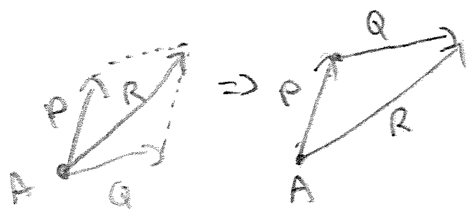
Chapter 2: Statics of Particles

Objective: Understand the effects of forces on particles (dot without dimensions → dimensions considered negligible)
- Replace multiple forces acting on a particle with a equivalent or resultant force
- Find the state of equilibrium for a particle when forces are acting on it

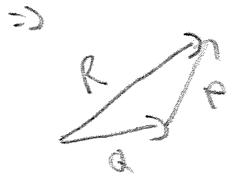
2.1 Forces and Equilibrium in a Plane (2D)

Resultant of two forces ($\vec{R} = \vec{P} + \vec{Q}$) (vector addition)



→ Put the forces from the tail to the arrowhead and the resultant is from the start to the last arrowhead (tail to tip fashion)

* Order is not important

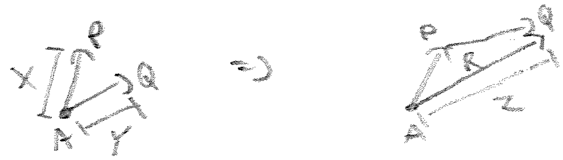


Find values?

three methods : - Graphical
- trigonometric
- Components

Graphical

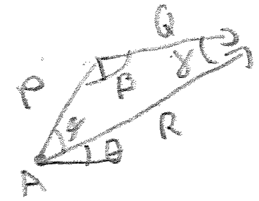
- Need a scale, ex: 1cm = 5N
- Put the vectors in tail to tip with the scale established and put the angle with your protractor.



- Measure the magnitude and direction of the resultant. (Reconvert in Newton with the scale)

* Approximation method

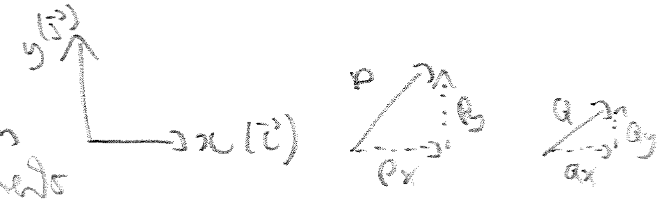
Trigonometric Laws



→ Apply the law of cosines ($R^2 = P^2 + Q^2 - 2PQ\cos\beta$), to find R (magnitu

→ Apply the law of sines ($\frac{\sin\alpha}{Q} = \frac{\sin\beta}{R} = \frac{\sin\gamma}{P}$), to find θ (direction)

Components



→ Cartesian coordinate system

→ Decompose in x and y components

→ Add the components in x to find the resultant in x

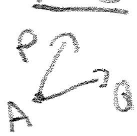
→ Add the components in y to find the resultant in y

→ Calculate the magnitude and direction of the resultant

$$\vec{R} = \vec{P} + \vec{Q} \qquad \left.\begin{array}{l} R_x = P_x + Q_x = \sum F_x \\ R_y = P_y + Q_y = \sum F_y \end{array}\right\} \vec{R} = R_x\vec{i} + R_y\vec{j}$$

Example:

FBD



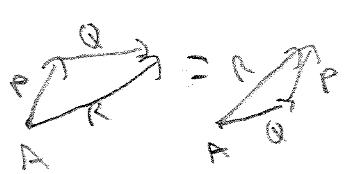
→ Replace the two forces with their resultant

