Carleton University

ECON 2020H – Intermediate Microeconomics I: Producers and Market Structure

Midterm Exam

February 11, 2015

Student Name : ________________

Student Number : ________________

Instructions

1. There are 5 questions
2. Read carefully before you provide your answers
3. Show all your work. Random guesses will receive NO credit
4. Label all your graphs with as much detail as possible
5. Budget your valuable time
6. Solutions for quantities need not be integers
7. Recall that the solutions to a quadratic function $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Good Luck

1. (20 points) Short questions:

   (a) In some firms, managers are given bonuses that are tied to the market share of a firm (i.e. the firm's share of total revenue in the market's total sales). Is this an efficient compensation strategy? Why?

   No, this is not an efficient compensation strategy. Bonuses based on revenue market shares provide an incentive for managers to maximize revenue rather than profit. Maximizing revenue will lead to managers overproducing and not maximizing profits. In the short run, when other inputs are fixed, this will lead to inefficient production.

   (b) Explain how increasing returns to scale can co-exist with diminishing marginal productivity (e.g. diminishing marginal productivity of labor or MP_L). Provide an example of a production function with increasing returns to scale and diminishing marginal returns.

   These concepts are different. The first refers to how the production function changes when the quantities of all inputs change, while the second refers to changes in the production function when the quantity of only one input changes. A production function exhibits increasing returns to scale if, when all inputs change in a given proportion, total output changes more than proportionally. Marginal productivity refers to how the output of a firm changes when the quantity of only one input changes. Diminishing marginal productivity means that as the amount of the input increases, the incremental output produced decreases.

   A general Cobb-Douglas production function with $q = L^a K^b$ with $a + b > 1$ while $a < 1$ and $b < 1$ exhibits IRS and diminishing marginal returns for each input. IRS means that $f(\lambda L, \lambda K) > \lambda f(L, K)$ for $\lambda > 1$. Using the above production function:

$$
\begin{aligned}
(\lambda L)^a (\lambda K)^b &> \lambda L^a K^b \\
\lambda^a L^a \lambda^b K^b &> \lambda L^a K^b \\
\lambda^{(a+b)} L^a K^b &> \lambda L^a K^b \\
\lambda^{(a+b)} &> \lambda
\end{aligned}
$$

   The above is true if $a + b > 1$ as we assumed. Diminishing marginal returns means that the marginal product of a factor is decreasing. The marginal product of labor is:

$$
MP_L = \frac{\partial q}{\partial L} = \frac{\partial L^a K^b}{\partial L} = aL^{a-1}K^b
$$

   To show that MP_L is decreasing we can show that the MP_L has a negative slope, i.e. that when L increases MP_L decreases.

$$slope = \frac{\partial MP_L}{\partial L} = a(a-1)L^{a-2}K^b$$

Because $a < 1$, then $a - 1 < 0$ and because all other terms above are positive, then the whole expression is negative which means that the MP_L is decreasing.

(c) You are an employer seeking to fill a vacant position on an assembly line. In determining whether to fill that vacant position, are you more concerned with the average product of labor or the marginal product of labor for the last person hired? If you observe that your average product is just beginning to decline, should you hire any more workers? What does this situation imply about the marginal product of your last worker hired?

In filling a vacant position, you should be concerned with the marginal product of the last worker hired, because firm's make optimal decisions on the margin: comparing marginal cost to marginal benefit. The marginal product measures the value of the additional worker by its effect on output, or total product. When deciding whether to hire one more worker the firm compares the value of this marginal product to the marginal cost of hiring the worker and will continue hiring so long as the value of the marginal product is larger than the marginal cost.

The point at which the average product begins to decline is the point where average product is equal to marginal product. As more workers are used beyond this point, both average product and marginal product decline. However, marginal product can still be positive –even if lower than the average product– so total product continues to increase. Thus, it may still be profitable to hire another worker. Mathematically, we can show that marginal product and not average product matters in two different ways but the verbal explanation is sufficient.

First, the optimal condition we derive from minimizing a firm's cost subject to the firm choosing a particular level of output is

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

This condition says that the optimal amount of labor hired has to be such that the last dollar spent on one input yields as much additional output as the last dollar spent on any other input. This condition depends on the marginal and not average product of labor.

