Mat 1330 Fall 2014 Assignment 7

Practice Integrals

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a) $$\int (17x-17)^{17}\,dx \underset{\substack{\uparrow\\ y=17x-17\\ dy=17\,dx}}{=} \int y^{17}\,\frac{dy}{17} = \frac{1}{17}\,\frac{1}{18}\,y^{18}+c = \frac{(17x-17)^{18}}{17\cdot 18}+c$$

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b) $$\int 5^{2x-3}\,dx = \int e^{(2x-3)\cdot\ln 5} \underset{\substack{\uparrow\\ y=\ln 5\,(2x+3)\\ dy=2\ln 5\,dx}}{=} \int e^{y}\,\frac{dx}{2\ln 5} = \frac{1}{2\ln 5}\,e^{y}+c$$

$$= \frac{1}{2\ln 5}\cdot 5^{2x-3}+c$$

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c) $$\int \sqrt[3]{12x+2014}\,dx \underset{\substack{\uparrow\\ y=12x+2014\\ dy=12dx}}{=} \int y^{1/3}\,\frac{dx}{12} = \frac{1}{12}\cdot\frac{3}{4}\,y^{4/3}+c$$

$$= \frac{1}{16}\sqrt[3]{(12x+2014)^4}+c$$

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d) $$\int \frac{t}{1+t}\,dt \underset{\substack{y=1+t\\ dy=dt}}{=} \int \frac{t+1-1}{y}\,dy = \int \frac{y-1}{y}\,dy = \int 1-\frac{1}{y}\,dy$$

$$= y-\ln|y|+c = 1+t-\ln|1+t|+c$$

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e) $$\int \frac{1}{x\,(\ln x)^2}\,dx \underset{\substack{\uparrow\\ y=\ln x\\ dy=\frac{1}{x}dx}}{=} \int y^{-2}\,dy = -\frac{1}{y}+c = -\frac{1}{\ln x}+c$$

f) $$\int e^{3x}\sqrt{2-e^{3x}}\,dx = -\frac{1}{3}\int \sqrt{y}\,dy = -\frac{1}{3}\,\frac{2}{3}\,y^{3/2} + c$$

$$y = 2 - e^{3x}$$
$$dy = -3e^{-3x}\,dx$$

$$= -\frac{2}{9}\sqrt{2-e^{3x}} + c$$

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g) $$\int \frac{5}{1+9x^2}\,dx = 5\int \frac{1}{1+y^2}\,\frac{dy}{3} = \frac{5}{3}\arctan(y) + c$$

$$y = 3x$$
$$dy = 3\,dx$$

$$= \frac{5}{3}\arctan(3x) + c$$

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h) $$\int \frac{1}{\sqrt{1-16x^2}}\,dx = \int \frac{1}{\sqrt{1-y^2}}\,\frac{dy}{4} = \frac{1}{4}\arcsin(y) + c$$

$$y = 4x$$

$$= \frac{1}{4}\arcsin(4x) + c$$

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i) $$\int \frac{x}{2}\cos(5x)\,dx = \frac{x}{10}\sin(5x) - \frac{1}{10}\int \sin(5x)\,dx = \frac{x}{10}\sin(5x) + \frac{1}{50}\cos(5x) + c$$

$$u(x) = \frac{x}{2},\quad v'(x) = \cos(5x)$$
$$u'(x) = \frac{1}{2},\quad v(x) = \frac{1}{5}\sin(5x)$$

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j) $$\int \sqrt{x}\,\ln(x)\,dx = \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{1/2}\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + c$$

$$u(x) = \ln x \qquad v'(x) = \sqrt{x}$$
$$u'(x) = \frac{1}{x} \qquad v(x) = \frac{2}{3}x^{3/2}$$

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k) $$\int \arcsin(x)\,dx = x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx = x\arcsin x - \frac{1}{2}\int \frac{dy}{\sqrt{y}}$$

$$u = \arcsin(x) \qquad v' = 1$$
$$u' = \frac{1}{\sqrt{1-x^2}} \qquad v = x$$

$$y = 1 - x^2$$
$$dy = 2x\,dx$$

$$= x\arcsin x - \sqrt{1-x^2} + c$$

l) $\int x\cdot 3^x\,dx = \int x\, e^{x\ln 3}\,dx =$

$u(x)=x \quad v'(x)=e^{x\ln 3}$

$u'(x)=1 \quad v(x)=e^{x\ln 3}/\ln 3$

$$= \frac{x}{\ln 3} e^{x\ln 3} - \int \frac{e^{x\ln 3}}{\ln 3}\,dx = \frac{x}{\ln 3} 3^x - \frac{1}{(\ln 3)^2} 3^x + c$$

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m) $\int x^2 e^{-x}\,dx = -x^2 e^{-x} + \int 2x e^{-x}\,dx =$

$u(x)=x^2 \quad v'(x)=e^{-x}$ $\qquad$ $u(x)=2x \quad v'(x)=e^{-x}$

$u'(x)=2x \quad v(x)=-e^{-x}$ $\qquad$ $u'(x)=2 \quad v(x)=-e^{-x}$

$$= -x^2 e^{-x} - 2xe^{-x} + 2\int e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c$$

Q2:

a) $F(t) = \int 3t^2+1\,dt = t^3+t+c$

$F(0)=1 \Rightarrow c=1.\quad F(1) = 1^3+1+1 = 3$

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b) $F(t) = \int \frac{1}{17t+12}\,dt = \frac{1}{17}\int y^{-1}\,dy = \frac{1}{17}\ln|y| + c = \frac{\ln|17t+12|}{17} + c$

$y = 17t+12,\ dy = 17\,dt$

$F(0)=1 \Rightarrow c = 1 - \frac{\ln 12}{17} \qquad F(1) = \frac{1}{17}\left(\ln(29) - \ln(12)\right) + 1$

Q2: c) $F(t) = \int 12e^{2t}dt = 6e^{2t} + c$

$F(0) = 1 \Rightarrow c = -5$. $\quad F(1) = 6e^2 - 5$

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d) $F(t) = \int (t+t^2)e^{-t}$

1) $\int te^{-t} = -te^{-t} - e^{-t} + c$

2) $\int t^2e^{-t} = (-t^2 - 2t - 2)e^{-t} + c$

$\Big\}$ see Q1, m)

so $F(t) = -e^{-t}(t^2 + 2t + 2 + t + 1) + c$

$= -e^{-t}(t^2 + 3t + 3) + c$

$F(0) = 1 \Rightarrow -3 + c = 1 \Rightarrow c = 4$

$F(1) = -\frac{7}{e} + 4$

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e) $F(t) = \int 3t\cos(t^2)dt = \frac{3}{2}\int \cos(y)\, dy = \frac{3}{2}\sin(y) + c$

$y = t^2$

$dy = 2t\,dt$

$= \frac{3}{2}\sin(t^2) + c$

$F(0) = 1 \Rightarrow c = 1$ so $F(1) = \frac{3}{2}\sin(1) + c$