MAT 1330, Fall 2014 Assignment 2
Due Thursday October 2 by 9:00pm.
Late assignments will not be accepted; nor will unstapled assignments. Professors in the math department will not lend you a stapler; do not ask for one.

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Student Name ____________________ Student Number ____________________
By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ____________________

QUESTION 1. Does the limit $\lim_{x\to 4} \frac{x-\sqrt{3x+4}}{4-x}$ exist? If so, what is its value?

Answer: $\frac{-5}{8}$

Justify your answer without using sequences of numerical values of x.

$$\lim_{x\to 4} \frac{x-\sqrt{3x+4}}{4-x} = \frac{4-\sqrt{16}}{4-4} = \frac{0}{0} \rightsquigarrow \text{undefined limit}$$

$$\lim_{x\to 4} \frac{x-\sqrt{3x+4}}{4-x} = \lim_{x\to 4} \frac{x-\sqrt{3x+4}}{4-x} \times \frac{x+\sqrt{3x+4}}{4+\sqrt{3x+4}}$$

$$= \lim_{x\to 4} \frac{x^2-(3x+4)}{(4-x)(4+\sqrt{3x+4})}$$

$$= \lim_{x\to 4} \frac{x^2-3x-4}{-(x-4)(4+\sqrt{3x+4})}$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(x+1)}{-\cancel{(x-4)}(4+\sqrt{3x+4})} = \lim_{x\to 4} \frac{x+1}{-(4+\sqrt{3x+4})} = \boxed{\frac{-5}{8}}$$

QUESTION 2. Let $G(x) = \frac{x^2-5x+6}{|3-x|} + |x-2|$

a) Find $\lim_{x\to 3^+} G(x)$.

$$\lim_{x\to 3^+} G(x) = \lim_{x\to 3^+} \frac{(x-2)\cancel{(x-3)}}{\cancel{x-3}} + |x-2|$$
$$= \lim_{x\to 3^+} (x-2) + |x-2|$$
$$= (3-2) + |3-2| = \boxed{2}$$

b) Find $\lim_{x\to 3^-} G(x)$.

$$\lim_{x\to 3^-} G(x) = \lim_{x\to 3^-} \frac{(x-2)(x-3)}{-(x-3)} + |x-2|$$
$$= \lim_{x\to 3^-} \frac{x-2}{-1} + |x-2|$$
$$= -1 + 1 = \boxed{0}$$

c) Does $\lim_{x\to 3} G(x)$ exists? Answer: NO.

Justify your answer.

Since right-hand limit and left-hand limit are not equal, then the limit doesn't exist.

$$\lim_{x\to 3^+} G(x) \neq \lim_{x\to 3^-} G(x)$$

$$|3-x| = \begin{cases} 3-x & \text{if } 3-x \geq 0 \Rightarrow (x \leq 3) \\ -(3-x) & \text{if } 3-x < 0 \Rightarrow (x > 3) \end{cases}$$

QUESTION 3. Use sequences of numbers to guess the limit as $x \to a$ of the following function or argue that the limit does not exist. Describe the behavior of the function near a as best as you can, using the terminology of (one-sided) limits from class. Show all your work.

a) $f(x) = \frac{\cos(\pi x/2)}{\ln(x)}$, $a = 1$.

b) $f(x) = \tan(x - \pi/2)$, $a = 0$.

(a)

Right-hand limit

$x>1$	1.5	1.3	1.1	1.01	1.0001	1.00001
$f(x)$	-1.7439	-1.7304	-1.6415	-1.5786	-1.5709	-1.5708

$\lim_{x \to 1^+} f(x) \approx -1.571$

Left-hand limit

$x<1$	0.5	0.7	0.9	0.99	0.9999	0.99999
$f(x)$	-1.0201	-1.2728	-1.4847	-1.5629	-1.5700	-1.5708

$\lim_{x \to 1^-} f(x) \approx -1.571$

Since $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$ then $\lim_{x \to 1} f(x)$ exists.

QUESTION 4. For a real number b, consider the function

$$f(x) = \begin{cases} \sin(x - b), & x > 0 \\ x^2 + 1, & x < 0. \end{cases}$$

Find the smallest possible positive value of b such that the limit $\lim_{x \to 0} f(x)$ exists. Show all your work.

- Right-hand limit: $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \sin(x - b)$
$= \sin(0 - b) = \sin(-b)$
$= -\sin(b)$

- Left-hand limit: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x^2 + 1)$
$= 0^2 + 1 = 1$

Then the limit $\lim_{x \to 0} f(x)$ exists if $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$-\sin b = 1 \Rightarrow$

$\sin b = -1 \Rightarrow b = \frac{3\pi}{2}$

Note: $b = 2k\pi - \frac{\pi}{2}$ $k \in \{1, 2, 3, \ldots\}$, Since we're looking for the smallest positive b, then $b = \frac{3\pi}{2}$.

(b) $f(x) = \tan(x - \frac{\pi}{2})$ $\quad a = 0$

$$f(x) = \frac{\sin(x - \frac{\pi}{2})}{\cos(x - \frac{\pi}{2})} \qquad a = 0$$

Right-hand limit:

$x > 0$	0.5	0.3	0.1	0.01	0.0001	0.00001
$f(x)$	-1.8305	-3.2327	-9.9667	-99.997	-10^4	-10^5

$$\lim_{x \to 0^+} f(x) = -\infty$$

Left-hand limit:

$x < 0$	-0.5	-0.3	-0.1	-0.01	-0.0001	-0.00001
$f(x)$	1.8305	3.2327	9.9667	99.997	10^4	10^5

$$\lim_{x \to 0^-} f(x) = \infty$$

Since $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$, then the limit

$\lim_{x \to 0} f(x)$ doesn't exist!