Second, we showed that the short-run marginal cost curve, when labor is the only variable input is

$$MC = \frac{w}{MP_L}$$

In equilibrium, we know that a competitive firm produces where $MC = p$, so the optimal amount of labor will be determined by $p = \frac{w}{MP_L}$ which also depends on the marginal product of labor and not the average product of labor.

(d) Why is there a social cost to monopoly power? If the gains to producers from monopoly power could be redistributed to consumers, would the social cost of monopoly power be eliminated? Explain briefly.

When the firm exploits its monopoly power by charging a price above marginal cost, consumers buy less at the higher price, and consumer surplus decreases. Some of the lost consumer surplus is not captured by the seller, however, because the quantity produced and consumed decreases at the higher price, and this is a deadweight loss to society. Therefore, if the gains to producers were redistributed to consumers, society would still suffer the deadweight loss.

(e) A monopolist firm faces a demand with constant elasticity of -2.0. It has a constant marginal cost of \$20 per unit and sets a price to maximize profit. If marginal cost should increase by 25%, would the price charged also rise by 25%?

We have shown that the monopolist's profit-maximizing pricing rule is:

$$\frac{p - MC}{p} = -\frac{1}{\epsilon}$$

or alternatively

$$p = \frac{MC}{1 + \frac{1}{\epsilon}}$$

Therefore, price should be set so that

$$p = \frac{MC}{1 + \frac{1}{-2}} = 2MC$$

With $MC = 20$, the optimal price is $P = 2(20) = \$40$.

If MC increases by 25% to \$25, the new optimal price is $P = 2(25) = \$50$, a 25% increase.

So if marginal cost increases by 25%, the price also increases by 25%.

2. (20 points) Suppose there are 1000 identical wheat farmers. For each farmer, the cost function is $C(q) = 160,000 + q^2$, where q is the output produced by the farmer. Farmers operate in a competitive wheat market. The market demand is $Q = 600,000 - 100p$.

   (a) What is each farmer's supply curve expressed as quantity as a function of price $q = q(p)$?

   A competitive firm's supply curve is its marginal cost curve above the minimum average variable cost. Hence, the farmer's supply curve is:

   $MC = \frac{dC}{dq} = 2q \rightarrow p = 2q$ is the inverse supply curve and $q = 0.5p$ is the supply curve

   The average variable cost is

   $$AVC = \frac{VC}{q} = \frac{q^2}{q} = q$$

   For positive values of q, the MC is always above AVC.

   (b) What is the total market supply curve? Derive the short-run equilibrium Q, q, and p. Does the typical farmer earn a short-run profit? If so, what is it?

   The firm's supply is $q = 0.5p$; because there are 1,000 identical farmers, the market supply is $Q = 1,000q = 1,000(0.5p) = 500p$.

   The market equilibrium can be found where supply equals demand:

   $$S(p) = D(p)$$
   $$500p = 600,000 - 100p$$
   $$600p = 600,000 \rightarrow p = 1,000 \rightarrow Q = 500,000$$

   At this price, each farmer produces $q = 0.5p = 500$. And the farmer's profit

   $$\pi = (500 \times 1,000) - (160,000 + 500^2) = \$90,000$$

   So each firm earns a positive profit.

(c) What is the long-run equilibrium in this market? (*Hint: Depending on how you approach this problem, you might want to multiply by q to find the solution more easily)*

In the long run, other farmers will enter the market until each of them makes zero profits, i.e. price is equal to the minimum average cost which occurs at the level of output where average cost is equal to marginal cost.

$$AC = \frac{C(q)}{q} = \frac{dC(q)}{dq} = MC$$
$$\frac{160,000}{q} + q = 2q$$
(multiplying by q) $160,000 + q^2 - 2q^2 = 0$
$$160,000 - q^2 = 0$$
$$160,000 = q^2 \rightarrow q = \sqrt{160,000} = 400$$

At this level of output, the long-run equilibrium price is, $p = MC = AC = \$800$.

(d) Is it different from the short-run equilibrium? Why or why not?

The long-run equilibrium is indeed very different from the short-run equilibrium because farmers were making very large profits in the short run. Hence, other farmers had the incentive to enter which expands total output and lowers the equilibrium price.

3. (20 points) A monopolist faces the inverse demand for its output: $p = 30 - Q$. The monopolist also has a constant marginal and average cost of \$4/unit. The government is seeking ways to collect tax revenue from the monopolist and faces two proposals:

   i. Impose a specific tax of τ on the monopolist.

   ii. Impose an ad valorem tax of α on the monopolist.