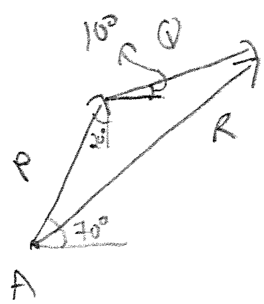
$\vec{P} = 50N \angle 70°$

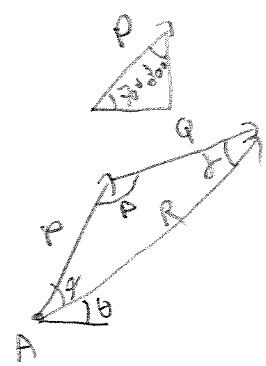
$\vec{Q} = 30N \angle 10°$

Trigonometric Laws

Triangle of Forces







Find R : Law of cosines

$R^2 = P^2 + Q^2 - 2PQ\cos\beta$ → $\beta = 20° + 10° + 90° = 120°$

$$R=\sqrt{(50N)^2+(30N)^2-2(50N)(30N)\cos 120°} = 70N$$

Find direction (θ): Law of sines

$$\frac{\sin\alpha}{Q}=\frac{\sin\beta}{R} \Rightarrow \sin\varphi = \frac{\sin\beta\cdot Q}{R} \Rightarrow \varphi=\sin^{-1}\left(\frac{\sin\beta\cdot Q}{R}\right)$$

$$\varphi=\sin^{-1}\left(\frac{\sin(120°)\cdot 30N}{70N}\right)$$

$$\varphi = 21.8°$$

$$\theta = 70° - 21.8° = 48.2°$$

$$\Rightarrow \boxed{\vec{R} = 70N \angle 48.2°}$$

★ Important → write units (→ vectors...)
→ write magnitude and direction

Components

$\cos\theta = \frac{adj}{hyp}$

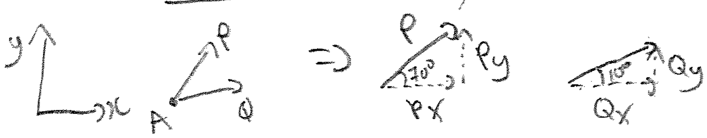
$\sin\theta = \frac{opp}{hyp}$

$$\vec{P} = P_x\hat{i} + P_y\hat{j} = P\cos 70°\hat{i} + P\sin 70°\hat{j}$$

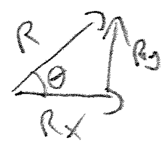
$P_x = ?$ $\cos 70° = \frac{P_x}{P} \Rightarrow P_x = P\cos 70°$

$P_y = ?$ $\sin 70° = \frac{P_y}{P} \Rightarrow P_y = P\sin 70°$

FBD



$$R_x = \Sigma F_x = P_x + Q_x = P\cos 70° + Q\cos 10° = 46.6N$$
$$R_y = \Sigma F_y = P_y + Q_y = P\sin 70° + Q\sin 10° = 52.2N$$



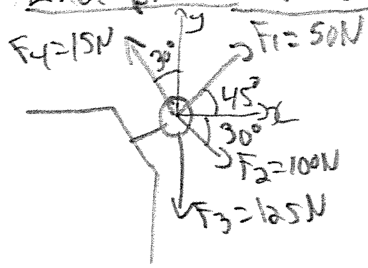
$$\tan\theta = \frac{R_y}{R_y} \Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{52.2N}{46.6N}\right) = 48.2°$$

Pythagorean theorem

$$R^2 = R_x^2 + R_y^2 \Rightarrow R = \sqrt{(46.6N)^2+(52.2N)^2} = 70N$$

$$\Rightarrow \boxed{\vec{R} = 70N \angle 48.2°}$$

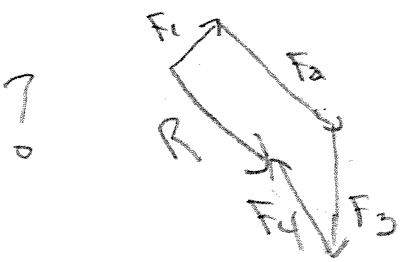
Example 2: Addition of several forces



Resultant = ?

Trigonometric laws

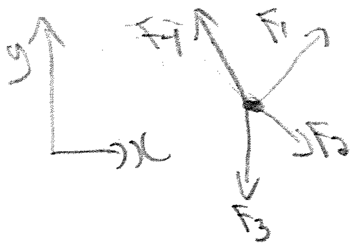
?



How to use the laws??

