

Carleton University
Department of Civil and Environmental Engineering
Engineering Economics (ECOR 3800B)
ASSIGNMENT # 2

Issued March 02, 2014 Due Date: March 14, 2014 at 12:00 Noon

Drop off location: Filing cabinet near the entrance to the Civil and Environmental Engineering office.

The cabinet located to the right of room 3424 ME.

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Question (1) (15 marks)

(A)

If your credit card calculates interest based on 12.5% APR,

(a) What are your monthly interest rate and annual effective interest rate?

(b) If your current outstanding balance is \$2,000 and you skip payments for two months, what would be the total balance two months from now?

Solution

APR= 12.5%

a) $i/\text{month} = i_{m=r/12} = 12.5\%/12 = 1.042\%$

Annual effective interest rate :

$I_a = (1+r/M)^M - 1 = 13.2\%$

b) $F = 2000(1+0.01042)^2 = 2041.90$

B)

College Financial Sources, which makes small loans to college students, offers to lend \$500. The borrower is required to pay \$400 at the end of each week for 16 weeks. Find the interest rate per week. What is the nominal interest rate per year? What is the effective interest rate per year?

Solution

$P = A(P/A, i, N)$

$P/A = 500/40 = 12.5,$

From the table we can get $P/A = 12.5, i = 3\%$

Nominal = $r = i_w(52)$ week/year = 156%

$I_a = (1+i/M)^M - 1 = 365.1\%$

(C)

College Financial Sources, which makes small loans to college students, offers to lend \$500. The borrower is required to pay \$400 at the end of each week for 16 weeks. Find the interest rate per week. What is the nominal interest rate per year? What is the effective interest rate per year?

Solution

$$F = P(1+i)^n \quad F = 450, P = 400, n=1 \quad \text{Therefore } i = 12.5\%$$

a) **Nominal** = $i_n = i \times 52 \text{ weeks} = (0.125)52 = 650\%$

b) **Effective** = $i_e = (1+6.5/52)^{52}-1 = 45702.2$

a) $AE = [-10,000 + 3,000(P/A, 7\%, 2) + 4000(P/A, 7\%, 2)(P/F, 7\%, 2) + 2000(P/F, 7\%, 5) + (2,000 + 200)(P/F, 7\%, 6)]$

$$= [-10,000 + 3,000(1.808) + 4000(1.808)(0.8734) + 2000(0.7130) + (2200)(0.6663)] \cdot (0.2098)$$

$$= 4632.29$$

$$AE(7\%) = 4632.29(A/P, 7\%, 6) = 4632.29 \cdot 0.2098$$

$$= \$971.85$$

b) **PROJECT A:**

$$\text{Payback period } P/A = \$2500/\$300 = 8.33 \text{ year}$$

Therefore project does not pay back

PROJECT B:

$$\text{Payback period} = 1 + \$1000/\$1500 = 1.667 \text{ years}$$

PROJECT C:

$$\text{Payback period} = 2 + \$1500/\$2000 = 2.75 \text{ years}$$

PROJECT D

$$\text{Payback period} = \$4000/\$5000 = 0.8 \text{ years, of initial payment}$$

Project D can be viewed as two separate projects, where the first investment is recovered at the end of year 1 and the investment that were made in year 2 and 3 will be recovered at the end of year 6.

Project A: i=9%

Period	Cash Flow (\$)	Cost of Funds (\$)	Cumulative Cash Flow (\$)
0	-2500	0	-2500
1	300	-2500(0.09)= -225	-2425
2	300	-2425(0.09)=-218.25	-2343.25
3	300	-2343.25(0.09)=-210.89	-2254.14
4	300	-2254.14(0.09)=-202.87	-2157.07
5	300	-2157.01(0.09)=-194.13	-2051.14
6	300	-2051.14(0.09)=-184.60	-1935.74
7	300	-1935.74(0.09)=-174.22	-1809.96
8	300	-1809.96(0.09)= -162.90	-1672.86

Therefore discounted payback period is larger than 8 years

PROJECT B: i=9%

Period	Cash Flow (\$)	Cost of funds (\$)	Cumulative cash flow(\$)
0	-3000	0	-3000
1	2000	-3000(0.09)=-270	-1270
2	1500	-1270(0.090)=-114.3	115.7
3	1500		
4	500		
5	500		
6	1500		

