) Find the unpaid balance after 15 years.
- c) Find the total amount of interest paid on the mortgage.

Q9 (10 Marks) A car rental company has 4 red cars and 5 blue cars.

- a) In how many ways can a fleet of 3 cars be selected.
- b) If the fleet of 3 cars is selected at random, what is the probability that all the cars are blue?
- c) If the fleet of 3 cars is selected at random, what is the probability that 1 is blue and the other 2 are red?

Q10 (10 Marks) The board of directors of a company consists of 6 women and 4 men.

- a) If the Chair and Vice-Chair of the board are selected at random, what is that probability that both are women?
- b) If the members of a 3 person committee are selected at random, what is the probability that the majority of members are men?

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Question #1

Q A manufacturer of Tablets

$$P(x) = 360 - 20x$$

$$C(x) = 300 + 95x \quad \text{Domain } 1 \leq x \leq 15$$

a) Find the revenue function $R(x)$ + sketch graph $R(x)$

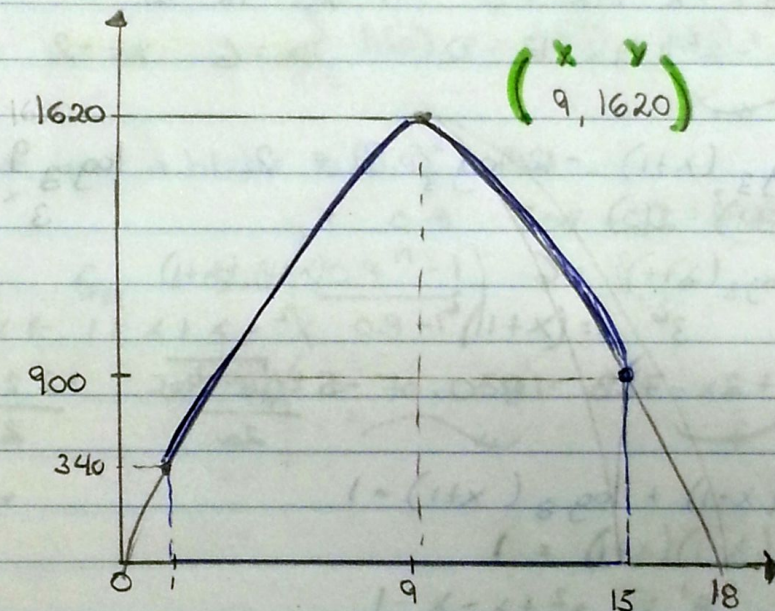
$$R(x) = px = (360 - 20x)x$$

$$R(x) = -20x^2 + 360x$$

$$a = -20 \quad b = 360 \quad c = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-360 \pm 360}{-40}$$

$$v \left(\frac{-b}{2a}, \frac{-\Delta}{4a} \right) \rightarrow v \left(\frac{-360}{-40}, \frac{12960}{-40} \right) \rightarrow (9, 1620)$$



b) With output would produce the max revenue

$$x = 9 \text{ M tablets}$$

What is max revenue

$$R(9) = \$1620 \text{ M}$$

What is the wholesale price that produces max R

$$P(9) = 360 - 20(9) = 180\$$$

c) Loss, profits \rightarrow breakeven point $R(x) = C(x)$

$$-20x^2 + 360x = 300 + 95x$$

$$= 20x^2 - 265x + 300 = \frac{265 \pm \sqrt{46225}}{40}$$

So

⊖ Loss occurs $[1, 1.25) \cup (12, 15)$

$$x_1 = 1.25 \quad x_2 = 12.5$$

⊕ Profits $(1.25, 12) \$\$\$$

Question #2

$x = a^y \Rightarrow \log_a x = y$

a) $x^2 - x = \log_2 4$
 $x^2 - x = 2$
 $x^2 - x - 2 = 0$ $(x-2)$ $\otimes -2$
 $\hookrightarrow x=2$ $x=-1$ $(x+1)$ $\oplus -1$