Components

FBD



$$R_x = \Sigma F_x = F_1 \cos 45° + F_2 \cos 30° - F_4 \sin 30° = 114.5N$$

$$R_y = \Sigma F_y = F_1 \sin 45° - F_2 \sin 30° - F_3 + F_4 \cos 30° = -126.6N$$

do not put the ⊖

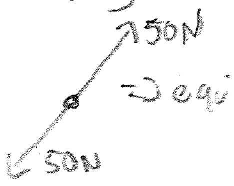
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{126.6N}{114.5N}\right) = 47.9°$$

$$R = \sqrt{(114.5N)^2 + (-126.6N)^2} = 170.7N$$

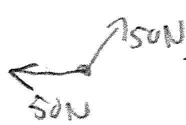
$$\boxed{\vec{R} = 170.7N \measuredangle 47.9°}$$

Equilibrium

When? A particle is in equilibrium when the resultant of all forces acting on it is zero. $\vec{R} = \Sigma\vec{F} = 0$

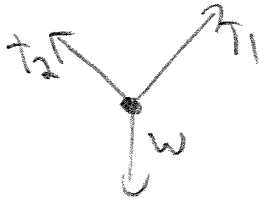


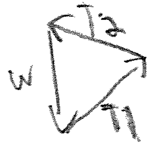
⇒ equilibrium



⇒ not in equilibrium

⇒ Two forces: - equal magnitude
- same line of action
- opposite sense

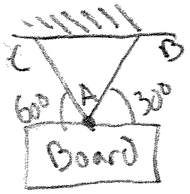


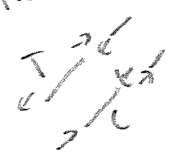


Several forces: - closed polygon
- $\sum \vec{F} = 0$

Example:

*Two cables/ropes





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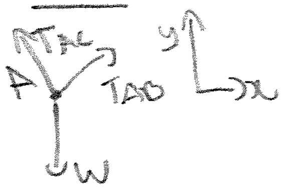


Board's mass: 10 Kg

*A cable can not be in compression (ex: sheet)
→ Tension pulls on the particle
→ Compression pushes on the particle

$$W = mg = (10\,kg)(9.81\,m/s^2) = 98.1\,N$$

Equilibrium

$$\overset{+}{\rightarrow}\sum F_x = 0 = T_{AB}\cos 30° - T_{AC}\cos 60° \quad (1)$$
$$+\uparrow\sum F_y = 0 = T_{AB}\sin 30° + T_{AC}\sin 60° - W \quad (2)$$

From (1) $\quad T_{AB} = T_{AC}\dfrac{\cos 60°}{\cos 30°} \quad (1')$

(1') in (2)

$$0 = \left(T_{AC}\frac{\cos 60°}{\cos 30°}\right)\sin 30° + T_{AC}\sin 60° - W$$

$$0 = 0.29\,T_{AC} + 0.87\,T_{AC} - 98.1\,N$$
$$\Rightarrow \boxed{T_{AC} = 84.6\,N}$$

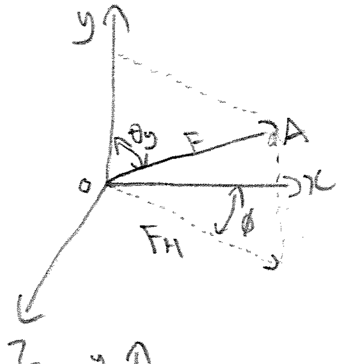
From (1') $\quad \boxed{T_{AB} = 48.8\,N}$

*To verify, put the values of T_{AC} and T_{AB} in (1) and (2), you should get
$0 = 0$

2.2 Forces and Equilibrium in a Space (3D)

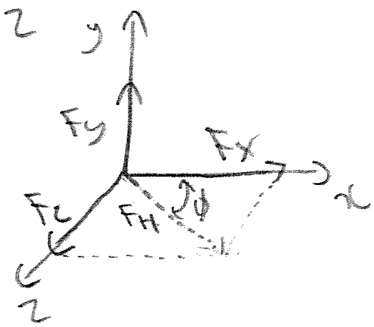
How to find the components of a force in 3D?

* If θ_y and ϕ are given



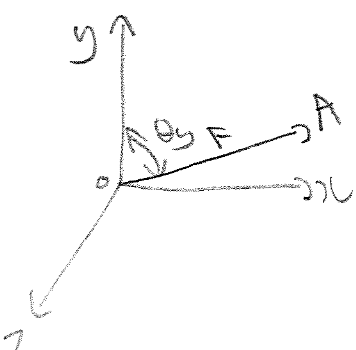
$F_y = F\cos\theta_y$
$F_H = F\sin\theta_y$

F_x and F_z = ??