Therefore discounted payback period = 1.917 years

PROJECT C: i=9%

Period	Cash flow (\$)	Cost of funds (\$)	Cumulative cash flow (\$)
0	-5500	0	-5500
1	2000	-5500(0.09)=-495	-3995
2	2000	-3995(0.09)=-359.55	-2354.55
3	2000	-2354.55(0.09)=-211.91	-566.46
4	5000	-566.46(0.09)=-50.98	\$4382.56

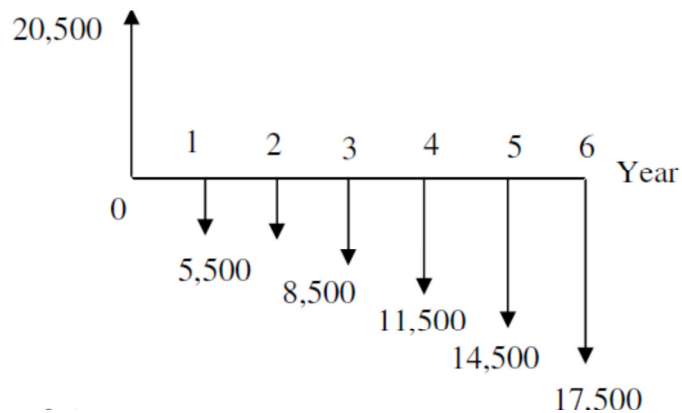
Therefore discounted payback period= 4 years

PROJECT D: i= 9%

Period	Cash flow (\$)	Cost of funds (\$)	Cumulative cash flow (\$)
0	-4000	0	-4000
1	5000	-4000(0.09)=-360	640
2	-3000	640(0.09)=57.6	-2302.40
3	-2500	-2302.40(0.09)=-207.22	-5009.62
4	1000	-5009.62(0.09)=-450.87	-4460.49
5	1000	-4460.49(0.09)=-401.44	-3861.93
6	2000	-3861.93(0.09)=-347.57	-2209.50
7	3000	-2209.5(0.09)=-198.86	591.64

Therefore discounted payback period= 0.862 years

C) Consider the cash flow diagram. Compute the equivalent annual worth at $i = 11\%$



Solution

$$A (P/A, 0.11, 6) =$$

$$-\{5500(P/A, 0.11, 2) + 8500(P/F, 0.11, 3) + 11500(P/F, 11, 4) + 14500(P/F, 0.11, 5) + 17500(P/F, 0.11, 6)\}$$

OR

$$AE_1 (11\%) = +20500(A/P, 11\%, 6)$$

$$AE_2 (11\%) = -5000(P/A, 11\%, 6) (A/P, 11\%, 6)$$

$$AE_3 (11\%) = -3000(P/G, 11\%, 5) (P/F, 11\%, 1) (A/P, 11\%, 6)$$

OR

$$AE (9\%) = +20500(A/P, 11\%, 6) - 5500 - 3000(P/G, 11\%, 5) (P/F, 11\%, 1)(A/P, 11\%, 6)$$

Answer: $P_1 = 20500$

$$P_2 = -5500(P/A, 11\%, 6)$$

$$= -5500(4.233) \quad \text{Note} = 10\% \text{ ---} 4.355, 12\% \text{ ----} 4.111, 11\% \text{ ----} 4.233$$