   (a) Suppose the government imposes a 20% ad valorem tax on the monopolist ($\alpha = 0.2$). What price and quantity does the monopolist choose and how much revenue does the government generate from the tax?

   With a 20% ad valorem tax, the monopolist seeks to maximize profits:

$$\pi = (1-\alpha)p(Q)Q - C(Q) = (1-0.20)(30-Q)Q - 4Q$$
$$\pi = 30Q - Q^2 - 6Q + 0.2Q^2 - 4Q$$
$$\pi = -0.8Q^2 + 20Q$$

   The first order condition is:

$$\frac{d\pi}{dQ} = -1.6Q + 20 = 0$$
$$\frac{20}{1.6} = Q^*$$
$$12.5 = Q^*\text{, replacing in the demand function } \rightarrow p^* = \$17.5$$

   The government's revenue from the tax is: $TR^* = 0.2 \times 12.5 \times 17.5 = 43.75$

(b) Rather than an ad valorem tax, what is the government's revenue from a specific tax of τ imposed on the monopolist? Your answer should be in terms of τ.

A specific tax of t makes the monopolist's profit maximization:

$$\pi = (30 - Q)Q - 4Q - \tau Q$$

The first order condition is:

$$\frac{d\pi}{dQ} = 30 - 2Q - 4 - \tau = 0$$
$$30 - 4 - \tau = 2Q$$
$$\frac{26 - \tau}{2} = Q^*$$

The government's tax revenue is:

$$TR = Q^*\tau = \frac{26 - \tau}{2}\tau$$

(c) Show that a specific tax of $3.97/unit generates the same revenue as a 20% ad valorem tax (approximately).

Find τ^* where the tax revenue from both types of tax is the same:

$$\frac{26 - \tau}{2}\tau = 43.75$$
$$13\tau - \frac{\tau^2}{2} = 43.75$$
$$13\tau - \frac{\tau^2}{2} - 43.75 = 0$$

The solution to this quadratic function ($a\tau^2 + b\tau + c = 0$) is

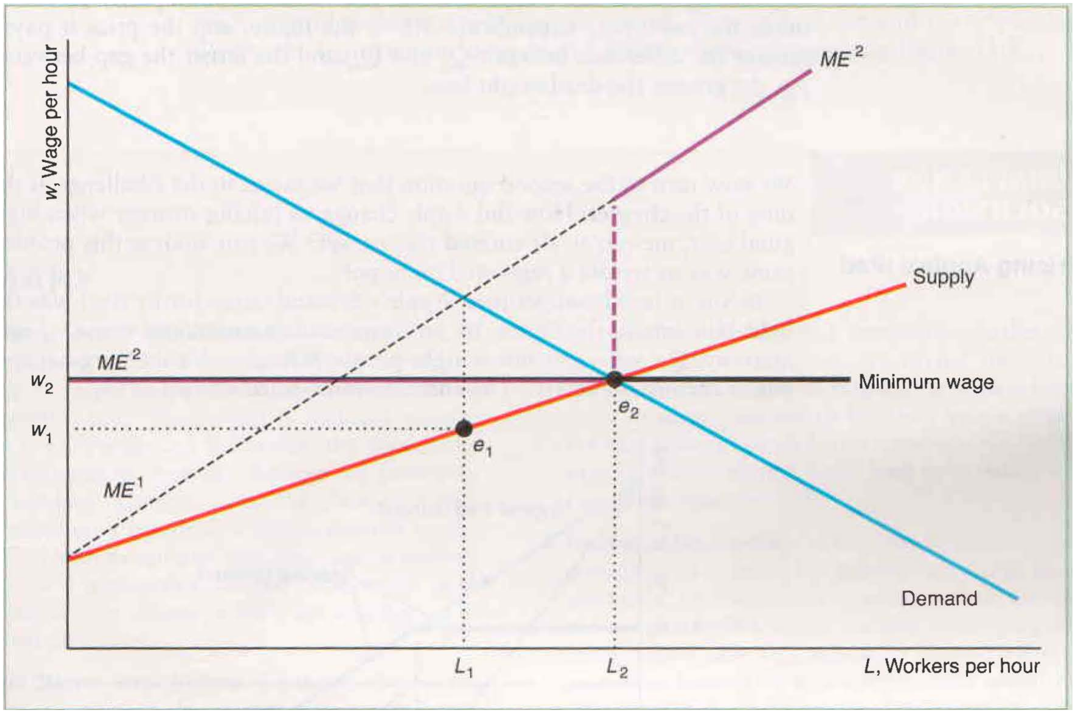
$$\tau = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-13 \pm \sqrt{13^2 - 4(-1/2)(-43.75)}}{2(-1/2)} = 3.97$$

Or simply replace $\tau = \$3.97$ in the solution from the previous question.