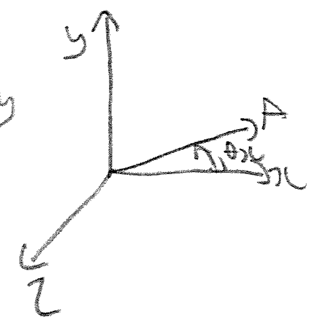


$F_x = F_H\cos\phi = \boxed{F\sin\theta_y}\cos\phi$
$F_z = F_H\sin\phi = F\sin\theta_y\sin\phi$

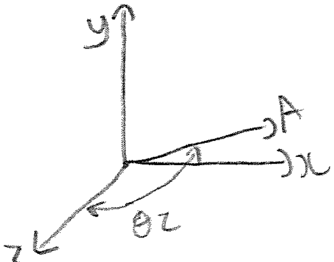
* OR (If all angles between the force and the axes are given)



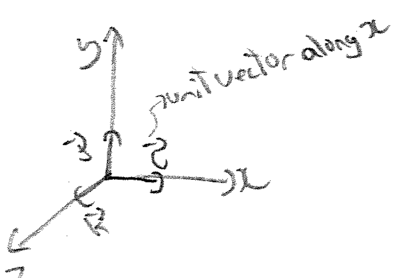
$F_y = F\cos\theta_y$



$F_x = F\cos\theta_x$



$F_z = F\cos\theta_z$



$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

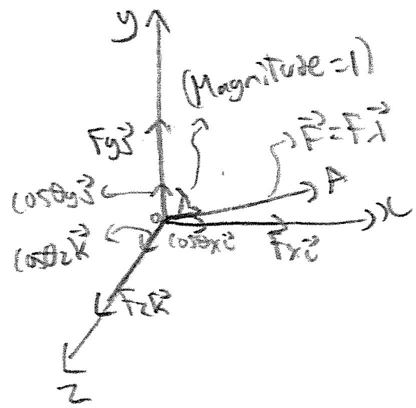
$\vec{F} = F\cos\theta_x\vec{i} + F\cos\theta_y\vec{j} + F\cos\theta_z\vec{k}$
$\vec{F} = F(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k})$

By defining $\vec{\lambda}$ equals at $\cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$

* Extremely important equation in 3D $\Rightarrow$ $\boxed{\vec{F} = F\vec{\lambda}}$

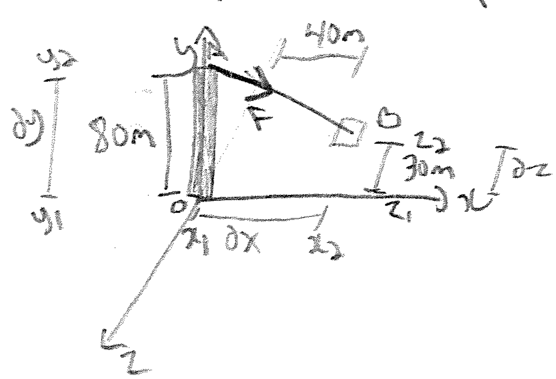
Thus, we can state that: $\lambda_x = \overbrace{\cos\theta_x}^{\frac{F_x}{F}}$, $\lambda_y = \overbrace{\cos\theta_y}^{\frac{F_y}{F}}$, $\lambda = \overbrace{\cos\theta_z}^{\frac{F_z}{F}}$

* $\vec{\lambda}$ is a unit vector along F $\sqrt{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z} = 1$



However, in reality, it is often the distances that are given!!!

By assuming a pole is held in place by a cable



$dx = x_2 - x_1$
$dy = y_2 - y_1$
$dz = z_2 - z_1$

The position vector is: $\vec{AB} = dx\vec{i} + dy\vec{j} + dz\vec{k}$

$$\vec{\lambda} = \frac{\vec{AB}}{|AB|} = \frac{1}{d}(dx\vec{i} + dy\vec{j} + dz\vec{k})$$

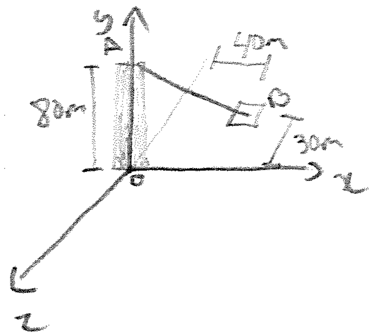
$\hookrightarrow$ Pythagoras in 3D $d = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