b) $36^{2x} - 6^{x^2-12} = 0$
 $6^{4(2x)} - 6^{x^2-12} = 0$ $(x-6)$ $\otimes -12$
 $2(2x) - (x^2-12) = 0$ $(x+2)$ $\oplus -4$
 $4x - x^2 + 12 = 0 \rightarrow x^2 - 4x - 12$
 $-x^2 + 4x + 12 = 0$ $x=6$ $x=-2$

c) $2 \log_3 (x+1) - 2 \log_3 9 = 2$ $\log_3 9 = y$
 $\log_3 (x+1)^2 - 2(2) = 2$ $3^x = 9$ $x=2$
 $\log_3 (x+1)^2 = 6$ $(x+1)(x+1)$
 $3^6 = (x+1)^2$ $x^2 + x + x + 1 \rightarrow x^2 + 2x + 1$
 $x^2 + 2x - 728 = 0$ $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\frac{-2 \pm 54}{2}$ $x=26$
 ~~$x=28$~~
 rejected $\ominus \log$

d) $\log_8 (x-1) + \log_8 (x+1) = 1$
 $\log_8 (x-1)(x+1) = 1$
 $8^1 = x^2 + x - x - 1$
 $= x^2 - 9$ $(x+3)(x-3)$

e) $\log_{10} (x+7) - \log_{10} (x-2) = 1$
 $\log_{10} \left(\frac{x+7}{x-2} \right) = 1$
 $10^1 = \frac{x+7}{x-2}$
 $10x - 20 = x + 7$
 $9x = 27$
 $x = 3$ $x=3$ only

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Question # 3

Arithmetic sequence 100 terms

100 term \rightarrow 1000

Sum all terms is 50000 \leadsto What is the first term?

$$SUM = S_n = \frac{n}{2} (a_1 + a_n)$$

$$n = 100$$

$$a_{100} = 1000$$

$$S_{100} = 50000$$

\hookrightarrow find a_1

$$50000 = \frac{100}{2} (a_1 + 1000)$$

$$1000 = a_1 + 1000$$

$$0 = a_1$$

b) Geometric: if $h(n) = 10(0.9)^n$ compute the sum

$$\sum_{n=2}^{20} h(n) = h(2) + h(3) + \dots + h(20)$$

$$\text{First term} \rightarrow h(2) = 10(0.9)^2 = 8.1$$

$$\text{So } S_{19} = 8.1 \left(\frac{0.9^{19} - 1}{0.9 - 1} \right) \quad r = 0.9 \text{ and } 19 \text{ TERMS}$$

$$S_{19} \approx 70.0581$$

$$S_n = a_1 \left(\frac{r^n - 1}{r - 1} \right)$$

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Solve the following

*

$$\begin{array}{ccc|c} 3 & 6 & 9 & 3 \\ 2 & 3 & 4 & 3 \\ 3 & 6 & 3 & 9 \end{array}$$

$$\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 1 & 3 \end{array} \quad \begin{array}{l} L_2 - 2L_1 \rightarrow L_2 \\ L_3 - L_1 \rightarrow L_3 \end{array}$$

$$\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & -2 & 2 \end{array} \quad \begin{array}{l} L_1 + 2L_2 \rightarrow L_1 \\ L_2 \cdot (-1) \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -2 & 2 \end{array} \quad \begin{array}{l} L_3 - L_1 \rightarrow L_3 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -3 \end{array} \quad \begin{array}{l} L_1 + L_3 \rightarrow L_1 \\ L_2 - 2L_3 \rightarrow L_2 \end{array}$$

$$\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \end{array} \quad \begin{array}{l} x = -4 \\ y = 5 \\ z = -3 \end{array}$$

$$3(-4) + 6(5) + 9(-3) =$$

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Final

#5 loan \$150,000 interest 4% compounded monthly

a) After two years how much the company should pay

$$A = P(1+i)^n$$

$$A = 150,000 \left(1 + \frac{0.04}{12}\right)^{24}$$

$$A = 162,471.44 \$$$

$$A = 150,000 \$$$

$$n = 12 \cdot 2 = 24$$

$$i = \frac{0.04}{12} =$$

b) Second loan of \$8000 in 5 years
 $i = \frac{0.03}{12}$ \rightarrow Amount of payments \rightarrow How much interest would be paid

