Functions in 2 Variables

A function f of two variables is a rule that assigns to each ordered pair of real numbers (x,y) in a set D a unique real number denoted by $f(x,y)$. D is the domain of f and the set of values $f(x,y)$ is called the range of f

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Write $z = f(x,y)$.

z: dependent variable; x, y: independent variables

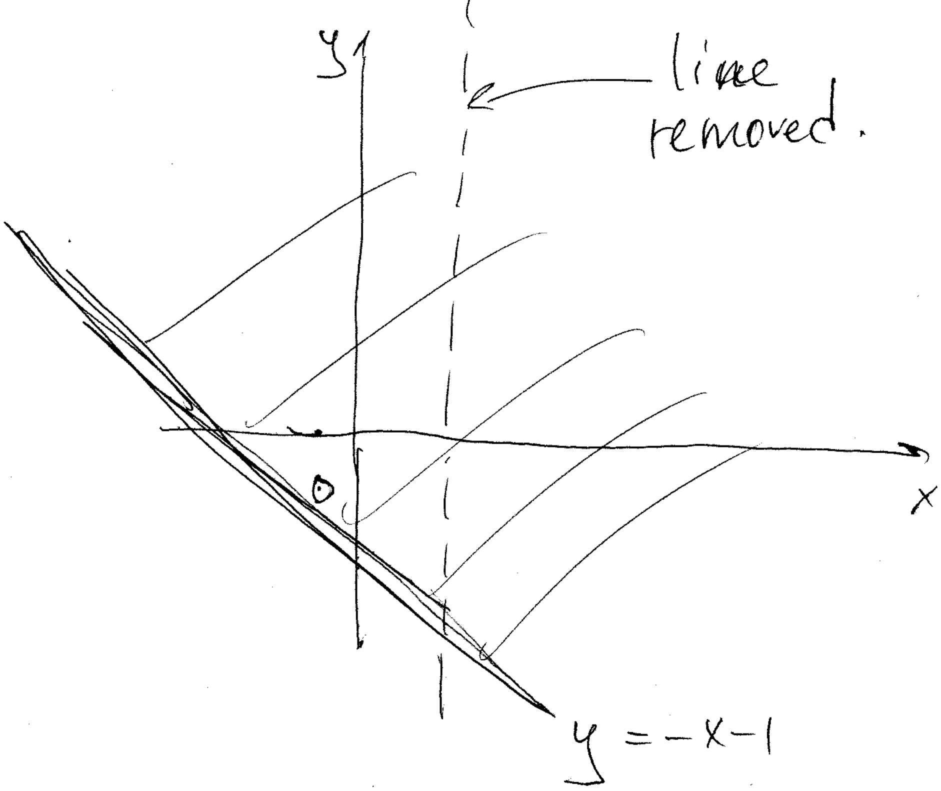
Ex. Evaluate $f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$ at $(3,2)$ and find the domain of f.

$$f(3,2) = \frac{\sqrt{3+2+1}}{3-1} = \frac{\sqrt{6}}{2}$$

Domain: f is defined if the denominator is not 0 and $x+y+1 \geq 0$ $D = \{(x,y) \mid x+y+1 \geq 0,\ x \neq 1\}$

We sketch it.

(17-3)



$x + y + 1 \geq 0$

$y \geq -x - 1.$

$x \neq 1.$

(17-4)

Ex Function can be given by a table of values

Wind speed. $\vec{v}$

T \ v	5	10	15	20
5	4	3	2	1
0	-2	-3	-4	-5
-5	-7	-9	-11	-12

Actual temperature ↓ x

z.

$f(T, v)$ = the wind-chill index.

$f(5, 15) = 2.$

Graphs (17-5)

Let $f(x,y) = z$ be a function in 2 variables.
The graph of f is the set of all points $(x,y,z) \in \mathbb{R}^3$ such that $z = f(x,y)$ and $(x,y) \in D$.

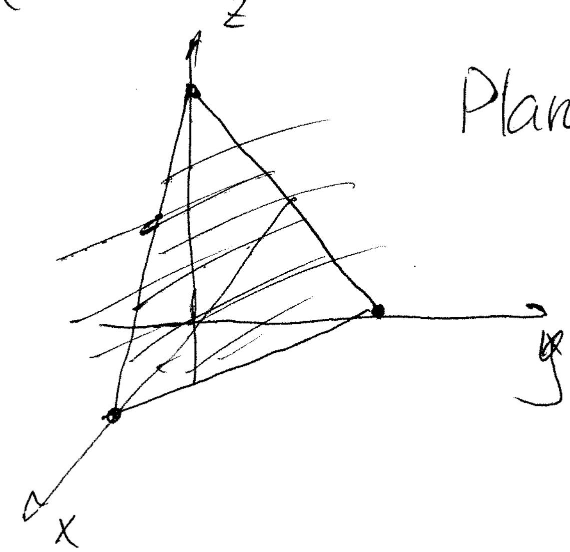
Ex. Sketch the graph of $f(x,y) = 6-3x-2y$

We have $z = 6-3x-2y$ or equivalently

$3x+2y+z=6$ represents a plane with normal vector $(3,2,1)$.

Find its intersections with OX, OY, OZ: (17-6)

$y = z = 0$ $\quad 3x = 6$ $\quad x = 2$	$x = z = 0$ $\quad 2y = 6$ $\quad y = 3$	$x = y = 0$ $\quad z = 6$
$(2,0,0)$	$(0,3,0)$	$(0,0,6)$.



Plane that passes through these 3 points.

Ex. Find the domain and range and sketch the graph of (17-7)

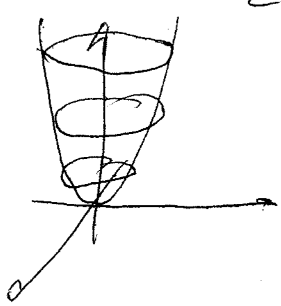
$h(x,y) = 4x^2 + y^2$.

$D =$ anything $= \mathbb{R}^2$

range $h(x,y) \geq 0$ so is $[0, +\infty)$.

Graph: $z = 4x^2 + y^2$ or $4x^2 + y^2 - z = 0$.

elliptic paraboloid.



Level Curves (17-8)

The level curve of $z = f(x,y)$ consists of points (x,y) such that $f(x,y) = k$ where k is fixed (k is in the range of f).

Ex. • Level curves can be seen on topographic maps as curves showing elevation above sea level.

• Or as isothermal curves showing the temperature

Ex. Sketch the level curves of the function: $g(x,y) = \sqrt{9-x^2-y^2}$ for $k = 0, 1, 2, 3$.

$\sqrt{9-x^2-y^2} = 0$ $\qquad x^2+y^2 = 9$

$= 1$ $\qquad x^2+y^2 = 8$

$= 2.$ $\qquad x^2+y^2 = 9-2^2 = 5.$

$\qquad x^2+y^2 = 0.$