As before but this time with distances instead of with forces

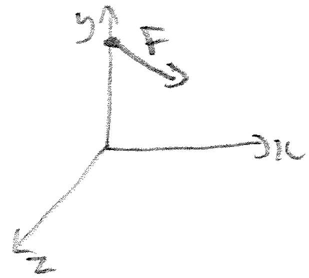
$$\vec{\lambda} = \overbrace{\frac{dx}{d}}^{\cos\theta_x}\vec{i} + \overbrace{\frac{dy}{d}}^{\cos\theta_y}\vec{j} + \underbrace{\frac{dz}{d}\vec{k}}_{\cos\theta_z}$$

Example

$F_{AB} = 2000N$



FBD



$d_x = 40m - 0m = 40m$
$d_y = 0m - 80m = -80m$
$d_z = -30m - 0m = -30m$

$$\Rightarrow \vec{AB} = 40m\vec{i} - 80m\vec{j} - 30m\vec{k}$$

We know,

$$\vec{F} = F\vec{\lambda} \Rightarrow \vec{F_{AB}} = F_{AB}\vec{\lambda_{AB}} \quad \text{where } \vec{\lambda_{AB}} = \frac{\vec{AB}}{AB}$$

$$AB = \sqrt{(40m)^2 + (-80m)^2 + (-30m)^2} = 94.3m$$

$$\Rightarrow \vec{\lambda_{AB}} = \frac{1}{94.3m}(40m\vec{i} - 80m\vec{j} - 30m\vec{k}) = 0.42\vec{i} - 0.85\vec{j} - 0.32\vec{k}$$

$$\Rightarrow \vec{F_{AB}} = F_{AB}(0.42\vec{i} - 0.85\vec{j} - 0.32\vec{k})$$

$$= 2000N \cdot (0.42\vec{i} - 0.85\vec{j} - 0.32\vec{k})$$

$$\boxed{\vec{F_{AB}} = 840N\vec{i} - 1700N\vec{j} - 640N\vec{k}}$$ * Do not forget the units (N)

* To sum up, $\vec{\lambda}$ gives the direction whereas F gives the magnitude!

Vector Addition in 3D

- The reason why we need to put the forces into components (x, y, z) as shown by the last technique, is to be able to add it

* Components method

-> Good luck with the other two.

$$\vec{R} = \Sigma\vec{F}$$

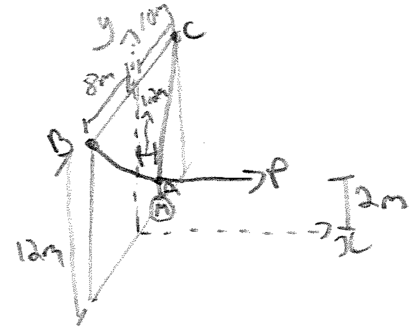
$$R_x\vec{i} + R_y\vec{j} + R_z\vec{k} = (\Sigma F_x)\vec{i} + (\Sigma F_y)\vec{j} + (\Sigma F_z)\vec{k}$$

Particle Equilibrium in 3D

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$$

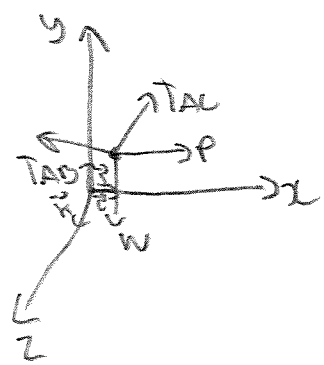
→p.75 modified

Example:



P along $\vec{i}$
$M = 100\,Kg$
$P = ?$ and $T_{AC}, T_{AB} = ?$

FBD



1st step

Find the $\vec{i}, \vec{j}, \vec{k}$ components in order to be able to add them and therefore, find the equilibrium.