$$PMT = \frac{PV \cdot i}{1 - (1+i)^{-n}}$$

$$PMT = \frac{8000 \cdot \frac{0.03}{12}}{1 - (1 + \frac{0.03}{12})^{-60}} = 143.68 \$$$

$$8620.69 - 8000 = 620.69 \$$$

\rightarrow interest

#6 Invest 5000 \$ \rightarrow at the end of 30 years
earns 8% COMPOUNDED annually

a) How much money at the end of this time

$$FV = \frac{PMT (1+i)^n - 1}{i}$$

$$5000 \cdot \frac{(1+0.08)^{30} - 1}{0.08} = 566,416.05 \$$$

$$i = \frac{0.08}{5} = 0.016$$

$$r = 0.08 \quad m=1 \quad t=30$$

$$i = \frac{r}{m} = \frac{0.08}{1} = 0.08$$

$$n = m t = 30$$

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Max. length

- H6b) Purchase an annuity pays equal monthly amounts for 10 years
 6% compounded monthly
 ⇒ Amount of the monthly payments

$$PMT = \frac{PV \cdot i}{(1 - (1+i)^{-n})}$$

$$r = \frac{0.06}{12} = 0.005$$

$$n = 120$$

$$= \frac{2832}{0.4503} = 6289\$$$

- H7) Extremeze the function $P(x, y) = 10x + 12y$

$$2x + 3y \leq 12$$

$$y \leq -2x + 12$$

$$0 = -2x + 12$$

$$-12 = -2x$$

$$6 = x$$

$$12 = y$$

$$4x - 5y \leq 2$$

$$-5y \leq 2 - 4x$$

$$y \geq \frac{-2 + 4x}{5}$$

$$0 = \frac{-2 + 4x}{5}$$

$$\frac{2}{5} = \frac{4x}{5}$$

$$10 = 20x$$

$$\frac{1}{2} = x \quad y = -2/5$$

$$x \geq 1$$

$$y \geq 1$$

x	y
0	4
3	2
6	0

x	y
0	-0.4
1/2	0
3	2

Intersection point

$$\textcircled{1} \quad 2x + 3y = 12 \quad \times 2$$

$$\textcircled{2} \quad 4x - 5y = 2$$

$$4x + 6y = 24$$

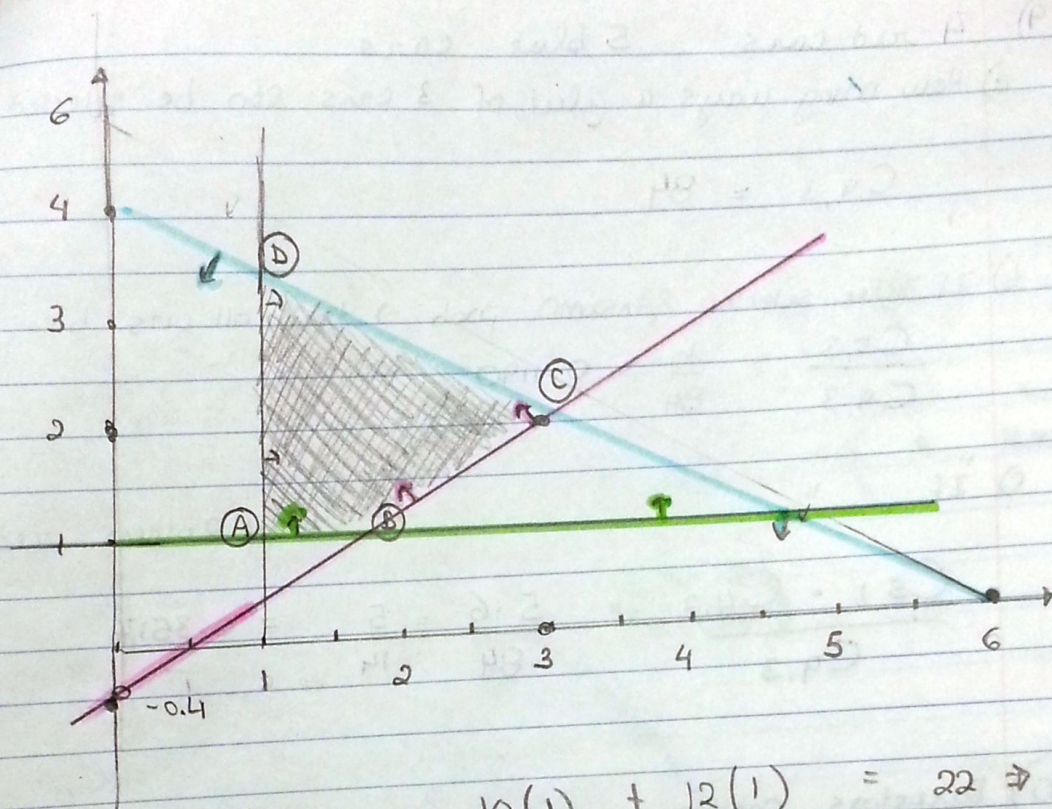
$$- \quad 4x + 5y = 2$$

$$11y = 22$$

$$y = 2$$

$$x = 3$$

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A (1, 1)	$10(1) + 12(1)$	$= 22 \Rightarrow \text{MIN}$