$$= 23281.5$$

$$P_3 = -3000(P/G, 11\%, 5) (P/F, 11\%, 1)$$

$$= -3000\{(6.862 + 6.397)/2\}\{(.9091 + .8929)/2\}$$

$$= -3000(6.624)(0.901)$$

$$= 17904.67$$

$$AE_1 = 20500(A/P, 11\%, 6) = 20500\{(0.2296 + 0.2432)/2\} = 20500(0.2364) = 4846.2$$

$$AE_2 = -23281.5(0.2364) = 5503.75$$

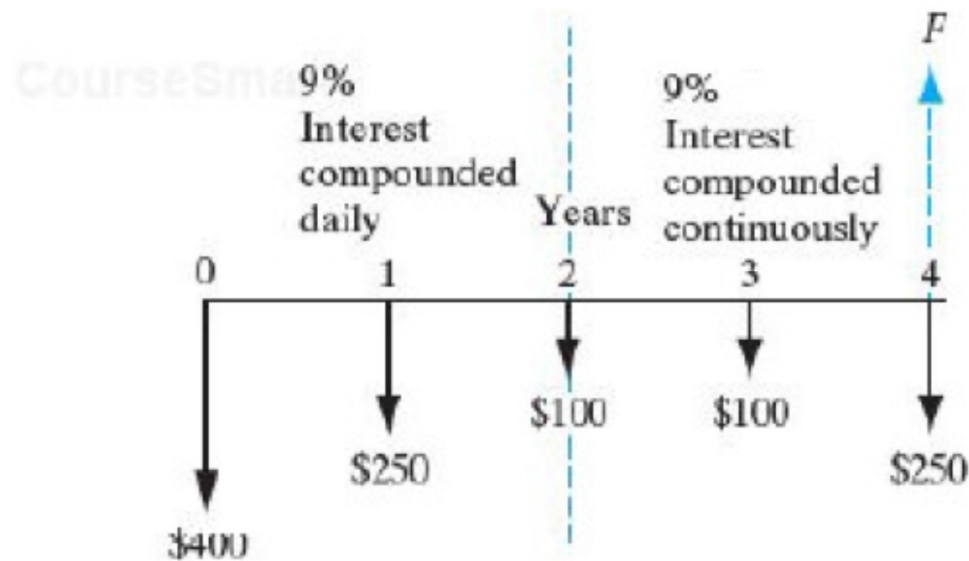
$$AE_3 = -17904.67(0.2364) = 4232.67$$

$$AE = 4846.2 - 5503.75 - 4232.67 = 4890.22$$

Question (3) (15 marks)

(A)

Compute the future worth of the cash flows with different interest rates specified. The cash flows occur at the end of each year over four years.



Solution

$$i_{a1} = 9\% \text{ daily} = (1+r/M)^M - 1 = (1+0.09/365)^{365} - 1$$

For years 3 and 4 $i = 9\%$ continuous

$$i_{a2} = e^{r/k} - 1 = e^{0.09/1} - 1 = 0.09417$$

F =

$$400(F/P, 9.416, 2)(F/P, 9.417, 2) + 250(F/P, 9.416, 1)(F/P, 9.417, 2) + 100(F/P, 9.417, 2) + 100(F/P, 9.417, 1) + 250$$

$$= 400(1.1972)(1.1972) + 250(1.0942)(1.1972) + 100(1.1972) + 100(1.0942) + 250$$

$$= 573.32 + 327.49 + 119.72 + 109.42 + 250$$

$$= \$1379.95$$

(B)

Georgei Rostov deposits \$15,000 in a saving account that pays 4% interest compounded monthly. Three years later. He deposits \$14,000. Two years after the \$14,000 deposit. He makes another deposit in the amount of \$12,500. Four years after the \$12,500 deposit, half of the accumulated fund is transferred to a fund that pays 5% interest compounded quarterly. How much money will be in each account six years after the transfer?

Solution

The balance just before transfer: $i=4/12=0.33\%$

$$F_9 = 15000(F/P, 0.33\%, 108) + 14000(F/P, 0.33\%, 72) + 12500(F/P, 0.33\%, 48) = \$53,799.61$$

$$\text{Half transferred } 53799.61/2 = 26899.805$$

Balance at 6 years for the fund not transferred

$$F_{15} = 26889.805 (1.0033)^{72} = \$34101.04$$

The fund that have been transferred compounding at 5% quarterly ($i=5/4=1.25\%$)

Balance for transferred fund at 6 years

$$F_{15} = 26899.805(1.0125)^{6 \times 4} = \$36243.48 \quad (A = P(1 + r/n)^{nt})$$

Question (4) (15 Marks)

(A)

Your firm is considering purchasing an old office building with an estimated remaining service life of 25 years. Recently, the tenants signed a long-term lease, which leads you to believe that the current rental income of \$150,000 per year will remain constant for the first 5 years. Then the rental income will increase by 10% for every 5-year interval over the remaining life of the asset. For example, the annual rental income would be \$165,000 for years 6 through 10, \$181,500 for years 11 through 15, \$199,650 for years 16 through 20, and \$219,615 for years 21 through 25. You estimate that operating expenses, including income taxes, will be \$45,000 for the first year and that they will increase by \$3,000 each year thereafter. You also estimate that razing the building and selling the lot on which it stands will realize a net amount of \$50,000 at the end of the 25-year period. If you had the opportunity to invest your money elsewhere and thereby earn interest at the rate of 12% per annum, what would be the maximum amount you would be willing to pay for the building and lot at the present time?