(d) Which tax has a greater distortion on the monopoly output?

If $\tau = \$3.97$ the output is $Q^* = \frac{26-\tau}{2} \approx 11$ with the specific tax versus 12.5 with the ad valorem tax. So the specific tax generates a greater distortion by reducing output more than the ad valorem tax.

4. (20 points) Show with a graph what is the effect of the government imposing a minimum wage equal to the competitive wage in a monopsony market.



- The monopsony equilibrium is at e_1
  - The monopsony amount of labor hired L_1 is determined by the intersection of the marginal expenditure curve ME^1 (the diagonal ME^1 curve that starts with dashed lines and becomes purple later) and the demand curve
  - The monopsony wage is determined by the supply curve for that level (L_1) of workers hired: w_1
- In the competitive equilibrium e_2, the amount of labor hired L_2 and the equilibrium wage w_2 are determined by the intersection of supply and demand
- The monopsony creates a deadweight loss equal to the triangle between L_1 and L_2 and the supply and demand curves.
- When the government implements a minimum wage equal to the competitive wage w_2, the monopsonist's ME curve becomes ME^2 which is split into two parts:
  - It is equal to the minimum wage up to the competitive equilibrium quantity and then goes back to the original ME^1 curve
  - Given this new ME^2 curve, the new equilibrium for the monopsony is the competitive equilibrium, which increases the amount of labor hired and the wage rate and eliminates deadweight loss.

5. (20 points) Sam and Erica are starting a new restaurant in Portland, Oregon. While Sam plans to do the cooking himself, he will need to employ workers and machinery to produce food. He estimates his production function as:

$$q = 15L^{0.25}K$$

where L is the number of workers and K is units of capital (machinery). Sam is able to accumulate \$10,000 to finance the business. Workers cost \$10 per worker and capital costs \$50 per unit.

(a) If Sam wishes to produce the most output with the finances available, how much labor and capital should Sam employ and how much output can he produce? Use a Lagrangian to solve this problem.

Sam will maximize production subject to his cost constraint of \$10,000.

$$\mathcal{L} = 15L^{0.25}K + \lambda(10,000 - wL - rK) = 15L^{0.25}K + \lambda(10,000 - 10L - 50K)$$

First order conditions:

$$\frac{d\mathcal{L}}{dL} = 15 \times 0.25L^{-0.75}K - \lambda 10 = 0 \rightarrow 15 \times 0.25L^{-0.75}K = \lambda 10$$
$$\frac{d\mathcal{L}}{dK} = 15L^{0.25} - \lambda 50 = 0 \rightarrow 15L^{0.25} = \lambda 50$$
$$\frac{d\mathcal{L}}{d\lambda} = 10,000 - 10L - 50K = 0 \rightarrow 10,000 = 10L + 50K$$

Divide the first and second equations:

$$\frac{15 \times 0.25L^{-0.75}K}{15L^{0.25}} = \frac{\lambda 10}{\lambda 50}$$
$$\frac{0.25K}{L} = \frac{1}{5}$$
$$K = \frac{L}{5 \times 0.25} = \frac{4L}{5}$$

Replace this result in the third first order condition:

$$10,000 = 10L + 50K = 10L + 50\frac{4L}{5} = 50L \rightarrow L = 10,000/50 = 200$$

Replace this result back into the solution for K

$$K = \frac{4L}{5} = \frac{4 \times 200}{5} = 160$$

With $L = 200$ and $K = 160$, production is $q = 15L^{0.25}K = 15 \times 200^{0.25}160 = 9,025.4$

(b) Does this bundle of capital and labor also minimize the costs?

Yes, the first two first order conditions yield the MRTS = w/r condition, which is the same as for the cost-minimization problem. If a firm is producing the maximum output for a given amount of costs, then it must also be producing that amount of output at the lowest cost (duality).