B ($7/4, 1$)	$10(7/4) + 12(1)$	$= 29.5$
C (3, 2)	$10(3) + 12(2)$	$= 54 \Rightarrow \text{MAX}$
D ($1, 10/3$)	$10(1) + 12(10/3)$	$= 50$

#8 \$ 130,000 25 years mortgage at 4.25% compounded monthly
 a) Find monthly payment

$$i = \frac{0.0425}{12} = 0.00354$$

$$Pv = PMT \frac{1 - (1+i)^{-n}}{i} \quad PMT = \frac{Pv \cdot i}{1 - (1+i)^{-n}}$$

$$n = 12 \cdot 25 = 300$$

$$PMT = \frac{130,000 \cdot 0.00354}{(1 - (1 + 0.00354)^{-300})} = \frac{460.2}{0.63} = 704.11$$

b) The unpaid balance after 15 years

$$n = 12 \cdot 15 = 180$$

$$FV = 700.63 \frac{(1 + 0.00354)^{180} - 1}{0.00354}$$

unpaid balance after 15 years

$$\text{So } 25 - 15 = 10 \text{ left } 130,000 - 104,771.14 \$$$

the total interest

$$25,228.86 \$$$

$$210,189 - 130,000 = 80,189 \$$$

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High English

9) 4 red cars 5 blue cars

a) How many ways a fleet of 3 cars can be selected

$$C_{9,3} = 84$$

b) If 3 cars selected RANDOM prob \rightarrow that all cars blue

$$\frac{C_{5,3}}{C_{9,3}} = \frac{10}{84} = 0.1190$$

c) If " " \rightarrow 1 blue / 2 others used

$$\frac{C_{5,1} \cdot C_{4,2}}{C_{9,3}} = \frac{5 \cdot 6}{84} = \frac{5}{14} = 0.357$$

10) Directors company 6 W and 4 M

a) Chair & Vice selected at random prob both woman

PERMUTATION \rightarrow ORDER MATTERS

$$\frac{P_{6,2}}{P_{10,2}} = \frac{30}{90} = \frac{1}{3} = 0.333...$$

b) 3 person committee selected random

what is the prob that they may all be men

2 men + 1 woman & 3 men

$$\frac{C_{4,2} \cdot C_{6,1}}{C_{10,3}} + \frac{C_{4,3}}{C_{10,3}}$$

$$\frac{36}{120} + \frac{4}{120} = \frac{1}{3} \approx 0.3333$$