Solution

$$\begin{aligned} P &= 150000(P/A, 12\%, 25) + 150000 (P/A, 12\%, 20)(P/F,12\%,5) + 165000(P/A, 12\%, 15) (P/F,12\%,10) \\ &\quad + 18150(P/A, 12\%, 10) (P/F,12\%,15) + 199650(P/A, 12\%, 5)(P/F, 12\%, 20) + \\ &\quad 50000(P/F, 12\%, 25) - 45000 (P/A, 12\%, 25) - 3000(P/G, 12\%, 25) \\ &= \$793111.46 \end{aligned}$$

(B)

A large food-processing corporation is considering using laser technology to speed up and eliminate waste in the potato-peeling process. To implement the system, the company anticipates needing \$2.5 million to purchase the industrial-strength lasers. The systems will save \$1,200,000 per year in labor and materials. However, it will require an additional operating and maintenance cost of \$275,000. Annual income taxes will also increase by \$145,000. The system is expected to have a 10-year service life and will have a salvage value of about \$225,000. If the company's MARR is 10%, justify the economics of the project based on:

(a) PE method

(b) FE method

(c) AE method

Solution

$$PE(10\%) = -2.5 \times 10^6 + 1.2 \times 10^6 - 275000 - 145000 (P/A, 10\%, 10) + 225000 (P/F, 10\%, 10) \\ = \$ 2379525.50$$

$$FE(10\%) = PE(10\%) (P/F, 10\%, 10) \\ = \$ 6171775.29$$

$$AE(10\%) = FE(10\%) (A/P, 10\%, 10) \\ = \$ 387148.80$$

Question (5) (20 Marks)

(A)

Consider the following project balances for a typical investment project with a service life of 4 years

N	A _n	Project Balance
0	-1000	-1000
1	()	-1100
2	()	-800
3	460	-500
4	()	0

(a) Construct the original cash flows of the project.

(b) Determine the interest rate in computing the project balance.

(c) At $i=15\%$, would this project be acceptable?

Solution

At year 3 cost of funds = -800 i

$$-500 = [460 + 800i] + (-800) = 0.20 = 20\%$$

$$A_1 = -1100 + 1000(1.2) = \$100$$

$$A_2 = -800 + 1100(1.20) = \$520$$

$$A_3 = 0 + 500(1.2) = \$600$$

(C)

FE(15%) =

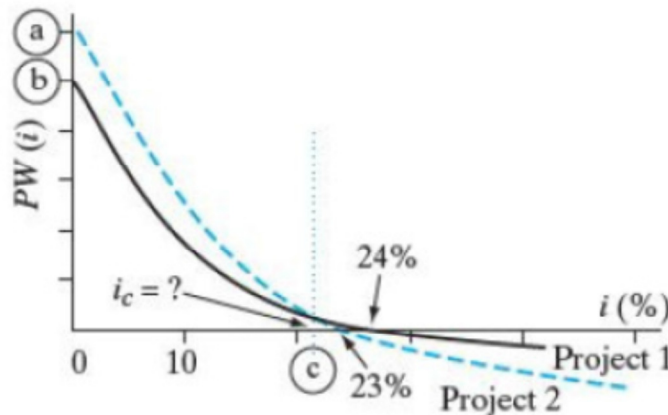
$$-1000(F/P, 15\%, 4) + 100(F/P, 15\%, 3) + 520(F/P, 15\%, 2) + 460(F/P, 15\%, 1) + 600 \\ = \$ 219.78 \quad FE(15\%) > 0 \text{ the project is acceptable}$$

Question (5) (20 Marks)

Consider the following cash flows and present-worth profile:

Year	Net Cash Flows (\$)	
	Project 1	Project 2
0	-\$1000	-\$1000
1	400	300
2	800	Y
3	X	800

- Determine the values of X and Y .
- Calculate the terminal project balance of project 1 at $MARR = 24\%$
- Find the values of a , b , and c in the NPW plot.



Solution

$$PW(24\%) = -1000 + 400(P/F, 24\%, 1) + 800(P/F, 24\%, 2) + X(P/F, 24\%, 3)$$

$$X = \$ 299.58$$

$$PW(23\%) = -1000 + 300(P/F, 23\%, 1) + 800(P/F, 23\%, 2) + Y(P/F, 23\%, 2)$$

$$Y = \$493.49$$

$$FW(24\%) = -1000(1.24)^3 + 400(1.24)^2 + 800(1.24)^2 + 299.49 = 0$$

INTEREST i for Project

$$400(1+i_1)^{-1} + 800(1+i_1)^{-2} + 229.58((1+i_1)^{-3}) = 300((1+i_1)^{-1}) + 493.49(1+i_1)^{-2} + 800((1+i_1)^{-3})$$

$$100((1+i_1)^{-2}) + 30651((1+i_1) - 500) = 0$$

$$i = 0.1783 \text{ OR } 17.83\%$$

$$\text{Point A} = -1000 + 300 + 493.5 + 800 = 593.49$$

$$\text{POINTB} = -1000 + 400 + 800 + 300 = 500$$

Question (6) (20 Marks)