$$\vec{P} = P\vec{i}$$

$$\vec{W} = -(mg)\vec{j} = -(100Kg)(9.81\,m/s^2)\vec{j} = -981\,N\vec{j}$$

$\vec{F} = F\vec{\lambda}$

$$\vec{T_{AB}} = T_{AB}\vec{\lambda_{AB}} \text{ where } \lambda_{AB} = \frac{\vec{AB}}{|AB|} \quad \vec{AB} = (-1.2m\vec{i} + 10m\vec{j} + 8m\vec{k})$$

$$AB = \sqrt{(-1.2m)^2 + (10m)^2 + (8m)^2} = 12.86m$$

$$= T_{AB} \cdot \frac{1}{12.86m} \cdot (-1.2m\vec{i} + 10m\vec{j} + 8m\vec{k}) = T_{AB} \cdot (-0.093\vec{i} + 0.776\vec{j} + 0.622\vec{k})$$

$$\Rightarrow \vec{T_{AB}} = -0.093T_{AB}\vec{i} + 0.776T_{AB}\vec{j} + 0.622T_{AB}\vec{k}$$

$$\vec{T_{AC}} = T_{AC}\vec{\lambda_{AC}} \text{ where } \vec{\lambda_{AC}} = \frac{\vec{AC}}{|AC|} \quad \vec{AC} = (-1.2m\vec{i} + 10m\vec{j} - 10m\vec{k})$$

$$AC = \sqrt{(-1.2m)^2 + (10m)^2 + (-10m)^2} = 14.19m$$

$$= T_{AC} \cdot \frac{1}{14.19m} \cdot (-1.2m\vec{i} + 10m\vec{j} - 10m\vec{k}) = T_{AC}(-0.085\vec{i} + 0.705\vec{j} - 0.705\vec{k})$$

$$\hookrightarrow |\lambda| = \sqrt{(\,)^2 + (\,)^2 + (\,)^2} = 1 \rightarrow \text{unit vector}$$

$$\vec{T_{AC}} = -0.085T_{AC}\vec{i} + 0.705T_{AC}\vec{j} - 0.705T_{AC}\vec{k}$$

2nd step

Equilibrium

$$\xrightarrow{+} \Sigma F_x = 0 = P - 0.093T_{AB} - 0.085T_{AC} \quad (1)$$
$$+\uparrow \Sigma F_y = 0 = -981N + 0.776T_{AB} + 0.705T_{AC} \quad (2)$$
$$+\swarrow \Sigma F_z = 0 = 0.622T_{AB} - 0.705T_{AC} \quad (3)$$

From (3) $T_{AB} = 1.13T_{AC}$ (3')

(3') in (2)

$$981N = 0.776(1.13T_{AC}) + 0.705T_{AC}$$
$$\Rightarrow \boxed{T_{AC} = 620.1N}$$

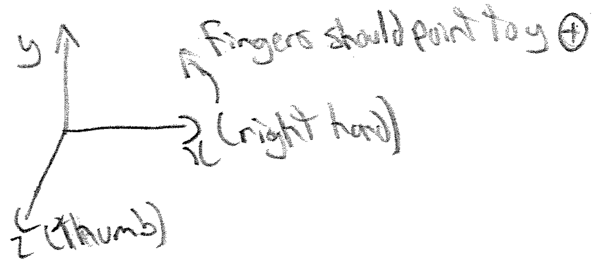
From (3') $\boxed{T_{AB} = 700.8N}$

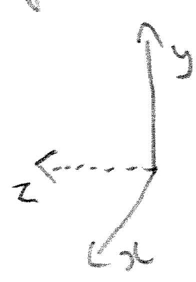
From (1) $\boxed{P = 117.9N}$

* To verify, put the values in (1), (2) and (3)
0=0

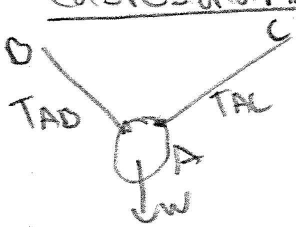
Additional Notes

Right Hand Rule (RHR) for axis placement



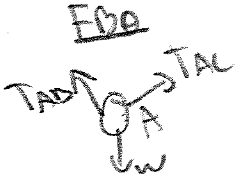


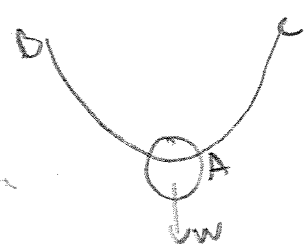
Cables and rings



$T_{AB} \neq T_{AC}$ ← not necessarily equal
↳ two different cables
↳ As the previous example

FBD





$T_{AB} = T_{AC}$
↳ in magnitude
↳ Same cable so it' the same force since it is one cable

FBD

* be careful in proble
↳ it will be writt passes through ring