(A)

An electric motor is rated at 10 horsepower (HP) and costs \$835. Its full load efficiency is specified to be 85%. A newly designed, high-efficiency motor of the same size has an efficiency of 90%, but costs \$1300. It is estimated that the motors will operate at a rated 10 HP output for 1500 hours a year, and the cost of energy will be \$0.07 per kilowatt-hour. Each motor is expected to have a 15-year life. At the end of 15 years, the first motor will have a salvage value of \$75. Consider the MARR to be 7%. (Note: 1HP = 0.7457 kW.)

(a) Determine which motor should be installed based on the PE criterion.

(b) In (a), what if the motors operated 2500 hours a year instead of 1500 hours a year? Would the same motor in (a) be the choice?

Required Energy

$$\text{Motor A} = 10/0.85 = 11.765 \text{HP}$$

$$\text{Motor B} = 10/0.90 = 11.11 \text{HP}$$

Annual Energy Cost

Motor A

$$11.765(0.7457)(1500)(0.07) = 921.15/\text{Y}$$

Motor B

$$11.11(0.7457)(1500)(0.07) = 869.98/\text{Y}$$

Motor A

$$P_w(7\%) = -835 - 921.15(P/A, 7\%, 15) + 75(P/F, 7\%, 15) = -9197.56$$

Motor B

$$P_w(7\%) = -1300 - 869.98(P/A, 7\%, 15) = -9223.72$$

Motor A is preferred.

With 2500 operating hours:

Motor A

$$\text{Energy cost} = 11.765(0.7457)(2500)(0.07) = 1535.25$$

$$P_w(7\%) = -835 - 1535.25(P/A, 7\%, 15) + 75(P/F, 7\%, 15) = -14790.73$$

Motor B:

$$\text{Energy cost} = 11.11(0.7457)(2500)(0.07) = 1449.83$$

$$P_w(7\%) = -1300 - 1449.83(P/A, 7\%, 15) = -14506.20$$

Motor B is preferred.

(B)

Consider the following investment projects

Project's Cash Flow					
<i>n</i>	A	B	C	D	E
0	-\$1,800	-\$5,200	-\$3,800	-\$4,000	-\$6,500
1	-500	2,500	0	500	1,000
2	900	-4,000	0	2,000	3,600
3	1,300	5,000	4,000	3,000	2,400
4	2,200	6,000	7,000	4,000	
5	-700	3,000	12,000	1,250	

a) Compute the future worth at the end of life for each project $i=12\%$

b) Determine the acceptability of each project

Solution

$$\begin{aligned}FW(12\%)_A &= -1800(F/P, 12\%, 5) - 500(F/P, 12\%, 4) + 900(F/P, 12\%, 3) + 1300(F/P, \\ &12\%, 2) + \\ &2200(F/P, 12\%, 1) - 700 \\ &= \$700.18\end{aligned}$$

$$\begin{aligned}FW(12\%)_B &= -5200(F/P, 12\%, 5) + 2500(F/P, 12\%, 4) - 4000(F/P, 12\%, 3) + 5000(F/P, \\ &12\%, 2) + \\ &6000(F/P, 12\%, 1) + 3000 \\ &= 5141.19\end{aligned}$$

$$\begin{aligned}FW(12\%)_C &= -3800(F/P, 12\%, 5) + 4000(F/P, 12\%, 2) + 7000(F/P, 12\%, 1) + 12000 \\ &= \$18160.7\end{aligned}$$

$$\begin{aligned}FW(12\%)_D &= -4000(F/P, 12\%, 5) + 500(F/P, 12\%, 4) + 2000(F/P, 12\%, 3) + 3000(F/P, \\ &12\%, 2) + \\ &4000(F/P, 12\%, 1) + 1250 \\ &= \$6040.45\end{aligned}$$

$$\begin{aligned}FW(12\%)_E &= -6500(F/P, 12\%, 3) + 1000(F/P, 12\%, 2) + 3600(F/P, 12\%, 1) + 2400 \\ &= \$-1446.5\end{aligned}$$

All projects are acceptable except D